Henry Scott

LOGARITHMICK ARITHMETICK

CONTAINING A NEW AND CORRECT

TABLE OF LOGARITHMS

OF THE NATURAL NUMBERS FROM 1 TO 10.000,

EXTENDED TO SEVEN PLACES BESIDES THE INDEX; AND SO CONTRIVED, THAT THE LOGARITHM MAY BE EASILY FOUND TO ANY NUMBER BETWEEN 1 AND 10,000,000.

ALSO,

AN EASY METHOD

OF CONSTRUCTING A TABLE OF LOGARITHMS, TOGETHER WITH THEIR NUMEROUS AND IMPORTANT USES IN THE MORE DIFFICULT PARTS OF

ARITHMETICK.

TO WHICH ARE ADDED A NUMBER OF

ASTRONOMICAL TABLES,

BY WHICH THE DIFFERENT PHASES OF THE MOON,—THE TIMES OF HER OPPOSITION AND CONJUNCTION, MAY BE COMPUTED WITH THE GREATEST EASE AND EXACTNESS: AND AN EASY METHOD OF CALCULATING

SOLAR AND LUNAR ECLIPSES;

ILLUSTRATED WITH

GEOMETRICAL PROJECTIONS:

DESIGNED FOR THE INSTRUCTION OF YOUTH IN THE

SCHOOLS AND ACADEMIES

OF NEW ENGLAND,

BY ELIJAH HINSDALE BURRITT.

WILLIAMSBURGH:
PRINTED BY EPHRAIM WHITMAN.
1818.

DISTRICT OF MASSACHUSETTS, to wit :

District Clerk's Office.

Be it remembered, That on the fifteenth day of September, A. D. 1818, and in the forty-third year of the Independence of the United States of America, Elijah Hinsdale Burritt, of the said district, has deposited in this office the title of a book, the right whereof he claims as author, in the words following, to wit :

Logarithmick Arithmetick, containing a new and correct Table of Logarithms of the natural numbers from 1 to 10,000, extended to seven places besides the index ; and so contrived, that the Logarithm may be easily found to any number between 1 and 10,000,000. Also, an easy method of constructing a Table of Logarithms, together with their numerous and important uses in the more difficult parts of Arithmetick. To which are added a number of Astronomical Tables, by which the different phases of the Moon, the times of her opposition and conjunction, may be computed with the greatest ease and exactness : and an easy method of calculating Solar and Lunar Eclipses ; illustrated with Geometrical Projections : Designed for the instruction of Youth in the Schools and Academies of New England. By Elijah Hinsdale Burritt.

In conformity to the act of the congress of the United States, entitled, " An act for the encouragement of learning, by securing the copies of maps, charts and books, to the authors and proprietors of such copies, during the times therein mentioned :" and also to an act entitled, " An act supplementary to an act, entitled, an act for the encouragement of learning, by securing the copies of maps, charts and books, to the authors and proprietors of such copies during the times therein mentioned ; and extending the benefits thereof to the arts of designing, engraving and etching historical, and other prints." JOHN W. DAVIS,

Clerk of the District of Massachusetts.

PREFACE.

"THE task of an author;" says Dr. Johnson, is to teach what is not known, or to recommend known truths by his manner of adorning them." To attempt the former is sufficiently irksome to enervate endeavour, and to succeed in the latter, he must surmount obstcles that no sagacity can avoid, and encounter difficulties which prevision cannot alleviate. He must appeal to judges prepossessed by passions or biased by prejudices: to some who are more willing to go wrong by their own judgment, than to be indebted for a better or a safer way to the wisdom of another; and to others, whose good taste and perspicacity will not allow them to read any thing until its reputation is eetablished by the "Ministers of Criticism." But of those who are more inclined to be pleased, who may be persuaded to remit their censure at those errours into which the author has inadvertently or unavoidably fallen, and who will pass with a friendly eye over those imperfections that are inseperably connected with all human productions; it is hoped, though the reader should not find his feelings imperceptably stolen from him by the enchantment of novelty, or his understanding captivated by the profundity of invention, that if his patience can endure a careful perusal of the following work, he will dismiss it with, at least this "negative encomium," that it is a laudable attempt to improve the instruction of the unlearned, and furnish schools in general with a useful and comprehensive system of Practical Arithmetick.

He who is resolved to attain any degree of eminence in the science of Mathematicks, dooms himself to wade through a toilsome course of severe, uninteresting labour, unaccompanied with any of those charms that can engage the feelings by their novelty, or delight the imagination by their lusture; without any hope of honorable reward to stimulate to exertion, or soften the asperities of study. But the writer of Common Arithmetick has a much easier task: The labours of many that have gone before him as "pioneers," who were distinguised by profound investigation and mathematical researches, have rendered his path comparatively easy, even where their feet have never trodden; "as the sun dissipates the shades of darkness, and spreads a twilight beyond the immediate influence of his direct beams." Indeed, it may be said that the only necessary resourse of the writers of the present age, is to copy the best examples of their predecessors with such modifications and amendments

as the differnnt modes of reckoning in business, the fluctuation of coins, weights, and measures, and as the invention of means to facilitate the same solutions require.

The practical systems of Vulgar Arithmetick already extant are very numerous; and the authors of some of them have exhibited great knowledge and ability. it may therefore be expected, that he who ventures to add to their number should produce a substantial plea for such an obtrusion on the publick notice. It may justly be expected that he should be either an accurate schollar or a profound mathematician. The writer has no claim either to the one or the other of these distinctions; and must therefore plead another apology.

Having been repeatedly solicited by a number of gentlemen respectable for their understanding and skill in Mathematical Science, to publish his TABLE of LOGARITHMS constructed only for private use, together with their numereus and important uses in the more difficult parts of Arithmetick Trigonometry &c. and believing, notwithstanding the endless variety of books now extant on Vulgar Arithmetick, there is yet room to introduce usefulness with novelty, the author of the following work humbly ventures to assume the responsibility of endeavouring to avoid the redundencies of some, and to supply the deficiencies of others.

It is believed that a more accurate and extensive SYSTEM of LOGARITHMS for natural numbers was never before published in this country. The best American Tables are carried only to six places besides the index, which, though capable of giving answers sufficiently exact in most solutions, are, notwithstanding, deficient where great accuracy is required. In constructing the following table the author has pursued the "*Differential Method*" hinted at by Mr. Henry Briggs, Professor of Geometry in Gresham College London, and Dr. Hutton's "*Practical Rule for the Construction of Logarithms.*"

He was also solicited to publich with this, a Table of Logarithmick Signs, Tangents, &c. carried to seven places decimal, which should correspond with those for natural numbers; but as this would render the work too voluminous for a common school-book, it was thought unadvisable to augment the expense without proportionably increaseing the advantage. Should their occur sufficient reason for publishing such a system of Logarithms and Logarithmick Sines and Tangents, together with other Logarithmick and Mathematical Tables, they may be given to the publick at some future period.

THE METHOD OF COMPUTING BY LOGARITHMS, where it can be adopted, aa in the evolving of roots, is manifestly the most expiditious of any that human powers have hitherto invented. The learner, who before was unacquainted with their proper-

ties, who has had the patience and assiduity to toil through the tedious course of evolution by the usual process, will admire to find that so much labour should ever be reduced to so much ease!

The author has endeavoured, though in some instances at the expense of deviating from ancient usage, to arrange the several parts of Vulgar Arithmetick according to their relative importance, and their mutual dependence upon each other; and to render the whole as easy and familliar as the nature of the subjects would admit.

From what experience the author has had in the instruction of youth, and the general complaint of the want of some work adapted to accompany those elementary treatises on Astronomy which are used in our Schools and Academies, the writer was induced to subjoin a short, though imperfect sketch of Practical Astronomy. And in this, he has been more desirous to be useful than to appear original. All the Tables except the II, XVIII and XIX, together with the method of calculating the time of New and Full Moons and Eclipses, were taken from Dr. Brewster's edition of Fugeson's Astronomy lately published. The method of Projecting Eclipses is purely mathematica, and is not subject to those inacuracies which the use of the sector is liable to produce.

Whatever merit justice may award to the following prime tial pages, the author cannot forbear repeating, that it was the hope only of being useful, that induced their submission to publick scrutiny. If he has failed in the attempt, the disquietude of disappointment will be allayed by the reflection, that he is not the first "*qui magnis excidet ausis*" who have misjudged in their abilities. To suppoae the work wholly exempt from *errours*, whether of the press or of the pen, would be to suppose what the most vigilant care has seldom performed. And whether these will be found to have a counterbalance of good, is refered to the candour of those who may read to decide;—they are the constituted judges, and to them he submits with defference and respect.

E. H. B.

Williams College, October, 1818.

CONTENTS.

Explanation of Characters used in this book.

= Equal to, as 12d. = 1s. signifies that 12 pence are equal to 1 shilling.

+ More, the sign of addition, as 5 + 7=12, signifies that 5 and 7 added together, are equal to 12.

— Minus, or less, the sign of subtraction, as 6 — 2 = 4, signifies that 2 subtracted from 6, leaves 4.

× Multiply, or with the sign of Multiplication ; as 4 × 3=12 signifies that 4 multiplied by 3, is equal to 12.

÷ The sign of division ; as 8 ÷ 2 = 4, signifies that 8 divided by 2, is equal to 4 ; or thus, $\frac{8}{2}$=4, each of which signifies the same thing.

:: Four points set in the middle of four numbers, denote them to be proportional to one another, by the rule of three ; as 2 : 4 :: 8 : 16 ; that is, as 2 is to 4, so is 8 to 16.

$\sqrt{}$ Prefixed to any number, supposes that the squre root of that number is required.

$\sqrt[3]{}$ Prefixed to any number, supposes the cube root of that number is required.

$\sqrt[4]{}$ Denotes the biquadrate root, or fourth power, &c.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

To learn this table : Find your multiplier in the left hand column, and the multiplicand a-top, and in the common angle of meeting, or against your multiplier, along at the right hand, and under your multiplicand, you will find the product or answer.

1. *Sterling Money.*

4 farthings make	1 penny,	*d.*
12 pence,	1 shilling,	*s.*
20 shillings,	1 pound,	£

2. *Troy Weight.*

24 grains *(gr.)* make	1 penny-weight, marked	*pwt.*
20 penny-weights,	1 ounce,	*oz.*
12 ounces,	1 pound,	*lb.*

3. *Avoirdupois Weight.*

16 drams *(dr.)* make	1 ounce,	*oz.*
16 ounces,	1 pound,	*lb.*
28 pounds,	1 quarter of a hundred weight,	*qr.*
4 quarters,	1 hundred weight,	*cwt.*
20 hundred weight,	1 ton,	*T.*

By this weight are weighed all coarse and drossy goods, grocery wares, and all metals except gold and silver.

4. *Cloth Measure.*

4 nails *(na.)* make	1 quarter of a yard,	*qr.*
4 quarters,	1 yard,	*yd.*
3 quarters,	1 Ell Flemish,	*E. Fl.*
5 quarers,	1 Ell English,	*E. E.*
6 quarters,	1 Ell French,	*E. Fr.*

5. *Dry Measure.*

2 pints *(pt.)* make	1 quart,	*qt.*
8 quarts,	1 peck,	*pk.*
4 pecks,	1 bushel,	*bu.*

This measure is applied to grain, beans, flak-seed, salt, roots oysters, coal, &c

6 *Wine Measure.*

4 gills *(gi.)* make	1 pint,	*pt.*
2 pints,	1 quart,	*qt.*
4 quarts,	1 gallon,	*gal.*
31½ gallons,	1 barrel,	*bl.*
42 gallons,	1 tierce,	*tier.*
63 gallons,	1 hogshead	*hhd.*
2 hogsheads,	1 pipe,	*pi.*
3 pipes,	1 tun,	*T.*

All brandies, spirits, mead, vinegar, oil &c. are measured by wine measure. *Note.*— 231 solid inches, make a gallon.

7 *Long Measure.*

3 barley corns *(b. c.)* make	1 inch, marked	*in.*
12 inches,	1 foot,	*ft.*
3 feet,	1 yard,	*y.*
5½ yards,	1 rod, pole, or perch,	*rd.*
40 rods,	1 furlong,	*fur.*
8 furlongs,	1 mile,	*m.*
3 miles,	1 league,	*lea.*
69⅓ statute miles,	1 degree on the earth.	
360 degrees, the circumference of the earth.		

The use of long measure is to measure the distance of places, or any other thing, where length is considered, without regard to breadth.

N. B. In measuring the height of horses, 4 inches make 1 hand. In measuring depths, 6 feet make 1 fathom, or French toise. Distances are measured by a chain, four rods long, containing one hundred links.

8. *Land or Square Measure.*

144	square inches make	1 square foot.
9	square feet,	1 square yard.
30¼	square yards, or	1 square rod.
272¼	square feet,	
40	square rods,	1 square rood.
4	square roods,	1 square acre
640	square acres,	1 square mile.

9 *Solid or Cubic Measure.*

1728	solid inches make	1 solid foot.
40	feet of round timber, or	1 ton or load.
50	feet of hewn timber,	
128	solid feet, or 8 feet long,	1 cord of wood.
	4 wide, and 4 high,	

All solids, or things that have length, breadth and depth, are measured by this measure. N. B. The wine gallon contains 231 solid or cubic inches, and the beer gallon, 282. A bushel contains 2150,42 solid inches.

10. *Time.*

60	seconds (*S.*) make	1 minute, marked	*S. M.*
60	minutes,	1 hour,	*h.*
24	hours,	1 day,	*d.*
7	days,	1 week,	*w.*
4	weeks,	1 month,	*mo.*
13	months, 1 day, and 6 hours,	1 Julian year	*yr.*

Thirty days hath September, April, June, and November, February twenty-eight alone, all the rest have thirty-one.

N. B. In bissextile or leap-year, February hath 29 days.

11 *Circular motion.*

60	second's (″) make	1 minute, marked	′
60	minutes,	1 degree,	°
30	degrees	1 sign,	*S.*

12 signs, or 360 degrees, the whole great circle of the Zodiac.

ARITHMETICK.

IS THE Science of Numbers, and exhibits the method, or art of computing by them : it is divided into five parts, viz. Notation, Addition, Subtraction, Multiplication, and Division.

NOTATION.

NOTATION teaches how to read and write numbers represented by the following characters, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 ; each of which has a simple value, and also a local value, according to the order of their combination, as in the following table.

Hundreds of Thousands of Trillions.	Tens of Thousands of Trillions.	Thousands of Trillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Thousands of Billions.	Tens of Thousands of Billions.	Thousands of Billions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Thousands of Millions.	Tens of Thousands of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
9	7	8	9	6	4	3	2	4	0	2	7	9	0	4	9	7	1	5	2	6	3	8	5

To know the value of any number of figures.

RULE.

Numerate from the right hand figure to the left as in the table, and to the simple value of each figure apply its local value, according as it is removed from the place of units towards the right.

EXAMPLES.

Read the following numbers.

75 Seventy five.
937 Nine hundred and thirty seven.
4617 Four thousand six hundred and seventeen.
59000028 Fifty nine millions and twenty eight.
679244321 Six hundred and seventy nine millions two hundred and forty four thousand three hundred and twenty one.

To write numbers.

RULE.

Beginning on the right hand, write unites in their place, tens in the tens place, and so on towards the left hand, writing each figure according to its value in numeration, and supplying those places of the natural order with cyphers which are omitted in the question.

EXAMPLES.

Write down the following numbers.

Twenty four	24
Three hundred and five	305
Seven thousand and ninety	7090
Eight millions and eight	8000008

SIMPLE ADDITION.

Is the uniting into one number several smaller ones of the same denomination.

RULE.

Write the numbers distinctly, units under units, tens under tens, &c. Then reckon the amount of the right hand column. If it be less than ten, write it down. If it exceed ten, write the units only and carry the tens to the next place.

In like manner carry the tens of each column to the next, observing to write down the full sum of the left hand column.

METHOD OF PROOF.

1. Draw a line between the first and second lines of figures to cut off the first number.

2. Add the other numbers together and set their sum under the sum of all the numbers.

3. Add the numbers last found with the numbers cut off, and if their sum be the same as that of the first addition, the work is right.

EXAMPLES.

	(1)	(2)	(3)	
	49643	32014	43214	
	64532	13100	1960	
	98760	19264	420	
	14000	83400	40	
Sum	226935	147778	45634	Sum
	177292	115764	2420	
Proof	226935	147778	45634	Proof

4. Add 8655, 2194, 7421, 5063, 2194, and 1245. *Ans.* 26754.

5. How many days in the 12 calender months? *Ans.* 365.

6. From the building of Rome to the death of Alexander, was 683 years, from the death of Alexander to the Christian era, was 65 years; how many years from the building of Rome to the present date, 1818? *Ans.* 2566.

At the late census of 1810, the inhabitants of New England were as follows.

Of District of Maine 228,705, New Hampshire 214,460, Vermont 217,896, Massachusetts 472,040, Rhode-Island 76,931, and of Connecticut 261,942, what was the sum total?

Ans. 1,471,974.

FEDERAL MONEY.

By an act of Congress, all the accounts of the United States, the Sallaries of Officers, all Revenues, &c. are to be reckoned in Federal Money, which mode of reckoning in point of simplicity is the nearest allied to whole numbers, as it increases like them in ten fold proportion.

ADDITION OF FEDERAL MONEY.

RULE.

Write the numbers according to their value, that is, dollars under dollars, dimes under dimes, cents under cents, mills under mills, &c. then proceed the same as in whole numbers, observing to place the seperatrix in the sum total directly under the seperating points above.

EXAMPLES.

(1)				(2)				(3)			
$	D.	C.	M.	$	D.	C.	M.	$	D.	C.	M.
365	5	4	1	439	3	0	4	136	5	1	4
487	0	6	0	416	3	9	0	125	0	9	0
694	6	7	0	168	9	3	4	200	9	0	9
439	0	8	9	239	0	6	0	304	0	0	6
742	5	0	0	143	0	0	5	111	1	9	1
Ans. 2728	8	6	0	1406	6	9	3	877	7	1	0 *Ans.*

4. What is the sum total of $ 367,14, $ 117,09, $ 37,75, $ 11,05, and $ 96,50? *Ans.* $ 629,53.

* 5. What is the amount of $ 102,12½ cts. $ 9,67, 45 cts. 67 cts. and $ 1,08 cts.? *Ans.* $ 113,99,5.

6. Received of A. B. and C. a sum of money; A. paid $ 123,08 cts. B. paid four times as much as A. and C. paid as much as A. together with twice as much B. Required the sum of payments? *Ans.* $ 1723,12 cts.

SIMPLE SUBTRACTION.

Is the operation by which we take a lesser number from a greater of the same denomination, and thereby find their difference, or remainder.

The lesser number is called the SUBTRAHEND, the greater number the MINUEND, and the number found by the operation the REMAINDER, or DIFFERENCE.

RULE.

1. Place the less number under the greater, so that units may stand under units, tens under tens, &c. and draw a line under them.

2. Beginning at the right, take each figure in the subtrahend from the figure over it, and set the remainder under the line.

3. If the lower figure be greater than the one over it, add ten to the upper figure, from which figure so increased, take the lower, and write the remainder, carrying one to the next figure in the lower line, and thus proceed till the whole is finished.

PROOF.

Add the remainder to the less number, and if the sum be equal to the greater, the work is right.

* If the cents are less than 10 place a cypher in the tens place, or place of dimes; example, write 10 dols. and 3 cents, $ 10,03.

EXAMPLES.

(1)		(2)	
From	3 2 8 7 6 2 5	From	5 3 2 7 4 6 7
Take	2 3 4 3 7 5 6	Take	1 0 0 8 4 3 8
Rem.	9 4 3 8 6 9	Rem.	4 3 1 9 0 2 9
Proof	3 2 8 7 6 2 5	Proof	5 3 2 7 4 6 7

3. From 2637804, take 2376982. *Ans.* 260822.
4. From 3762162, take 826541. *Ans.* 2935621.
5. From 78213606, take 27821890. *Ans.* 50391716.
6. From ninety seven thousand six hundred and thirteen, subtract thirty thousand nine hundred and nine. *Ans.* 66704.
7. From the destruction of Carthage to the year of our Lord 1818, was 1965 years, how long before the Christian era was the city destroyed? *Ans.* 147 years.
8. Gun powder was invented by a Monk of Cologne, in 1330, how long has it been in use to this date 1818? *Ans.* 488 years.
9. The Arabian method of Notation was first known in England in the year 1150; how long thence to 1818? *Ans.* 668 years.

SUBTRACTION OF FEDERAL MONEY.

RULE.

Place the numbers according to their value, and subtract as in whole numbers.

EXAMPLES.

	(1) $	D.	C.	M.		(2) $	D.	C.	M.
From	48	9	6	4	From	940	6	4	9
Take	37	8	9	3	Take	804	9	6	3
Ans.	11	0	7	1	*Ans.*	135	6	8	6

3. From 125 dols. take 9 dols. and 9 cts. *Ans.* $ 115 91 cts.
4. From 127 dols. 1 cent, take $ 41 10 cts. *Ans.* $ 85 91.
5. From 365 dolls. 12 cts. take $ 187 35 cts. *Ans.* $ 177 77.
6. From 100 dolls. take 99 cts. *Ans.* $ 99 01 cent.
7. A. owes B. 1000 dols. and pays him in part as follows, viz: at one time $ 236 10 cts. at another $ 108 25 cts. and lets him have fifty bushels of wheat at 2 dollars pr. bushel, together with a horse worth 85 dols. and a plated harness valued at $ 60 75 cts.; it is required to find how much is yet due?

Ans. $ 409 90 cts.

SIMPLE MULTIPLICATION.

Is the operation by which we increase, or repeat one of two numbers of the same denomination, as often as there are units in the other.

The number to be multiplied, is called the MULTIPLICAND.
The number by which we multiply, is called the MULTIPIER.
The number found by the operation, is called the PRODUCT.
The Multiplicand and Multiplier, are both called FACTORS.

RULE.

1 Place the Multiplier under the Multiplicand, so that units may stand under units, tens under tens, &c. and draw a line under them.

2. Begin at the right, multiply each figure in the Multiplicand by the Multiplier, carry one for every ten, and you will have the product.

PROOF.

Multiply the Multiplier by the Multiplicand.

EXAMPLES.

(1)		(2)	
Multiply	436 Multiplicand.	Multiply	90036
by	5 Multiplier.	by	7
2180	Product.	*Ans.*	630252

(3)		(4)	
Multiply	34293	Multiply	32745654478
by	74	by	234
	137172		130982617892
	240051		98236963419
			65491308946
Ans.	2537682	Product	7662483146682

5. Multiply 364111 by 56. *Ans.* 20390216.
6. Multiply 7128368 by 96. *Ans.* 684323328.
7. Multiply 123456789 by 1440. *Ans.* 177777776160.

CONTRACTIONS.

When there are cyphers on the right of one, or both the factors.

RULE.

Proceed as before. Multiply by the significant figures, neglecting the cyphers, and on the right of the product place as many cyphers as were neglected in both the factors.

EXAMPLES.

1. Multiply 1234500 by 7500

	61725
	86415
Product.	9258750000

(2) 360000 by 1200000.

	1200000
	432000000000 *Ans.*

3. Multiply 461200 by 72000. *Ans.* 33206400000.
Multiply 815036000 by 70300 *Ans.* 57297030800000.

MULTIPLICATION OF FEDERAL MONEY.

RULE.

Multiply the given price by the quantity, and carry as in whole numbers. The seperatrix will be as many figures from the right hand in the product, as in the given price.

EXAMPLES.

1. What will 36 yds. of broad cloth come to at six dollars twenty five cents pr. yd.?

	$ 6 25	
	36 yards.	
	37 50	
	187 5	
Ans.	$ 225 00	225 dolls. 0 cents.

2. What cost 15 yds. calico at 67 cts. pr. yd.? *Ans.* $ 10 05.
3. What cost 35 yds. kerseymere at $ 1 67 cts. pr. yd.? *Ans.* $ 58 45.
4. What is the value of 75 yds. satin at $ 3 75 cts. pr. yd.? *Ans.* $ 281 25.
5. What cost 131 bushels wheat at $ 1 67 cts. per bush.? *Ans.* $ 218 77.
6. What cost 126 gallons molasses 33 cts. per gal.? *Ans.* $ 40 96.
7. What will 66 bushels oats cost at 38 cts. pr. bush. *Ans.* $ 18 48.
8. What cost 97 lb. sugar at 12½ pr. lb.? *Ans.* $ 12 12½

Mr. William Williams,

Bought of James Merchant.

10 lbs. Green Tea,	at	$ 2	15 cts. pr. lb.
36 " Coffee,	"		21¼ cts. pr. lb.
15 " Loaf Sugar,	"		25 cts. pr. lb.
3 cwt. of Malaga Raisins,	"	7	00 pr. cwt.
36 gallons Wine,	"	2	12 cts. pr. gal.
91 " Molasses,	"		36 cts. pr. gal.

$ 136, 58

Received payment.

JAMES MERCHANT.

New-York, the 1st May, 1818.

SIMPLE DIVISION.

Teaches to find how often one number is contained in another of the same denomination; or to find a quotient which multiplied into the divisor will produce the dividend.

The number to be divived, is called the DIVIDEND.

The number to divide by, is called the DIVISOR.

The number of times the dividend contains the divisor, is called the QUOTIENT.

RULE.

1. Assume as many figures on the left hand of the dividend as contain the divisor once or oftener; find how many times they contain it, and place the answer on the right for the first figure of the quotient.

2. Multiply the divisor by the figure you have found, and place the product under that part of the dividend from which it was obtained.

3. Subtract the product from the figures above it; then bring down the next figure of the dividend and place it at the right hand of the remainder; divide the number it makes up as before, and proceed in this manner until the whole is finished.

EXAMPLES.

How many times are 5 contained in 137906 ?

PROOF.

Multiply the divisor into the quotient, add the remainder if there be any, to the product ; if the work is right the sum will be equal to the dividend.

Dividend,	Quotient.
Divisor 5)137906	(27581
10	5
—	—
37	proof. 137906
35	
—	
29	
25	
—	
40	
40	
—	
6	
5	
—	
1	remainder.

Divide 33489 by 9.

Divisor,	Dividend.	
9)	33489	(3721 *Ans.*
27		9
—		—
64	33489	proof.
63		
—		
18		
18		
—		
9		
9		

Divide 11680 by 32

32)11680	(365 *Ans.*
96	
—	
208	
192	
—	
160	
160	
—	

* Proof by addition 11680

Divide 1893312 by 912, *Ans.* 2076.

Divide 1893312 by 2076, *Ans.* 912.

How often does 761858465 contain 90001 ? *Ans.* 8465.

Divide 280208122081 by 912314. *Ans.* $307140\frac{1\,2\,3}{9\,1\,2\,3\,1\,4}$

CONTRACTIONS.

To divide when there are cyphers at the right hand of the divisor.

RULE.

Cut off the cyphers from the divisor, and just the same number of digits from the right of the dividend ; then divide the remaining figures as usual, the quotient will be the answer.—

* Add the remainder and all the products of the several quotient figures (multipled by the divisor) and the sum, if the work be right, will be equal to the dividend.

To the remainder (if there be any) annex those figures cut off from the dividend, and you will have the true remainder.

EXAMPLES.

(1)	(2)
Divide 460000 by 1200	Divide 7600 by 40
12(00)4600)00(383 *Ans.*	4(0)760)0(190 *Ans.*
36	4
100	36
96	36
40	0
36	

400 true remainder.

3. Divide 7380964 by 23000. *Ans.* $320\frac{2964}{23000}$
4. Divide 11659112 by 890000. *Ans.* $131\frac{112}{890000}$
5. Divide 9187642 by 9170000. *Ans.* $1\frac{17642}{9170000}$
6. Divide 29628754963 by 35000. *Ans.* $846535\frac{24963}{35000}$

SHORT DIVISION.

RULE.

Find how many times the divisor is contained in the first figure or figures of the dividend, place the result under, and carry as many tens to the next figure as there are ones over.

EXAMPLES.

	Dividend	(2)
1. Divisor	3)764329	4)1134152
Quotient	254776+1 rem.	283538

(3)	(4)
5)649871923	6)1027182341

(5)	(6)
7)25000321792	8)11297653009

(7)	(8)
9)8701256620	10)1097654321

(9) 11)3076259862

(10) 12)175634589

SECONDLY.

When the divisor is a composite number, or the product of two or more numbers in the TABLE.

RULE.

Divide successively by the component parts of the given divisor.

EXAMPLES.

1. Divide 9125 by 25.

5X5=25; the component parts of the given divisor then, are 5 and 5. Thus,

5)9125

5)1825

365 quotient.

2. Divide 178464 by 16. *Ans.* 11154
3. Divide 79638 by 36.* *Ans.* $2212\frac{6}{36}$
4. Divide 957387 by 54. *Ans.* $17359\frac{1}{54}$
5. Divide 93975 by 84. *Ans.* $1118\frac{63}{84}$
6. Divide 145260 by 108. *Ans.* 1345
7. Divide 1575360 by 144. *Ans.* 10940

To Divide by 10, 100, 1000, 10000, &c.

RULE.

Cut off so many figures from the right of the dividend, as there are cyphers in the divisor;—that part cut off from the dividend is the *remainder*, the other figures in the dividend are the *quotient*.

EXAMPLES.

(1)

Divide 600065 by 1000

1(000)600)065(600 quotient, and 65 remainder.

2. Divide 65 by 10. *Ans.* 36 and 5 rem.
3. Divide 5762 by 100. *Ans.* 57 and 62 rem.
4. Divide 90764 by 1000. *Ans.* 90 and 764 rem.
5. Divide 876432 by 10000. *Ans.* 87 and 6432 rem.

* The total remainder is found by multiplying the last remainder by the first divisor, and adding in the first remainder.

SUPPLEMENT TO MULTIPLICATION.

To multiply by a mixed number, that is a whole number and a fraction.

RULE.

Multiply by the whole number, and take $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, &c. of the multiplicand, and add it to the product.

EXAMPLES.

1. Multiply 43 by $12\frac{1}{2}$

$\frac{1}{2})43$

12

$21\frac{1}{2}$

516

Ans. $537\frac{1}{2}$

(2)

Multiply $\frac{1}{2})24$ by $3\frac{3}{4}$

$\frac{1}{2})12 = \frac{1}{2}$

$6 = \frac{1}{4}$

72

Ans. 90

3. Multiply 2464 by $8\frac{1}{3}$ *Ans.* $20533\frac{1}{3}$
4. Multiply 6497 by $5\frac{1}{7}$ *Ans.* $33413\frac{1}{7}$
5. Multiply 12248 by $9\frac{1}{4}$ *Ans.* 1191418
6. Multiply 345 by $19\frac{1}{8}$ *Ans.* $6598\frac{1}{8}$

Practical Questions in Multiplication and Division.

1. In 36 pieces of broad cloth each containing $24\frac{1}{4}$ yds. how many yds. ? *Ans.* 873
2. What is the product of 430 multiplied into itself ? *Ans.* 184900
3. What number multiplied by 9 will make 225 ? *Ans.* 25
4. What cost 9 yds. cloth at $7 pr. yd ? *Ans.* $63
5. If a man spend $600 pr. year, what is that pr. calender month ? *Ans.* $50
6. Sold a ships cargo for $7940, required to find 1-4 of the amount ? *Ans.* $1985
7. The quotient of a certain number is 11940, and the divisor 20, what the dividend ? *Ans.* 238800
8. How many feet are there in a mile, or 320 rods, allowing each rod contains $16\frac{1}{2}$ feet ? *Ans.* 5280
9. How many yards in a mile, if $5\frac{1}{2}$ yds. make one rod ? *Ans.* 1760
10. How many yards of broadcloth, at $7\frac{1}{2}$ per. yd. may be bought for 37 yds. of do. at $3\frac{3}{4}$ per. yrd. ? *Ans.* $18\frac{1}{2}$ yrds.

11. How much wine, at $1\frac{3}{4}$ doll. pr. gal. with molasses, at $\frac{1}{2}$ doll. pr. gal. and of each an equal quantity, must be had in exchange, for 35 gallons of brandy, at $ 2,25, pr. gal. and 10 gallons of old spirits, at $ $1,12\frac{1}{2}$ pr. gal.? *Ans.* 40 gallons.

COMPOUND ADDITION.

Teaches to unite several numbers of different denominations—as pounds, shillings, pence, &c. into one sum.

RULE.*

1. Place numbers of the same denomination under each other.

2. Add the figures in the right hand column, and find how many of the next denomination are contained in the sum, which carry to the next denomination; observing to set down the remainder under the column added, and thus proceed with all the columns excepting the last, where the whole sum is to be written down.

STERLING MONEY.

(1)			(2)				(3)			
£.	s.	d.	£.	s.	d.	qr.	£.	s.	d.	qr.
17	13	4	84	17	5	2	47	13	6	0
13	10	2	75	13	4	1	19	2	9	2
10	17	3	51	17	8	3	14	10	11	1
8	8	7	20	10	10	1	12	9	1	3
3	3	4	17	15	4	2	8	7	6	2
	8	8	10	10	11	0	1	2	7	1
54	1	4	261	5	8	1				
36	8	0	176	8	2	3				
54	1	4	261	5	8	1				

* The reason of this rule will be obvious, if we consider, that 1 in the column of pence, is equal to 4 in the column of farthings, and 1 in that of shillings, to 12 in the column of pence &c.

(4)

£.	s.	d.	qr.
11	9	6	1
19	8	4	3
99	11	10	2
6	0	4	1

(5)

£.	s.	d.	qr.
144	9	12	1
160	19	10	3
140	4	0	2
910	0	4	3

(6)

£.	s.	d.	qr.
987	6	9	1
17	19	11	2
6	4	0	3
89	6	10	2

2. TROY WEIGHT.

lb.	oz.	pwt.	gr.
19	11	19	23
11	10	11	20
6	9	17	10
4	8	4	9
3	7	12	17

lb.	oz.	pwt.	gr.
11	9	6	4
20	7	3	20
10	9	16	17
9	8	14	23
17	4	19	3

lb.	oz.	pwt.	gr.
9	11	19	23
8	10	18	22
7	9	17	21
19	8	16	20
18	7	15	19

3. AVOIRDUPOIS WEIGHT.

cwt.	qr.	lb.
2	3	27
4	2	18
6	1	17
9	0	16
3	3	14

lb.	oz.	dr.
25	13	15
24	10	14
23	11	13
18	9	10
27	7	8

T.	cwt.	qr.	lb.	oz.
90	4	2	17	14
100	4	3	27	15
86	19	2	19	12
14	13	1	0	0
96	10	3	17	14

4. CLOTH MEASURE.

Yd.	qr.	na.
70	3	3
13	2	1
90	0	0
8	1	3
6	3	1

E.E.	qr.	na.
44	2	2
60	1	3
90	1	3
30	3	3
20	0	3

E.F.	qr.	na.
90	2	3
108	1	3
76	3	2
40	1	3
95	0	2

5. DRY MEASURE.

pk.	qt.	pt.
1	7	0
2	6	1
3	5	1
1	4	0
2	3	1

bu.	pk.	qt.
70	3	4
60	2	1
55	3	7
60	1	6
9	1	6

bu.	pk.	qt.	pt.
95	2	1	1
76	2	7	0
40	2	6	1
25	2	4	0
3	2	5	1

9 WINE MEASURE.

Gal.	*qt.*	*pt.*	*hhd.*	*gal.*	*qt.*	*pt.*	*pipe*	*hhd.*	*gal.*
39	3	1	42	61	3	1	34	2	2
36	3	0	20	35	2	0	11	1	3
35	2	1	19	24	1	1	7	2	2
22	3	0	17	11	3	0	19	1	3
11	2	1	10	9	0	1	45	1	1

7. LONG MEASURE.

Yds.	*ft.*	*in.*	*bc.*	*m.*	*fur.*	*pol.*	*le.*	*m.*	*fur.*	*pol.*
4	2	11	2	46	3	16	85	2	7	27
1	2	9	2	91	1	29	75	2	5	19
2	1	8	1	67	3	18	25	1	4	23
1	0	7	1	60	7	33	95	1	6	11
3	1	10	1	35	2	21	11	2	3	15

8. LAND, OR SQUARE MEASURE.

Acres	*roods*	*rods*	*acres*	*roods*	*rods*	*acres*	*roods*	*rods*
440	3	37	11	3	17	990	3	39
760	2	38	97	2	16	760	0	14
600	1	14	20	3	18	17	3	38
976	2	35	36	0	24	32	2	20
37	0	20	25	0	8	203	3	34

9. SOLID MEASURE.

T.	*ft.*	*cords.*	*ft.*	*ft.*	*inches.*
40	90	3	127	20	1440
203	7	20	220	26	1259
23	23	35	29	22	1440

10. TIME.

Yrs.	*mo.*	*w.*	*dys.*	*Yrs.*	*dys.*	*h.*	*m.*	*sec.*
57	7	3	26	22	300	23	59	34
230	6	2	5	33	327	23	44	43
3	10	4	30	28	364	20	43	58
19	9	3	9	34	303	23	34	38

II. CIRCULAR MOTION.

S.	°	′	″	S.	°	′	″	S.	°	′	″
3	29	17	14	11	29	50	40	17	0	13	50
2	23	57	44	10	20	30	45	34	25	49	35
4	22	20	40	37	18	48	29	42	10	38	7
1	20	33	40	23	17	59	57	20	12	36	54
4	14	45	55	99	29	40	50	44	20	0	33
6	24	42	0	8	17	39	42	5	27	15	42

COMPOUND SUBTRACTION.

Teaches to find the difference between any two sums of different denominations.

RULE.

Place numbers of the same denomination under each other, the less below the greater; begin with the least denomination, and if it exceed the figure above it, borrow as many units as make *one* of the next greater; subtract it therefrom, add the upper figure to the difference; always adding one to the next higher denomination for that which you borrowed. *PROOF*—the same as in Simple Subtraction.

1. STERLING MONEY.

EXAMPLES.

	£.	s.	d.	qr.	£.	s.	d.	qr.
From	345	16	5	3	494	17	9	3
Take	128	17	4	2	479	19	10	2
Difference	217	19	1	1				

2. TROY WEIGHT.

lb.	oz.	pwt.	oz	pwt.	gr.	lb.	oz.	pwt.	gr.
12	10	19	10	19	23	120	10	16	19
9	11	18	6	17	19	134	11	15	17

3. AVOIRDUPOIS WEIGHT.

lb.	*oz.*	*dr.*	*cwt.*	*qr.*	*lb.*	*T.*	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
9	7	15	7	3	13	3	10	3	24	15	13
4	9	16	5	2	15	2	12	2	29	14	15

4. CLOTH MEASURE.

Yd.	*qr.*	*na.*	*E. E.*	*qr.*	*na.*	*E. F.*	*qr.*	*na.*
70	3	3	64	2	2	190	2	3
13	2	1	40	1	3	08	1	3

5. DRY MEASURE.

bu.	*pk.*	*qt.*	*bu.*	*pk.*	*qt.*	*bu.*	*pk.*	*qt.*
7	1	1	70	3	4	95	2	1
6	2	0	60	2	1	76	2	7

6 WINE MEASURE.

Gal.	*qt.*	*pt.*	*gi.*	*hhd.*	*gal.*	*qt.*	*pt.*	*hhd.*	*gal.*	*qt.*	*pt.*
39	3	1	2	261	3	1	2	34	2	2	1
36	3	0	0	20	35	2	0	08	11	1	0

7. LONG MEASURE.

Yds.	*ft.*	*in.*	*bc.*	*m.*	*fur.*	*pol.*	*le.*	*m.*	*fur.*	*pol.*
4	2	11	2	96	3	16	85	2	7	27
1	2	9	2	41	1	29	75	2	5	19

8. LAND, OR SQUARE MEASURE.

Acres	*roods*	*rods*	*acres*	*roods*	*rods*	*feet.*	*inches.*
760	2	38	97	2	16	960	14
440	3	37	11	3	17	790	33

9. SOLID MEASURE.

T.	*ft.*	*cords.*	*ft.*	*T.*	*ft.*	*inches.*
240	90	3	127	1440	20	238
03	7	2	220	26	18	125

10. TIME.

Yrs.	*mo.*	*w.*	*dys.*	*Yrs.*	*dys.*	*h.*	*m.*	*sec.*
230	7	3	26	32	300	23	59	34
57	6	2	5	23	327	23	44	43

11. CIRCULAR MOTION.

S.	°	′	″	*S.*	°	′	″	*S.*	°	′	″
3	29	17	14	11	29	50	40	37	0	13	50
2	23	57	44	10	20	30	45	14	25	49	35

COMPOUND MULTIPLICATION.

Shows how to find the amount of any given number of diverse denominations, by repeating it any proposed number of times.

FEDERAL MONEY.

RULE.

Multiply as in whole numbers ; and place the seperatrix as many figures from the right hand in the product, as it is in the multiplicand.

EXAMPLES.

(1)

	$	cts.		
Multiply	17	18	by	25
		25		
	85	90		
	343	6		
Product.	$ 428,	50		

(2)

	dol.	d.	c.	m.		
Multiply	7	1	3	9	by	14
			1	4		
	28	5	5	6		
	71	3	9			
Product.	$ 99,	9	4	6		

3. Multiply 11 mills by 40 *Ans.* $ 0 44
4. Multiply 41 cents 5 mills, by 150 *Ans.* 62 25

5. Multiply 9 dols. by 50 *Ans.* 450 00

6. The number of inhabitants in the United States is 7 millions; if each should pay the sum of 8 cents yearly, for 9 years, how many dollars would be raised? *Ans.* 5 millions.

WEIGHT, MEASURE, STERLING MONEY, &c.

RULE.

Write the multiplicand, and place the quantity under the least denomination for the multiplier, observe the same rules, for carrying as in compound addition.

1. What cost 9 lb. of sugar, at 2*s*. 8*d*. 2*qr*. pr. lb.?

2*s* 8½*d*
9

Ans. 1*l*. 4*s* 4½*d*

2. 3 lb. of green tea, at 9*s*. 6*d*. pr. lb.? *Ans.* £ 1, 8*s* 6*d*

3. 5 lb. of loaf sugar, as 1*s*. 3*d*. pr. lb.? *Ans.* £0 6 3

4. 9 cwt. of cheese, at 1*l*. 11*s*. 5*d*. pr. cwt. *Ans.* 14*l* 2*s* 9*d*

When the multiplier exceeds 12;

Multiply successively by its component parts, instead of the whole number.

EXAMPLES.

1. 16 cwt. of sugar, at 1*l*. 18*s*. 8*d*. pr. cwt.

(1)

1*l* 18*s* 8*d*
4

7 14 8
4

£ 30 18*s* 8*d*

(2)

28 yds. of broad cloth, at 19*s*. 4*d*. pr. yd. *Ans.* £ 27, 1*s*. 4*d*.

3. 182 yds. Irish linen, at 2*s*. 4*d*. pr. yd. *Ans.* £ 15, 8*s*

4. 144 reams of paper, at 13*s*. 4*d*. pr. ream. *Ans.* £ 96, 0

5. 96 bushels wheat, at 1*l*. 3*s*. 4*d*. pr. bush. *Ans.* £ 112, 0

If no two numbers multiplied together will exactly equal the multiplier, multiply by any two numbers that come the nearest, then multiply the upper line by the remainder, which added to the last product gives the answer.

EXAMPLES.

1. What will 47 yds. broad cloth come to, at 17*s.* 9*d.* pr. yd. ?

	£	s.	d.	
	0,	17,	9	price of 1 yd.
			5	
	4	8	9	price of 5 yds.
			9	
	39	18	9	price of 45 yds.
	1	15	6	price of 2 yds.
Ans.	41,	14	3	price of 47 yards.

2. 29 yds. of cambricd, at 0*l.* 13*s.* 7*d.* pr. yd.
Ans. £19. 13*s.* 11*d.*

3. 111 yds. broad cloth, at 1*l.* 2*s.* 6*d.* pr. yd.
Ans. £124, 17*s.* 6*d.*

4. 23 ells, at 0£ 3*s.* 6½*d.* pr. ell ? *Ans.* £ 4, 1*s.* 5*d.* ½

5. 117 cwt. Malaga raisins, at £1, 2*s.* 3*d.* pr. cwt.
Ans. £130, 3*s.* 3*d.*

6. 59 yds. tabby velvet, at 7*s.* 10*d.* pr. yd. *Ans.* £ 23, 2*s.* 2*d.*

7. What is the weight of 7 hhds. of sugar, each weighing 9 cwt. 3 qr. 12 lb. ? *Ans.* 69 cwt.

8. In 9 fields, each containing 14 acres, 1 rood, and 25 poles, how many acres ? *Ans.* 129*a.* 2*qr.* 25*rods.*

9. In 6 parcels of wood, each containing 5 cords and 96 feet, how many cords ? *Ans.* 34½ cords.

	S.	°	′	″		S.	°	′	″
10. Multiply	1	15	48	24	by 5.	*Ans.* 7	19	2	0

11. Multiply 3 cords, 87 feet, by 8 *Ans.* 29 cords 56 feet.

COMPOUND DIVISION.

Teaches to find how often one number may be had in another of different denominations.

RULE.*

Write down the given sum in cents, and divide as in whole numbers;—the quotient will be the answer in cents.

* NOTE. If the cents in the given sum are less than 10, place a cypher on their left, or in the ten's place.

EXAMPLES.

Divide $ 674 19 cents by 24

(1) dol. cts. m.

24)67419(28, 09 $1\frac{1}{4}$

48

194

192

219

216

30

24

6

To bring cents into dollars, you need only point off 2 figures on the right, the rest will be dollars.

When there is a remainder, add a cypher, again divide, and you will have the mills.

2. Divide 4 dols. 9 cents, or 409 cents by 6. *Ans.* ,68, cts.
3. Divide 9 dols. 24 cents, by 12. *Ans.* ,77 cts.
4. Divide 2 dollars into 33 equal parts. *Ans.* ,66, cts.
5. Divide 1000 cents by 25. *Ans.* ,40 cts.
6. Divide 999 cents; by 9. *Ans.* 111 cts.
7. Divide 1 dol. by 2 mills. *Ans.* ,500 mills.
8. Divide 125 dols. by 500. *Ans.* 25 cts.
9. Divide 10 cents, by 10 mills. *Ans.* 10 mills.

STERLING MONEY, WEIGHTS, MEASURES, &c.

RULE.

Begin at the left hand, as in simple division, and if any thing remains, determine how many of the next denomination the remainder is equal to, which add to the next denomination, continuing to divide, and to carry the remainder, as before, till the whole is finished.

(1)

	£.	s.	d.	qr.	
Divide	19	4	10	3	by 4.
	4)19	4	10	3	
	£. 4	16	2	2	$\frac{3}{4}$

Ans.

		£.	s.	d.	qr.
2. Divide 31*l.* 14*s.* 9*d.* 2*qr.* by 17.	*Ans.*	1	17	4	0
3. Divide 119*l.* 12*s.* 2*d.* 3*qr.* by 9.	*Ans.*	13	5	9	2
4. Divide 1*l.* 19*s.* 8*d.* 0*qr.* by 11.	*Ans.*	0	3	7	$\frac{1}{4}$

NOTE. When the divisor exceeds 12, and is the product of 2 numbers; divide by one of these numbers first, and the quotient by the other, the last quotient will be the answer.

5. Divide 128*l.* 9*s.* 0*d.* 0*qr.* by 42. *Ans.* 3 1 2 0
6. Divide 5*l.* 10*s.* 3*d.* 0*qr.* by 81. *Ans.* 0 1 4 $\frac{1}{4}$
7. Divide 6 tons, 11 cwt. 3qrs. 19lb. by 4.
Ans. 1 *T.* 12 *cwt.* 3*qr.* 25*lb.* 12*oz.*
8. A. piece of cloth containing 24 yds. cost £. 18. 6*s.* what is it pr. yd.? Ans. 15*s.* 3*d.*
9. Divide 17 lea. 1 m. 4 fur. 21 pols. by 21.
Ans. 2 m. 4 fur. 1 pol.
10. From a piece of cloth containing 64 yds. and 2 nails, a taylor was directed to make 27 coats; what did each coat contain? *Ans.* 2 yds. 1 qr. 2 na.

REDUCTION.

Teaches to change numbers from one denomination to another, without altering their value.

Reduction is either Ascending, or Descending. It is Ascending, when numbers of a lower denomination are raised to a higher denomination. It is Descending, when numbers of a higher denomination are reduced to a lower denomination.

PROOF. Invert the order of the question.

REDUCTION DESCENDING.

RULE.

Multiply the highest denomination given, by so many of the next less, as make one of that greater; and thus continue till you have reduced it as low as the question requires.

EXAMPLES.

1. In 37*l.* 18*s.* 7*d* 3*qr.* how many farthings?

20			
753	Shillings.*		4)36175
12			
9043	pence.	*PROOF.*	12)9043 + 3qr.
4			20)75,3 + 7d.
Ans. 36175	farthings.		£37,18,7,3

2. In 23*l* 11*s.* 7½*l.* how many farthings? *Ans.* 22638.
3. In 47*l.* 19*s.* 3*d.* how many shillings, pence and farthings? *Ans.* 959*s.* 11511*d* and 46044*qr.*

*Note. In multiplying by 20, add in the shillings, by 12 add in the pence, and by 4, add in the farthings, if any, in all similar cases.

4. In 315 dolls. 50 cents, how many threepences and farthings ? *Ans.* 7572 threepences, 90864 *qr.*

5. In 121 French crowns, at 6*s.* 8*d.* each, how many pence and farthings ? *Ans.* 9680*d.* 38720 *qr.*

6. In 312*l* 8*s.* 6½*d.* how many sixpences and half-pences ?
Ans. 12497 sixpences, 149964 half pences.

REDUCTION ASCENDING.

RULE.

Divide the lowest denomination given, by so many of that denomination, as make one of the next higher, and so on through all the denominations, as far as the question requires.

PROOF. Multiply inversely by the several divisors.

EXAMPLES.

1. In 122318 farthings, how many pence, shillings, and pounds ?

Farthings in a penny = 4)122318.

Pence in a shilling = 12)30579 + 2qr.

Shillings in a pound = 2,0)254,8 + 3*d.*

Ans. £127,8s.3d.2qrs.

2. In 30329 farthings, how many pounds ?
Ans. £ 31, 11*s* 10*d.* 1*qr.*

3. In 46044 farthings, how many pence, shillings, and pounds ?
Ans. 11511*d.* 959*s.* 47*l.*

4. In 90864 farthings, how many dollars ? *Ans.* $ 315½.

5. In 20160 pence, how many pounds ? *Ans.* £ 84

2. TROY WEIGHT.

1. In a dozen of silver spoons, weighing 11lb. 3oz. 11pwt. how many grains ? *Ans.* 7464 grains.

2. In 10 ingots of gold, each weighing 9oz. 5pwt. how many grains ? *Ans.* 44400

3. How many table spoons weighing 23pwt. each, and ten spoons 4pwt. 6 grs. each, and an equal number of each sort, can be made from 4lb. 1oz. 1pwt. of silver. *Ans.* 36

REDUCTION ASCENDING AND DESCENDING.

3. AVOIRDUPOIS WEIGHT.

1. In 19 lb. 14oz. 11dr. how many drams ? *Ans.* 5099

2. In 5 tons, how many drams ? *Ans.* 2867200

3. Bring 5099 drams into pounds. *Ans.* 19*lb.* 14oz. 11dr.

4. A merchant has 5 hhds. of tobacco, each 8 cwt. 3 qrs. 14lb. and wishes to put it into boxes containing 70lb. each, how many boxes are requisite ? *Ans.* 71

4. CLOTH MEASURE.

1. In 5469 nails, how many yards? *Ans.* 341 yds. 3 qr. 1 na.
2. In 1220 nails, how many Ells English? *Ans.* 61.
3. In 28 Ells Flemish, how many quarters and nails?
Ans. 84 qr. 336 na.
4. How many coats containing $1\frac{3}{4}$ yds. each, can be made from $73\frac{1}{2}$ yards of broad cloth? *Ans.* 42.

5. DRY MEASURE.

1. In 68 bushels, how many peaks, quarts, and pints?
Ans. 272 pecks, 2176 qts. 4352 pts.
2. In 25 bush. 3 pks. 7qts. how many quarts? *Ans.* 831.
3. In 8704 pints, how many bushels? *Ans.* 136.
4. A gentleman has 1003 bush. 3 pks. of grain, and a team that consumes $2\frac{1}{4}$ bush. pr. day, how long will the grain last?
Ans. one year

9. WINE MEASURE.

1. In 9 tons of wine, how many quarts? Ans. 9072.
2. In 18144 pints of wine, how many hhds.? Ans. 36.
3. How many bottles containing $1\frac{1}{2}$ pint, can be filled from a pipe of wine? Ans. 672.

7. LONG MEASURE.

1. In 17 miles, how many inches? Ans. 1077120.
2. In 3 leagues, how many yards? Ans. 15840.
3. How many revolutions do the forward wheels of a stage describe in running from Hartford to New-Haven, it being 34 miles, allowing the wheels to be $14\frac{3}{4}$ feet in circumference?
Ans. $12114\frac{60}{177}$
4. What is the circumference of the globe in inches, it being 360 degrees? Ans. 1585267200

8. TIME.

1. In 1 year, or 3[illegible] 5h. 48′ 58″ how many seconds?
Ans. 31556938
2. In 655989 days, how many years, reckoning the year to contain 365d. 6h.? Ans. 1796
3. How many minutes were there from the brith of Christ to the year 1776, allowing the length of the year the same as in the first example? Ans. 934085364′ 48″

9. CIRCULAR MOTION.

1. In 9 signs, 13° 25′ how many seconds? Ans. 1020300
2. In 811480 seconds, how many signs?
Ans. 7*S.* 15° 24′ 40″
3. How many minutes in the whole of the Earth's orbit, or 12 signs? Ans. 21600

OF CURRENCIES.

RULE.

Divide the given sum reduced to shillings, to sixpences, or pence, by the number of shillings, or pence, in a dollar, in each state.

1. Reduce £63 15s. New England currency to Federal Money. Ans. $ 212 50 cts.

2. Reduce £481 New York currency to Federal Money. Ans. $ 1077 50

3. Reduce £37 10s. Pennsylvania currency to Federal Money. Ans. $ 100.

To change Federal Money to the currency of each state.

RULE.

Multiply the given sum in cents, by the number of pence in a dollar, and cut off two figures to the right of the product, what is left will be the answer in pence, and if the figures thus seperated, be multiplied by 4; and 2 figures again cut off as before, those at the left hand will be farthings.

EXAMPLES.

Reduce $ 438 42 cts. to New-York currency. Ans. £175 7s. 4½d.

Reduce $ 1971 96 cts. to New-England currency. Ans. £591 11s. 9d.

Reduce 85 dolls. 43 cts. to sterling money. Ans. £19 4s. 5d.

NOTE.—When the given sum is dollars, multiply by the number of shillings in a dollar.

Table of Coins current in the United States, with their Sterling and Federal value.

Names of Coins.	Standing weights.		Sterling money of Great-Britain.			Vermont, New-Hampshire, Massachusetts, Rhode-Island Connecticut and Virginia.			New-York and South-Carolina.			New-Jersey, Delaware and Maryland.			South-Carolina and Georgia.			Federal Value. Dollars. Cents. Mills.		
GOLD.	pwt.	gr.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	D.	C.	M.
A Johannes,	18	0	3	12	0	4	16	0	6	8	0	6	0	0	4	0	0	16	00	0
A Half Johannes,	9	0	1	16	0	2	8	0	3	4	0	3	0	0	2	0	0	8	00	0
A Doubloon,	16	21	3	6	0	4	8	0	5	16	0	5	12	6	3	10	0	14	93	3
A Moidee,	6	18	1	7	0	1	16	0	2	8	0	2	5	0	1	8	0	6	00	0
An English guinea,	5	6	1	1	0	1	8	0	1	17	0	1	15	0	1	1	9	4	66	7
A French Guinea,	5	5	1	1	0	1	7	6	1	16	0	1	14	6	1	1	5	4	60	0
A Spanish Pistole,	4	6	0	16	6	1	2	0	1	9	0	1	8	0	0	18	0	3	77	3
A French Pistole,	4	4	0	16	0	1	2	0	1	8	0	1	7	6	0	17	6	3	66	7
SILVER.																				
A Crown,	18	0	0	5	0	0	6	8	0	8	9	0	8	3	0	5	0	1	10	0
A Dollar,	17	6	0	4	6	0	6	0	0	8	0	0	7	6	0	4	8	1	00	0
An English Shilling,	3	18	0	1	0	0	1	4	0	1	9	0	1	8	0	1	1	0	22	2
A Pistarean,	3	11	0	0	$10\frac{3}{4}$	0	1	2	0	1	7	0	1	6	0	0	11	0	20	0

DUODECIMALS.

Duodecimals are so called, because they decrease by twelves, from the place of feet towards the right. Inches are sometimes called *primes*, and marked thus ' ; the next division, after inches, is called parts, or *seconds*, and is marked thus " ; the next is *thirds*, marked thus "' ; &c.

Multiplication of Duodecimals ; or Cross Multiplication.

RULE.

1. Under the multiplicand write the same denominations of the multiplier, that is, feet under feet, inches under inches, &c.

2. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, and write each result under its respective term, observing to carry an unite for every 12, from each lower denomination, to its next superiour.

3. In the same manner, multiply every term in the multiplicand by the inches in the multiplier, and set the result of each term one place farther toward the right of those in the multiplicand.

4. Proceed in like manner with the seconds and all the rest of the denominations, if there *be* any more ; and the sum of all the several products, will be the product required.

The products of the several denominations depend upon the principle, that—

Feet by feet give feet.	Primes by primes give seconds.
Feet by primes give primes.	Primes by seconds give thirds.
Feet by seconds give seconds.	Primes by thirds give fourths.
&c.	&c.
Seconds by seconds give fourths.	Thirds by thirds give sixths.
Seconds by thirds give fifths.	Thirds by fourths give sevenths.
Seconds by fourths give sixths.	Thirds by fifths give eights.
&c.	&c.

Or, in general

When feet are concerned, the product will be of the same denomination as the *term* by which the feet are multiplied. When feet are not concerned, the name of the product will be expressed by the *sum* of the *strokes*, or *marks* over both the factors.

EXAMPLES.

1. Required the contents of a ceiling 10ft. 4' 5" by 7ft. 8' 6"

ft.	'	"		
10	4	5		
7	8	6		
72	6	11		
6	10	11	4	
	5	2	2	6

Feet 79 11' 0" 6"' 6"'' Ans.

2. How many square feet in a board 17 feet 7 inches long, and 1 foot 5 inches wide? Ans. 24f. 10' 11"

3. How many solid feet in a stick of timber 12 feet 10 inches long, 1 foot 7 inches wide, and 1 foot 9 inches thick?
Ans. 35f. 6' 8" 6"'

4. Required the number of solid feet in a load of wood, that is 9¼ feet long, 3½ feet wide, and 3 feet 7 inches high?
Ans. 113f. 5' 8"

5. How many yards of painting in a room 20 feet in length, 14¾ feet in breadth, and 10½ feet in height, deducting a fire place of 4f. by 4½f. and 2 windows, each 6f. by 3⅝f.? Ans. 73$\frac{2}{7}$ yards.

In like manner may pounds, shillings, pence, &c. be multiplied into each other, by observing the following principle—that

Pounds by pounds give pounds.
Pounds by shillings give shillings.
Pounds by pence give pence,
&c.

Shillings by shillings, every 20 is 1 shilling, every 5 is 3d. and each 1 is 2 farthings, and 4 tenths of a farthing.

Shillings by pence, every 5 is a farthing, and each one 2 tenths of a farthing, &c.

Pence by pence, every 60 is a farthing, and every 6 one tenth of a farthing.

EXAMPLES.

1. Let it be required to multiply 3l. 5s. 6d. by 2l. 12s 9d.

£	*s.*	*d.*		
3	5	6		
2	12	9		
6	11	0	*qrs.*	*tenths.*
1	19	3	2	4
	2	5	1	9

Ans. 8l. 12s. 9d. 0qr. $\frac{3}{10}$

2. Let it be required to multiply 2s. 6d. by itself.

Ans. 3d. 3qrs.*

In the above example, *a pound* was considered the integer; but when a *shilling* is taken for the integer, observe the following precepts;—namely, that

Shillings by shillings give shillings.

Shillings by pence give pence.

Shillings by farthings give farthings, &c.

Pence by pence, every 12 is a penny, and each 3 a farthing.

Pence by farthings, every 12 is a farthing, and each 3 is $\frac{1}{4}$ of a farthing, &c.

Farthings by farthings, each 12 is $\frac{1}{4}$ of a farthing.

EXAMPLES

1. Let it be required to multiply 2s. 6d. by 2s. 6d. one shilling being taken for the integer.

s.	d.	
2	6	
2	6	
5	0	
1	3	0
£0	6	3 0

(2)

Multiply $2\frac{1}{2}$ feet by $2\frac{1}{2}$ feet.

2f.	6'	
2	6	
5	0	
1	3	0

Feet 6 3' Ans.

Where it is apparent, that if, instead of shillings, pence and farthings, we reckon feet inches and quarters, the result will be the same.

The two following questions are *Sexcessimals.*

3. If two places differ in longitude 2° 12'; what is their difference of time?

Mult. 2° 12' 00" 00'''
by 3' 59" 20''' the time in which the Sun passes through one degree.
Ans. 8' 46" 32'''

4. Two places differ in longitude 31° 27' 30", what is the difference in time of the Sun's coming to the meridian of those places, the Sun passing through 15° in an hour?

31° 27' 30"
4' 00" In 4 minutes of a solar day, the Sun passses 1 degree.
Ans. 1° 6' 30" 00'''

* NOTE. Whence it is manifest that fractions multiplied, become less, in the same proportion as integers, by multiplying, become greater.

SIMPLE PROPORTION, OR RULE OF THREE.

Teaches, that by having three proportionals given, to find a fourth, which multiplied into the first, shall be equal to the product of the other two: or the RULE OF THREE teaches, by having three numbers given, to find a fourth, which shall have to the second the same ratio, that the third has to the first.

PROPORTION IN COMMON ARITHMETICK, is generally considered direct, or inverse. It is direct, when more requires more, or less requires less, and inverse, when more requires less, and less requires more.

1. Observe that two of the given numbers in the question are always of the same kind, one of which must be the first number in stating, the other the third; consequently that number which is of the same kind with the answer or thing sought, will always be the second, or middle term in stating.

2. Observe farther; the third term is always a *demand*, and may be known by its asking the question.

RULE.

1. Write the numbers so that the term which asks the question may possess the third place, and that which is of the same kind with it, the first place, the remaining term will posses the second place.

2. Bring the first and third terms to the same denomination, and reduce the second, to the lowest name mentioned in it.

3. Multiply the second and third terms together, and divide their product by the first, the quotient will be the answer in the same denomination as the second term.*

---

* This rule is founded on the principle that if four numbers be proportional, the product of the EXTREMES, is equal to the product of the MEANS.

Thus 4 : 8 : : 16 : 32, here 4 X 32 = 128; and 8 X 16 = 128: whence it is evident, that dividing the product of the *means*, or the product of the *extremes* by the *first* extreme, the other extreme is obtained. Thus—dividing 128 the product of the means, by 4, gives 32 for the other extreme; and dividing the product of the *means*, or the product of the extremes by one of the *means*, the *other* mean is obtained. Thus—dividing 128 the product of the extremes, by 8, gives 16 for the other mean; hence the propriety of the rule, in multiplying together the 2d and 3d terms, or the *means*, and dividing by the first term, or the first extreme, to obtain the other; for it has been demonstrated, that the product of the means, divided by the first extreme, gives the other extreme.

This rule is applicable when the proportion is *direct*; but when the proportion is inverse; that is, when the conditions of the question require the answer to be greater, or less than the second term: Multiply the first and second terms together, and divide the product by the third. The quotient will be the answer in the same denomination as the middle term.

EXAMPLES.

1. If 6 yards cost $ 18, what will 12 yards cost at the same rate?

6 : 18 : : 12

12

6) 216 (36 dollars.
18

36
36

00

2. If 18 dollars buy 6 yards, how many yards will 36 dols. buy?

18 : 6 : : 36

6

18)216(12 Ans.
18

36
36

00

(3)

If $ 36 buy 12 yds. how many will 18 dollars buy.

36 : 12 : : 18

12

36)216 (6 Ans.
216

000

4. If 12 gallons brandy cost 25 dols. 44 cents, what will be the price of 2 pipes, at the same rate.

Gal. $ Pipes.
12 : 25,44 : : 2
2 hhds. in 1 pipe.
4
63 gal. in 1 hhd.
12
24

As. 12 : 2544 : : 252 do. in 2 pipes.
252

5088
12720
5088

12)631088 (525 90 $6\frac{2}{3}$ Ans. five hundred and twenty-five dollars, ninety cents, 6 mills and two thirds.
60
31
24
70
60
108
108
80
72
8
$\frac{8}{12} = \frac{2}{3}$

5. If 5 horses eat 10 bushels of corn in a week, how many will 35 eat in the same time? Ans. 70

6. If an ounce of silver is worth 90 cents, what is a cup worth that weighs 2lb. 10 oz. Ans. $ 37 80

7. If 15° of the equator revolve through the meridian in one hour, in what time will 150° 51' 15" revolve through?
Ans. 10h. 15' 25"

8. What is the tax upon $ 50 97, at ten cents on the dollar.
Ans. $ 509 70.

9. What will 4 casks of raisins, weighing 2 cwt. 2 qr. 25 lb.

come to, at 16 cents pr. lb.? Ans. $ 195 20

10. *At the rate of 15 deg. pr. hour, how much of the equator will revolve through the meridian in 12 hours 2 min. 26 sec.? Ans. 179 deg. 36 min. 30 sec.

11. When the Sun is on the meridian of London, what o'clock is it at Mexico North America, 100 degrees 5 min. 45 seconds.? Ans. 5 o'clock, 19 m. 37 sec. A. M.

12. What o'clock is it at Moscow 37 deg. 45 min. east long. when it is noon at London? Ans. 2 o'clock 31m. P. M.

13 If the Sun comes to the meridian of London, 4h. 45m. 20 sec. sooner than it does at the Meridian of Cambridge, what is the longitude of Cambridge? Ans. 71 deg. 20 m. W.

14. suppose a Gentleman has an income of $ 1940 a year, and he spends 3 dols. 46 cents pr. day, how much will he have saved, at the years end? Ans. $ 683 10

15. Sound uninterrupted, moves about 1142 feet in a second, how long then, after firing a cannon at Springfield before it will be heard at Hartford, it being 26 miles? Ans. 2 m. 0 sec. $\frac{120}{171}$

16. In a thunder storm it was observed, that it was 6 seconds between the lightning and thunder, at what distance was the explosion? Ans. 6852 ft. $= 1\frac{11}{120}$ mile.

17. Suppose a rocket was seen at the instant of discharge, 12 seconds before the report, at what distance was the gun. Ans. $2\frac{131}{240}$ miles.

18. If $ 100 in one year gain $ 6, what will $ 314 15 cts. gain in the same time? Ans. $ 18 84c. 9m.

19. If $ 212 25 c. gain $ 12 $37\frac{1}{2}$ in one year what is that pr. cent? Ans. 6

20. A owes B $ 1736 59 cts. but becoming a bankrupt, he is unable to pay more than 65 cents on the dollar, what does B receive for the debt? Ans. $ 1128 78c. $3\frac{5}{10}$m.

21. If a man buy merchandize to the amount of $ 560, and gain by the sale $ 190 40, how much will he gain by laying out 150 at the same rate? Ans. $ 50 00

22. If 30 men perform a piece of work in 11 days, how many men can accomplish another piece of work 4 times as large in, a fifth part of the time? Ans. 600

23. A wall that is to be built to the height of 27 feet, was raised 9 feet by 12 men in 6 days, how many men must be employed to finish the wall in 4 days, working at the same rate? Ans. 36

24. If a stick 8 feet long, cast a shadow on level ground 12

*Note. The equator may always be supposed to revolve through the meridian, at the rate of 15 degrees in 1 hour of solar time, without any sensible errour; though it is a fraction wide of the truth.

feet, what is the width of a river, over which a tower, known to be 180 feet in height casts its shade. Ans. 270 feet.

OF THE LEVER OR STEELYARD.

It is a principle in Mechanicks, that the power is to the weight, as the velocity of the weight, to the velocity of the power; therefore to find what weight may be raised or balanced by any given power, say;

As the distance between the body to be raised, or balanced and the fulcrum, or prop, is to the distance between the prop and the point where the power is applied: so is the power to the weight which it will balance.

If a man weighing 160lb. rest on the end of a lever 10 feet long, what weight will he balance on the other end, supposing the prop 1 foot from the weight?

The distance between the weight and the prop being 1 foot, the distance from the prop to the power is 10 — 1 = 9 feet, therefore,

ft. *ft.* *lb.* *lb.*
As 1 : 9 : : 160 : 1440 Ans.

If a weight of 1440 be placed 1 foot from the prop, at what distance from the prop must a power of 160lb. be applied to balance it?

As 160 : 1440 : : 1 : 9 feet. Ans.

At what distance from a weight of 1440lb. must a prop be placed, so that a power of 160lb. applied 9 feet from the prop, may balance it.

As 1440 : 160 : : 9 : 1 ft. Ans.

The celebrated Archimedes said he could move the Earth, if he had a place at distance from it to stand upon, to manage his machinerey.

Now suppose the Earth to contain in round numbers 4,000,000,000,000,000,000,00000 lb. or 400000 Trillions of lbs. and that Archimedes was suspended from the end of a lever 12,000,000.000,000,000,000,006,000 miles in length, and the *fulcrum*, or centre of motion of the lever to be 6000 miles from the Earths centre, how much must Archimedes weigh to balance the Earth? Ans. 200 lb.

OF THE WHEEL AND AXLE.

The proportion of the wheel and axle, (where the power is applied to the circumference of the wheel, and the weight to be raised is suspended by a cord, which coyls about the axle as the wheel turns round,) is as the diameter of the axle to the diame-

ter of the wheel, so is the power applied to the wheel, to the weight suspended from the axle.

Suppose a windless is constructed in such a manner, that 14lb. applied to the wheel will raise 224lb. suspended from the axle, which is 6 inches in diameter, what is the diameter of the wheel? Ans. 8 feet.

lb. in. lb. in.

Inversely, As 224 : 6 : : 14 : 96 = 8 feet.

Suppose the diameter of the wheel to be 8 feet, required the diameter of the axle, so that 14lb. suspended from the wheel, may balance 224lb. on the axle.

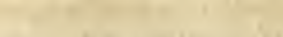
lb. in lb. in.

Inversely, As 14 : 96 : : 224 : 6 diameter required.

Suppose the diameter of the wheel 96 in. and that of the axle 6 in. what weight suspended from the axle will balance 14lb. upon the wheel?

Inversely, As 96 : 14 : : 6 : 224 weight required.

OF LOGARITHMS.



THE operations of Multiplication and Division, when they are to be often repeated, and the extracting of Roots, especially if they be from the higher powers, become so tedious, that it is an object which has long employed the skill and talents of the most profound mathematicians, to substitute in their place more expeditious, and easier methods of calculation.—To effect this, CERTAIN NUMBERS have been so contrived, and adapted to other numbers, that the addition and subtraction of the former, have been made to perform the office of multiplication and division in the latter, with imcomparable facility and expedition.

The invention of Logarithms is by some ascribed to Baron Napier. But the kind of Logarithms now in use, was invented by Mr. Henry Briggs, Professor of Geometry in Gresham College, London.

LOGARITHMS (from logos, *ratio* and arithmos, *number*) are the indices of the ratios of numbers to one another ; being a series of numbers in arithmetical progression, corresponding to others in geometrical progression.

Thus $\begin{cases} 0 & 1 & 2 & 3 & 4 & 5 \text{ indices or Logarithms.} \\ 1, & 10, & 100, & 1000, & 10000, & 100000, \end{cases}$

This is the most convenient series of numbers, to which most of the modern TABLES OF LOGARITHMS are calculated.

In which it is apparent that if any two indices, or Logarithms, be added together, their sum will be the index, or logarithm, of that number, which is equal to the product of the two terms, in the geometrick progression, to which those indices, or logarithms belong.

Thus, the logarithms 2 and 3, being added together, make 5, corresponding to 100000, the product of 100, into 1000, and the logarithms 1, and 4, being added together, make 5, the logarithm corresponding to 100000, the product of 10 into 10000. Whence it is evident that POWERS of the same ROOT may be *multiplied*, by adding their exponents, or logarithms. In like manner, if any one index, or logarithm, be *subtracted* from another, the difference will be the logarithm of that number, which is equal to the *quotient* of the two terms, to which those logarithms belong. Thus; if from 5, (the logarithm of 100000)

be subtracted 2, (the logarithm of 100) the difference 3, will correspond to 1000, the quotient of 100000 divided by 100.

Again ; if from 5, (the logarithm 100000) be subtracted 3, (the logarithm 1000) the difference is 2, answering to 100, the quotient arising from 100000 divided by 1000. Hence it is manifest, that.

A POWER may be divided by another power of the same root, by subtracting the logarithm of the divisor, from the logarithm of the dividend.

So also if the logarithm of any number be multiplied by the index of its power, the product will be equal to the logarithm of that power. Thus if 2, (the logarithm 100) be multiplied by 3, the product will be 6, equal to the logarithm of 1000000, or the 3d power of 100.

Again, if the logarithm of any number be divided by the index of its root, the quotient will be the logarithm of its root.

Thus, the index, or logarithm of 1000000, is 6, and if this number be divided by 3, the quotient will be 2, which is the logarithm of 100, or the cube root of 1000000.

In the following series, to wit.

10^4	10^3	10^2	10^1	0^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
10000	1000	100	10	1	10	100	1000	10000

Whose Logarithms are

4	3	2	1	0	—1	—2	—3	—4	&c.

It will be seen, that the logarithms of all the numbers between 1 and 10, are greater than 0, but less than 1 ; since by the series, it may be seen, that the logarithms of 1 and of 10, are 0, and 1.

Thus the logarithm

of 2 is	0.	3010300
of 5 =	0.	6989700
of 7 =	0.	8450980

Each number therefore between 1 and 10, has 0 for its index, with a decimal annexed.

For the same reason, if the given number be

between 10 and 100	the log. will be	1 and 2	1 + the decimal part
100 and 1000		2 and 3 i. e.	2 + the decimal part
1000 & 10000	between	3 and 4	3 + the decimal part

Thus the logarithm of the natural number

of 35 is	1.	5440680
of 175 is	2.	2430380
of 8795 is	3.	9442358

Whence we derive this *general truth*. The index of the logarithm, is always 1 less than the number of integral figures in the natural number, whose logarithm is required ; or the in-

dex shows how many figures to the left, the natural number extends from the place of units.

Thus the logarithm of 35, is 1. 5440680. Here the number of figures being *two*, the index or characteristick of the logarithm is 1.

The logarithm of 175 is 2 2430380. Here the number 175 consists of *three figures*, the first of which on the left hand is second from the place of units ; the index, or characteristick of the logarithm is therefore 2, and the logarithm 8795 is 3. 9442358. Which extending to three places counted from the unite figure, must have 3 for the index of its logarithm.

Integral numbers are said to form a geometrical series, increasing from unity towards the left; but decimals are supposed to form a like series, decreasing from unity towards the right; the indices of whose logarithms are *negative* as has been shown in the preceeding examples.

Whence it follows, that all numbers which consist of the *same figures*, whether integral, or fractional, or mixed, will have the decimal parts of their logarithms *the same*, differing only in the index, which will be *more, or less*, and *positive, or negative*, according as the first figure of the number is removed to the right, or left, from the place of units.

Thus the logarithm of 7359 is 3. 8668188 ; and the logarithm of $\frac{1}{10}$ or $\frac{1}{100}$ or $\frac{1}{1000}$ &c. part of it, will be as follows.

Numbers.	Logarithms.
7359	3. 8668188
735.9	2. 8668188
73.59	1. 8668188
7.359	0. 8668188
.7359	— 1. 8668188
.07359	— 2. 8668188
.007359	— 3. 8668188

Thus it appears that the negative index of a logarithm, shows how far the first significant figure of the natural number, is removed from the place of units on the right, in the same manner, as a positive index shows how far the first figure of the natural number, is removed from the place of units on the left. But when the index of the logarithm is *negative*, it is often more convenient to make it *positive* : and this is done by adding 10 to the *negative* index.

Thus, instead of — 1. 8668188 } write { 9. 8668188
of — 2. 8668188 } write { 8. 8668188
of — 3. 8668188 } write { 7. 8668188

Because — 1 + 10 = 9, and — 2 + 10 = 8, and — 3 + 10 = 7. Although this in truth, makes the index 10 too great ; yet by a little caution, it will produce no errour in the result :—observing

always, that when thesum, or product of the indices so increased, exceeds 10, 10 must be rejected.

Thus the sum of

$$\left.\begin{array}{r} -2.\ 8668188 \\ -3.\ 8668188 \end{array}\right\} \text{ becomes } \left\{\begin{array}{r} 8.\ 8668188 \\ 7.\ 8668188 \end{array}\right.$$

$$\overline{} \qquad \overline{}$$

* $-4.\ 7336376$ $\qquad$ $6.\ 7336276$

For $-4 + 10 = 6$

And the product of $-3.\ 8668188$ Multiplied by 2 $= 5.\ 7336376$

or $7.\ 8668188 \times 2 = 5.\ 7336376$

For $-5 + 10 = 5$

CONSTRUCTION OF LOGARITHMS.

The usual method of computing the logarithms to any of the natural numbers, 1. 2. 3. 4. 5. &c. is, I believe, as follows.

RULE.†

1. Take any two numbers whose difference is unity, or 1, and let the logarithm to the lesser number be known.

2. Divide the constant decimal, 868588964, &c. (or, $2 \div 2.3025$, &c.) by the *sum* of the two numbers, and reserve the quotient; divide the several quotients by the *square of the sum of the two nnmbers*, and reserve the quotient; divide this last quotient also, by the square of their sum, and again reserve the quotient; and thus proceed, continually dividing the last quotient, by the *square of the sum of the two numbers*, aslong as division can be made.

3. Then wiite these quotients in their order, under one another, the 1st uppermost; and divide them respectively by the prime, or odd numbers, 1. 3. 5. 7. 9. 11. 13. &c. as long as di-

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*The decimal parts of these logarithms are added as in simple numbers; but when you come to the left hand figure of each decimal, there is + 1 to carry to — 3, which equals — 2, and this added to the — 2 above it, gives — 4 for the sum of the indices.

† Yet there are many other ingenious methods of finding the logarithms of numbers, (see Introduction to Dr. Huttons Tables, and Baron Masores Scriptores, Logarithmici, also, Kiel, on Logarithms, Briggs Logarithms, Gardners, Taylors, Callets, and Sherwins, Mathematical Tables.

vision can be made, that is, divide the first reserved quotient by 1, the second by 3, the third by 5, the fourth by 7, and so on.

4. Add all these last quotients together, and the sum will be the logarithm of the greater number divided by the less; therefore to this logarithm, add also the logarithm of the lesser number, and their sum will be the logarithm to the greater, or proposed number.

EXAMPLES.

Example 1st. Let it be required to compute the logarithm of the number 2.

Here the given, or greater number is 2, and the next less number is 1, (whose Logarithm, in every System, is always 0) also the sum of 2 and 1 is 3. and its square 9; as follows.

3)	868588964	1)	289529654(	289529654
9)	289529654	3)	32169962(	10723321
9)	32169962	5)	3574440(	714888
9)	3574440	7)	397160(	56737
9)	397160	9)	44129(	4903
9)	44129	11)	4903(	446
9)	4903	13)	545(	42
9)	545	15)	61(	4
9)	61			

Log. of 2 ÷ 1.	.301029995
Add. log. of 1	.000000000
True Log. of 2.	.301029995

Example 2d.

Let it be required to compute the logarithm of 3

Here the given number is 3, and the next less is 2, whose logarithm by the first example is .301029995, and the sum also, of the 2 numbers 3 + 2 = 5 the square of which is 25, then the operation is as follows.

6)	868588963	1)	173717793(	173717793
25)	173717793	3)	6948712(	2316237
25)	6948712	5)	277948(	55590
25)	277948	7)	11118(	1528
25)	11118	9)	445(	50
25)	445	11)	18(	2
25)	18			

Log. of 3 ÷ 2	.176091260
Add Log. of 2	301029995
True log of 3 required.	477121255*

* See "Mr. Hutton's practical rule for the construction of Logarithms."

Then, because the sum of the Logarithms of numbers gives the logarithm of their product, and the difference of the logarithmsgives the logarithm of the quotient of the numbers; from the above two logarithms, and the logarithm of 10, which is 1, we may raise a great many logarithms, as in the following examples.

Example 3d.		*Example 4th.*	
Because 2 X 2 = 4, therefore		Because 2 X 3 = 6,	therefore
To Logarithm of 2	.301029995	To logarithm of 2	.301029995
Add Log. of 2	.301029995	Add log. of 3	.477121255
The sum is Log. of 4	.602059990	The sum is Log. of 6	.778151250

Example 5th.		*Example 6th.*	
Because 2 X 4 = 8, therefore		Because 10 ÷ 2 = 5, therefore	
To logarithm of 2	.301029995	From log. of 10	1.000000000
Add log. of 4	.602059990	Take log. of 2	.301029995
Gives the log of 8.	.903089985	Remains log. of 5	.698970005

Example 7th.		*Example 8th.*	
Because 5 X 8 = 40, therefore		Because 8 X 40 = 320 therefore	
To logarithm of 8	.903089985	To Logarithm 40	1.602059990
Add log. of 5	.698970005	Add log. of 8	.905089985
Logarithm of 40	1.602059990	Logarithm of 320	2.505149975

And thus computing by this general Rule, the Logarithms to the prime numbers, 2. 3. 7. 11. 13. 17. 19. 23. 29. 31. 37. 41. 43. 47. &c. and then by using composition and division, we may easily find as many logarithms as we please, or examine any logarithm in the table.

ANOTHER METHOD OF COMPUTING LOGARITHMS.

The construction of Logarithm according to the preceeding rules, given by the repeated extraction of Roots, is tedious; the simplest method yet known is the following.

* *To make a Table of Logarithms.*

1. Write for the logarithm of 1, a cypher for the index, and as many cyphers for the decimal part of the logarithm as you

* See "Mr. Briggs differential method of constructing logarithms."

would wish the logarithms to be extended: for the logarithm of 10, write an unit, with the same number of cyphers; for the logarithm of 100, put 2, with as many cyphers; for 1000, put 3 for the index, with as many cyphers; for 10000, put 4 for the index, &c.

2. Find the difference between some two logarithms above 1000, or rather 10000, that differ by unity; multiply the two numbers together, and that product by the constant decimal 434294481903251828896 &c.

3. Divide the last product by the Arithmetical mean between the two numbers, and the quotient will be the logarithm of the *difference* of the two numbers.

Thus; Let it be required to find the difference between the logarithm of 10000, and 10001. The product of these two numbers is 100010000, which multiplied by 4343, &c. gives 43434343, which divided by 10000.5, the Arithmetical mean between the two numbers, gives 4343. Now if to the *logarithm* of 1000 which is 4.0000000,4343 be added, we shall have 4.0000434, the true logarithm of 1000 to 7 places.

Having thus found the difference of the logarithms of any two numbers differing by unity, or 1, and consequently, some of the logarithms, by dividing the difference found by the Arithmetical mean of any two numbers differing by 1, we shall have the difference of their logarithms.

Thus; to find the difference between the logarithm of 274, and 275, divide 4343, the difference of the logarithm of 10000, and 10001, by 274.5 the quotient will be 1582, the difference required, which added to 2. 4377506 the logarithm of 274, gives 2. 4393327 for the true logarithm of 275.

5. Having by this Means found a few of the prime logarithms, the rest are made by Addition and Subtraction; and having made the canon upward, above 1000 to 10000, by consequence it is made for all inferior numbers.

Directions for taking Logarithms and their numbers from the Table.

To find the Logarithm of any number consisting of 4 figures.

Look for the number whose logarithm is required in the column of numbers, and against this number, its logarithm will be found.

Thus; the logarithm of 1234, is 3,0913151, so that any number under 10000, may be easily found by inspection.

But if the number is greater than 10,000, but less than 10,000,000

Cut off four figures on the left of the given number, and seek the logarithm in the table; add as many units to the index, as

there are figures remaining on the right : subtract the logarithm found, from the next following it in the table :—then as the difference of numbers in the Canon, is to the tabular distance of the logarithms answering to them, so are the remaining figures of the given number to the logarithmick difference : which if it be added to the logarithm before found, the sum will be the logarithm required. Thus, let the logarithm of the number 92375 be required. Cut off the four figures 9237, and to the index of the logarithm corresponding to them, add one unit, because one figure is cut off on the right.

Then from the logarithm of the next greater, number 9238, = 3.9655780

Subtract the logarithm of the required number 9237 = 39655309

10 471

*Then as 10 : 471 : : 5 : 235

Now to the log. of 4.9655309

Add 235 the difference found.

And the sum is the log. 4.9655544 required.

Or more briefly ; find the logarithm of the first four figures as before ; then multiply the common difference which stands against it, by the remaining figures of the given number, from the product cut off as many figures at the right hand, as you multiplied by, and add the remainder to the logarithm before found, fitting it with a proper index.

Thus, 471 X 5 = 2355, cut off 5, and add 235.

To find the Logarithm of a dicimal fraction.

The logarithm of a dicimal, is the same as that of a whole number excepting the *index*.

Take out then, the logarithm of a whole number consisting of the same figures, observing to make the negative index equal to the distance of the first significant figure of the fraction, from the place of units.

The log. of	0.07643	is — 2. 8832639	or 8. 8832639
of	0.00759	— 3. 4132998	or 7. 4132998
of	0.0006278	— 4. 7978213	or 6. 7978213

To find the logarithm of a mixed dicimal number.

Find the logarithm, in the same manner as if all the figures

*If one figure is cut off, say as 10 is to the diff. of Log. if two figures are cut off, as 100 is to the diff. if three, as 1000, &c.

were integers ; and then prefix the index which belongs to the integral part.

Thus, the logarithm of 39.68 is 1. 5985717

Here the index is *one*, becouse 1 is the index of the logarithm of every number greater than 10, and less than 100.

To find the logarithm of a vulgar fraction.

Subtract the logarithm of the denomenator from that of the numerator. The difference will be the logarithm of the fraction.

To find the logarithm of $\frac{37}{94}$

Logarithm of	37	1. 5682017
Logarithm of	94	1. 9731279
Diff. Log. of	$\frac{37}{94}$ —	1. 5950738

where the index 1 is negative.

To find in the Table the natural number to any logarithm.

This is to be done by the reverse method to the former, viz. by searching for the proposed logarithm among those in the Table, and taking out the corresponding number by inspection, in which the proper number of integers is to be pointed off, viz. 1 more than the units of the affirmative index.

To find the number corresponding to a logarithm greater than any in the Table. First, from the given logarithm subtract the logarithm of 10, of 100, or 1000, or 10,000, till you have a logarithm that will come within the compass of the table, find the number corresponding to this and multiply it by 10, or 100, or 1000, or 10,000, the product is the number required. Suppose for instance the number corresponding to the logarithm 7. 7589875 be required ; subtract the logarithm of the number 10.000 which, is 4.0000000 from 7.7589875, there remains 3 7589875, the number corresponding to which is 5741, this multiplied by 10,000 gives the number answering to the given logarithm.

METHOD OF CALCULATING BY LOGARITHMS.

MULTIPLICATION.

RULE.

Take out the logarithms of the factors from the table, add them together and their sum will be the logarithm of the product required. Then by means of the Table take out the natural number answering to the sum for the product saught.

EXAMPLES.

1. Multiply 45 by 27

Numbers.	Logarithms.
45	1. 6532125
27	1. 4313638
product. 1215	3. 0845763

2. Multiply 709 by 13

Numbers.	Logarithms.
709	2. 8506462
13	1. 1139434
Ans. 9217	3. 9645896

3. Multiply.

Numbers.	Logarithms.
23.14	1. 3643634
75.99	1 8807564
Ans. 1758.4086	3. 2451198

4. Multiply 3.7 by 3. 7

Numbers.	Logarithms.
3.7	0· 5682017
3.7	0. 5682017
Ans. 13.69	1. 1364034

5. Multiply 3.586, 2.1046, 0.8372, 0.0294, all together.

Numbers.	Logarithms.	
3 586	0. 5546103	
2.1046	0. 3231696	Here the 2 to be carried cancels the — 2 and there remains the —1 to be set down.
0.8372	— 1 9228292	
0.0294	— 2. 4683473	
0 1857618	*— 1. 2689564.	

6. What cost 87 pounds of grean tea, at $ 2 12 cts. pr. lb?

Numbers.	Logarithms
212	0. 3263359
87	1 9395193
Ans. $ 184. 44	2. 2658552

7. What cost 160 bushels oats, at 50 cts. pr. bushel ? Ans. $ 80
8. What cost 250 bushels of wheat, at $ 1 60 pr. bushel ? Ans. $ 400
9. What cost 1260 lb. rice, at 5 cts. pr. lb. ? Ans. $ 63

DIVISION BY LOGARITHMS.

RULE.

From the logarithm of the dividend, subtract the logarithm of the divisor, and the number answering to the remainder will be the quotient required.

* NOTE. In every operation what is carried from the dicimal part of the logarithm, to its index, is affirmative, and is therefore to be added to the index when it is affirmative, but subtracted when it is negative.

EXAMPLES.

1. Divide 15811 by 163

	Numbers.	Logarithms.
Dividend	15811	4. 1989593
Divisor	163	2. 2121876
Ans.	97	1. 9867717

2. Divide 163 by 8.18

	Numbers.	Logarithm.
Dividend	163	2. 2121876
Divisor	8.18	0. 9127538
quotient	19.926	1. 2994338

3. Divide 100000 by 100

	Number.	Logarithms.
Dividend	100000	5. 0000000
Divisor	100	2. 0000000
quotient	1000	3. 0000000

4. Divide 1000000 by 1000

	Number.	Logarithms.
Divid.	1000000	6. 0000000
Divisor	1000	3. 0000000
quotient	1000	3. 0000000

5. A ship took a prize worth $ 3960, it is required to divide it equally among the sailors, who are 264 in number.

What did each man share in the prize? Ans. $ 15

If 125 lb. of sugar cost $ 26. 25cts. what is that pr. lb.?

Ans. 21 cts.

PROPORTION BY LOGARITHMS.

RULE.

If the proportion be direct, add the logarithms of the second and third terms, and from the sum subtract the logarithm of the first term. The remainder will be the logarithm of the term required.

If the proportion be inverse, add the logarithms of the first and second terms, and from the sum subtract the logarithm of the third. The remainder will be the logarithm to the required term.

EXAMPLES.

Find a fourth proportional to 7964, 378, and 27960.

	Numbers.	Logarithms.
Second term	378	2.5774918
Third term	27960	4.4465372
		7.0240290
First term	7964	3.9011313
Fourth term	1327	3.1228977

Find a fourth proportional to 768, 381, and 9780.

	Numbers.	Logarithms.
Second term	381	2.5809250
Third term	9780	3.9903389
		6.5712639
First term	768	2.8853612
Fourth term	4852	3.6859027

*ARITHMETICAL COMPLEMENT.

The difference between a given number, and 10, or 100, or 1000, &c. is called the ARITHMETICAL COMPLEMENT of that number.

To obtain the ARITHMETICAL COMPLEMENT of a number, subtract the right hand significant figure from 10, and each of the other figures from 9.

N. B. In taking the Arithmetical Complement of a logarithm, if the index is negative, it must be added to 9; for adding a negative quantity, is the same as subtracting a positive one. The difference between — 4 and + 8, is not 4, but 12.

THE ARITHMETICAL COMPLEMENT.

of 5.2473621	is 4.7526379
of 1.9864362	is 8.0135638
of 0.6462310	is 9.3547689
of–2.7064923	is11.2935076

In the following proportion, the calculation is made in both ways.

1. If the profit on $ 2625 employed in trade, is $ 525, what is the profit on $ 7875 ?

*Note. The principal use of the Arithmetical Complement, is, in working proportions by Logarithms; for by this they may be performed by merely *adding* together the several terms of the proportion.

By the common method.			By the Arith'l Complement.		
Second term	525	2.7201593	Second term	525	2.7201593
Third term	7875	3.8962506	Third term	7875	3.8962506
		6.6164099	First term, a. c.		6.5808707
First term	2625	3.4191293			
			Fourth term	1575	3.1972806
Fourth term	$1575	3.1972806			

2. If $ 567 gain $ 81, what will be the gain on $ 1701 ?

As 567 stock	a. c.	7.2464169
Is to 81 profit		1 9084850
So is 1701 stock		3.2307043
To 243 profit		2.3856062

3. If the interest on $ 450 for one year, is $ 27, what will be the interest on $ 1150 for the same time. Ans. $ 69 ?

4. When a pipe of wine costs $ 252, what is the value of $17\frac{1}{2}$ gallons ? Ans. $ 35 ?

5. Bought 721 yards broad cloth, at the rate of $ 65 for every 13 yards, what did the whole come to ? Ans. $ 3605 ?

VULGAR FRACTIONS.

A vulgar fraction is any assignable part of a unit or integer, expressed by two numbers, placed one above the other with a line drawn between them, as $\frac{1}{4}$ one fourth, $\frac{2}{3}$ two thirds, &c.

The number above the line is called the numerator, and that below the line, the denominator.

A fraction is said to be in its lowest terms, when it is expressed by the least numbers possible, as $\frac{4}{8}$ when reduced to its lowest terms will be $\frac{1}{2}$; and $\frac{9}{12}$ is equal to $\frac{3}{4}$ &c.

CASE I.

To reduce fractions to their lowest terms.

RULE.

Divide both the numerator and denominator, by any number which will divide them without a remainder, and the quotients again in the same manner, till it appears that there is no number greater than 1, which will divide them again.

EXAMPLES.

1. Reduce $\frac{110}{238}$ to its lowest terms. Ans. $\frac{5}{12}$
2. Reduce $\frac{297}{891}$ to its lowest terms. Ans. $\frac{1}{3}$
3. Reduce $\frac{488}{560}$ to its lowest terms. Ans. $\frac{6}{7}$

4. Reduce $\frac{216}{288}$ to its lowest terms. Ans. $\frac{3}{4}$
5. Reduce $\frac{1344}{1536}$ to its lowest terms. Ans. $\frac{7}{8}$
Abbreviate $\frac{68966300}{307001600}$ as much as possible. Ans. $\frac{431105}{2293376}$

CASE II.

To reduce the value of a fraction to the known parts of an integer.

RULE.

Multiply the numerator by the common parts of the integer and divide by the denominator.

EXAMPLES.

1. What is the value of $\frac{2}{3}$ of a pound sterling ?

2
20 shillings in a pound.

Denominator 3)40(13s. 4d. Ans.
3

10
9

1
12 pence in a shilling.

3)12(4d.
12

2. What is the value of $\frac{3}{4}$ pound sterling ?
Ans. £0 15s. 0
3. Reduce $\frac{7}{9}$ of an hundred weight to its proper quantity.
Ans. 3qrs. 3lb. 1oz. $12\frac{4}{9}$dr.
4. Reduce $\frac{3}{5}$ of lb. troy to its proper quantity.
Ans. 7oz. 4pwt.
5. Reduce $\frac{4}{5}$ of a mile to its proper quantity.
Ans. 6 fur. 16 po.
6. Reduce $\frac{3}{5}$ of a month to its proper quantity.
Ans. 2 wks. 2 d. 19 h. 12 m.

CASE III.

To reduce a fraction of one denomination to that of another, but greater, retaining the same value.

RULE.

Reduce the given quantity to the lowest term mentioned, for a numerator ; then reduce the integral part to the same term, for a denominator ; which will form the fraction required.

EXAMPLES.

1. Reduce 15s. 8d. 2qrs. to the fraction of a pound.

20	Integral part.	15.	8	2 given sum.
12		12		
240		188		
4		4		
960	Denominator.	754	Num.	$\frac{754}{960} = \frac{377}{480}£$.

2. What part of a pound sterling is 13s. 4d. ? Ans. $\frac{2}{3}$
3. What part of a hundred weight is 3 qrs. 14lb. ? Ans. $\frac{1}{4}$
4. What part of a yard is 2 qrs. 1 nail ? Ans. $\frac{9}{16}$
5. What part of a common year is 3 weeks, and 4 days ? Ans. $\frac{5}{73}$
6. What part of a mile is 6 fur. 26 po. 3 yds. 2 ft. ?

fur.	*po.*	*yd.*	*ft.*		*feet.*	
6	26	3	2	=	4400	Num.
a mile				=	5280	Denom.

$\frac{4400}{5280} = \frac{5}{6}$

7. What part of a hhd. of wine is 54 gallons ? Ans. $\frac{6}{7}$
8. What part of a day is 16 h. 36 min. $55\frac{5}{13}$ sec. Ans. $\frac{6}{13}$
9. What part of a shilling is $4\frac{1}{2}$d. ? Ans. $\frac{3}{8}$
10. What part of an acre is 3 roods, and 20 rods ? Ans. $\frac{7}{8}$
11. What part of a pound troy is 10 oz. 11 pwt. 16 grs ? Ans. $\frac{127}{144}$
12. What part of a cord is $116\frac{4}{11}$ feet ? Ans. $\frac{10}{11}$

DECIMAL FRACTIONS.

A Decimal Fraction is that whose denominator is an unit, with a cyper, or cyphers annexed to it, thus $\frac{3}{10}$ $\frac{4}{100}$ $\frac{5}{1000}$ &c. &c.

As the integer is always divided either into 10, 100, 1000 &c. equal parts ; consequently the denominator will always be either 10, 100, 1000, 10,000, &c. which being implied, need not be expressed ; for the true value of a *Decimal Fraction* is properly expressed by writing the numerator, only with a point before it on the left.

Thus, instead of $\left\{\begin{array}{l}\frac{5}{10}\\ \frac{75}{100}\\ \frac{837}{1000}\end{array}\right\}$ write $\left\{\begin{array}{l}.5\\ .75\\ .837\ \text{&c.}\end{array}\right.$

But if the denominator has not so many places as the denominator has cyphers, prefix so many cyphers on the left as will make up the deficiency.

Thus, for $\left\{ \frac{5}{100} \; \frac{6}{1000} \; \frac{7}{10000} \right\}$ write $\left\{ \begin{matrix} .05 \\ .006 \\ .0007 \text{ &c.} \end{matrix} \right.$

Decimals are reckoned from the left hand towards the right, and the value of each figure is determined by its distance from the place of units, if it be in the first place after unit,s (or seperating point) it signifies tenth's; if in the second, hundreth's &c.decreasing towards the right in a tend fold proportion, as in the following

TABLE.

Millions	C. Thousands	X. Thousands	Thousands	Hundreds	Tens	Units	Tenth parts	Hundreth parts	Thousandth parts	X. Thousandth parts	C. Thousandth parts	Millionth parts
7	6	5	4	3	2	1	2	3	4	5	6	7
Whole Numbers.							*Decimals.*					

Cyphers placed at the right hand of a decimal fraction do not alter its value, since every significant figure continues to possess the same place; so, 5, 50, 500, 5000, are all of the same value, and each equal to $\frac{5}{10}$ or $\frac{1}{2}$.

But cyphers placed at the left hand of decimals, decrease their value in a ten fold proportion, by removing them farther from the decimal point. Thus, .5 .05 .005 .0005 &c. are 5 tenth, 5 hundredth, 5 thousandth paats, &c. respectively

ADDITION OF DECIMALS.

RULE.

1. Place the numbers, whether mixed, or pure decimals, under each other, according to their local value.

2. Find their sum as in whole numbers, and point off so many places for decimals, as are equal to the greatest number of decimal parts in any of the given numbers.

Note. The point prefixed is called the SEPERATRIX, from its separating the *Integral* from the *Decimal* part.

EXAMPLES.

1. What is the sum of 276, 39.213, 72014.9, 417, 5032 and 2214.298?

276.
39.213
72014.9
417.
5032.
2214.298

Ans. 79993.411

Hence we may observe that the Denominations of FEDERAL MONEY, as determined by an Act of Congress Aug. 8, 1786, are in a *decimal* ratio, and subject to one, and the same law of notation, and consequently of operation.

For since a dollar is the *integer* or unit, and a dime being the tenth, and a cent the hundredth, and a mill the thousandth part of a dollar, it is evident that any number of dollars, dimes, cents, and mills, is simply the expression of dollars, and decimal parts of a dollar: Thus, 15 dolls. 8 dimes, 3 cents, 5 mills is expres ed in decimals. 15,835, or $15.\frac{835}{1000}$

2. What is the sum of the following expressions of money, viz. $7530, $16.201, $3.0142, $957.13, $6.72819, $.03014? Ans. $8513.10353

3. Required the sum of .014, .9816, .32, .15914, .72913, and .0047, Ans. 2.20857

4. What is the sum total of 27.148, 918.73, 14016, 294304, .7138, and 221.7? Ans. 309488.2918

5. Required the sum of 312.984, 21.3918, 2700.42, 3.153, 27.2, and 581.06, Ans. 3646.2088

SUBTRACTION OF DECIMALS.

RULE.

1. Set the less number under the greater in the same manner as in addition.

2. Then subtract as in whole numbers, and place the decimal point in the remainder directly under the other points.

EXAMPLES.

(1) Dollars.		(2) Feet.	
From	21.481	From	125.64
Take	4.90142	Take	95.58756
Rem	16.57958	Rem.	30.05244

3. From 270.2 subtract 75.4075 Ans. 194.7925
4. From 2.73 subtract 1.9185 Ans. 0.8115
5. From .9173 subtract .2138 Ans. .7035
6. From 407, subtract 91.713 Ans. 315.287
7. From 800.135 subtract 16.37 Ans. 783.765
8. What number added to 9.999999 will make 10 ?
Ans. One millionth part of an unit.

MULTIPLICATION OF DECIMALS.

RULE.

1. Set down the factors under each other, and multiply them as in whole numbers.

2. And from the product, on the right, point as many figures for decimals, as there are decimal places in both the factors. But if there be not so many figures in the product as there ought to be decimals, prefix the proper number of cyphers to supply the defect.

EXAMPLES.

1. Multiply	91.78	2. Multiply	520.3
by	.381	by	.417
product	34.96818	product	216.9651

3. Multiply .217 by .0431 Ans. .0093527
4. Multiply 51.6 by 21 Ans. 1083.6
5. Multiply .051 by .0091 Ans. 0004641
6. What will 6.21 yards of cloth amount to, at 2 dollars 3 cents 5 mills pr. yard ? Ans. 2.325 X 6.21 = $36.9954
7. What cost 27.13 lb. of green tea, at $1 12½ pr. lb. ?
Ans. $30.52125
8. What cost 53¾ lb. sugar, at 14½ cents pr. lb. ?
Ans. $7.79375
9. What will 12.125 acres amount to, at $65.25 pr. acre ?
Ans. $791.15625
10. What is the value of .7584 ounces of gold, at $17.777 pr oz. ? Ans. $13.4820768

To multiply by 10, 100, 1000, &c. remove the separatrix, in the multiplicand, so many places to the right, as the multiplier has cyphers.

EXAMPLES.

The product of .7853 { Into 10 = 7.8530 ; " 100 = 78 5300 ; " 1000 = 785.3000 ; " 10000 = 7853.0000 } Or { 7.853 ; 78 53 ; 785.3 ; 7853 }

DIVISION OF DECIMALS.

RULE.

Divide as in whole numbers; and to know how many decimals to point off in the quotient, observe the following rule.

1. There must be as many decimals in the dividend, as in both the divisor and quotient; therefore point off for decimals in the quotient so many figures, as the decimal places in the dividend exceed those in the divisor.

2. If the figures in the quotient are not so many as the rule requires, supply the defect by prefixing cyphers.

3. But if the decimal places in the divisor exceed those in dividend, add cyphers as decimals to the dividend, till the number of decimals in the dividend be equal to those in the divisor, and the quotient will be *integers* till all these decimals are used. And, in case of a remainder, after all the figures of the dividend are used, and more figures are wanted in the quotient, annex cyphers to the remainder, to continue the division to any degree of exactness.

EXAMPLES.

1. Divide 3424.6056 by 43.6

```
43.6)3424·6056(78.546
     3052
     ----
      3726
      3488
      ----
       2380
       2180
       ----
        2005
        1744
        ----
         2616
         2616
            0 remains.
```

2. Divide 3877875 by.675 Ans. 5745000
3. Divide 7.25406 by 957 Ans. .00758
4. Divide 56 cts. by 1 doll. 12 cts. Ans .5
5. If 6.21 yards of cloth cost $14.45825, what cost one yard? Ans. $2.325
6. What is the value of 1 lb. green tea, when 27.13lb. cost $30.52125 Ans. $ .12½
7. If an ounce of gold be worth $17.777, what is the value of .7584 of an ounce at the same rate? Ans. $13.4820768

Note. When decimals, or whole numbers, are to be divided by 10, 100,1000, &c. remove the separating point in the dividend, so many places towards the left, as there are cyphers in the divisor.

EXAMPLES.

7853 divided by { 10 the quotient is 785.3 / 100 " " 78.53 / 1000 " " 7 853 / 10000 " " .7853

REDUCTION OF DECIMALS.

CASE I

To reduce a Vulgar Fraction to its equivalent Decimal.

RULE.

Annex cyphers to the numerator, and divide by the denominator, the quotient will be the decimal required.

Note. So many places must be pointed off in the quotient, as there were cyphers annexed to the numerator; but if there be not so many places of figures in the quotient, supply the defect by prefixing cyphers on the left of the said quotient.

EXAMPLES.

1. Reduce $\frac{1}{8}$ to its equivalent decimal.

8)1.000

.125 Ans.

2. Reduce $\frac{1}{4}$ to a decimal. Ans. .25
3. Reduce $\frac{1}{2}$ to a decimal. Ans. .5
4. Reduce $\frac{1}{5}$ to a decimal. Ans. .2
5. Rednce $\frac{17}{20}$ to a decimal. Ans. .85
6. What decimal is equivalent to $\frac{1}{40}$? Ans. .025
7. Bring $\frac{3}{32}$ to a decimal. Ans. .09375
8. Find the decimal expression of $\frac{9}{1125}$. Ans. .008

CASE II.

To reduce numbers of different denominations to their equivalent decimal values.

RULE.

Bring the given denominations to a vulgar fraction (by Case III. page 57) and reduce said fraction to its equivalent decimal value.

OR

Rule 2. Write the several denominations above each other in their order, placing the highest denomination at the bottom; then, beginning at the top, divide each denomination by its value in the next superiour denomination; the last quotient will be the decimal required.

EXAMPLES.

1. Reduce 15s. 9d. 3qrs. to the decimal of a pound.

15
12

189
4
---

20 X 12 X 4 = 960)759.000000(.790625 Answer.
6720

8700
8640

6000
5760

2400
1920

4800
4800

By RULE 2.

4	3.
12	9.75
20	15.8125

Decimal .790625 required.

2. Reduce 12s. 6d. 3qrs. to the decimal of a pound. Ans. .628125
3. Reduce 9s. to the decimal of a pound. Ans. .45
4. Reduce 19s. 5d 2 qrs. to the decimal of a pound. Ans. .9727916
5. Reduce 3s. 9d. to the decimal of a dollar. Ans. .625
6. Reduce 7 oz. 19 pwt. to the decimal of a lb. Troy. Ans. .6625
7. Reduce 3 qrs. 21 lb. to the decimal of an cwt. Ans. .9375

SIMPLE INTEREST.

INTEREST is the premium allowed for the loan of money, relative to which there are four particulars.

First, the PRINCIPAL, or sum at interest.

Second, the RATE PER CENT, or interest of £100, or dollars for one year.

Fourth, the amount, which is the sum of principal and interest, added together.

Interest is either Simple, or Compound.

SIMPLE INTEREST is that which arises from the principal only.

RULE.

1. Multiply the principal by the rate, and divide the product by 100. The quotient is the answer for one year.

2. Multiply the interest for one year by the given number of years, and the product is the answer for that time.

3. If there be parts of a year, as months, or days, work for the months by the aliquot parts of a year, and for the days by simple proportion.

Note. Solutions in SIMPLE INTEREST exhibit the principle, that the interest of $ 100, or £100, for 1 year, 2 years, 3 years, &c. correspond to a series of numbers in arithmetical proportion; from whence will naturally arise the following Theorem, that

If two ranks of numbers have the same ratio between every pair of correspondents, then the numbers themselves, their correspondent sums, and correspondent differences, will have the same common ratio.

			Principle.	Int.
Thus	1 : 6	or	100 :	6
	2 : 12		200 :	12
	3 : 19		300 :	18
	4 : 24		400 :	24
	10 : 60		1000 :	60

In the first pair of ranks, the ratio between any two correspondents, is 6; therefore, taking any number from the first rank,—suppose 2; then 2 : 12 : : 10 : 60; and 2 : 12 : : 8 : 48; or 2 : 12 : : 3 : 18. So also in the 2d. pair of ranks;—300 : 18 : : 400 : 24; and 300 : 18 : : 1000 : 60; or 300 : 18 : : 700 : 42 &c.

EXAMPLES.

What is the interest of £420 for 1 year, at £7 pr. cent, pr. annum. Ans. £29 8*s*.

(1)

420
7

29 | 40
20

3 | 00

Ans. £29 8*s*.

What is the interest of £745 16*s*. for 1 year, at 7 pr. cent? Ans. £52 4*s*. 1¼*d*.

(2)

745 16
7

52 | 20 12
20

4 | 12
12

1 | 44
4

1 | 76

3. What is the interest of £800, for 1 year, at 7 pr. cent, pr. annum? Ans. £56

4. What is the interest of £76, for 1 year, at 5 pr. cent? Ans. £3 16*s*.

5. Whst is the interest of £211 5*s*. for 1 year, at 7 pr. cent? Ans. £14 15*s*. 9*d*.

6. What is the interest of £472 1*s*. for 1 year, at 7 pr. cent, pr. annum? Ans. £33 0*s*. 10¼*d*.

7. What is the interest of £270 10*s*. 6*d*. for 1 year, at 5 pr. cent, pr. annum? Ans. £13 10*s*, 6¼*d*.

8. What is the interest of $ 542, for 1 year, at 7 pr. cent, pr. annum? Ans. $ 37 94cts.

9. What is the interest of $ 800, for one year, at 6 pr. cent? Ans. $ 48 00

10. What is the interest of $ 875 35cents, for one year, at 6 pr. cent? Ans. $ 52 52

11. What is the amount of a bond for $ 387 50 cents, for one year, at 6 pr cent? Ans. $ 410 75cts.

Note 1. When the principal consists of dollars, multiply by the rate pr. cent; the product will be the interest for 1 year, in cents.

Note 2. When the amount is required, add the principal to the interest.

CASE II.

1. If the interest required be for years, months, and days, take $\frac{1}{2}$ the number of months, and set it under the place of tens, take $\frac{1}{6}$ part of the number of days and put it under the place of units for a multiplier.

2. For the odd days, (if any) see what proportion they bear to the week, and divide the principal by this proportion, and then proceed to multiply as in whole numbers ; the product will be the interest for the whole time, in dollars, cents, and mills.

EXAMPLES.

Required the interest of $ 10 44cts. for 3 years 5 months and 10 days, at 6 pr. cent, pr. annum.

$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$ | 10 44 — *y. m. d.* 3 5 10

2 06 — 12

3 48 — $\frac{1}{2}$ | 41

3 48 — 20 + 1 = 30 days.

62 64 — 10

2088

2,15,7,60 Ans. 2 dol. 15 cts 7 mills, $\frac{60}{100}$m. — 6 | 40 | 6 — 36

4 odd days = $\frac{2}{3}$ of week.

2. What will 780 dols. amount to, at 6 pr. cent, in 5 years 7 months and 12 days? Ans. $ 975 99 cts.

3. What is the interest of $ 824 15 cts. for 22 weeks, at 7 pr. cent? Ans. $ 24 40 cts. 7 m.

4. What is the interest of $ 438 24 for 4 years 9 months and 14 days, at 7 pr. cent? Ans. $ 146 90cts. 7m.

CASE III.

When there is a fraction as $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$, &c. in the rate pr. cent.

RULE.

Multiply the principal by the rate pr. cent. to the product add $\frac{1}{4}$ $\frac{1}{2}$, &c. of said principal, and divide by 100 for the interest required.

EXAMPLES.

1. What is the interest of $ 428 for one year, at 6¼ pr. cent, pr. annum?

	6¼	
½	428	
	6¼	
	2568	
½	214	for half.
	107	for one fourth.

$ 28,89 cts. Ans.

2. What is the interest of 216*l.* 5*s.* for one year, at 5½ pr. cent? Ans. 11*l.* 17*s.* 10½

3. What is the interest of $ 300 for one year, at 6¼ pr. cent, pr. annum? Ans. $ 18 75 cts.

CASE IV.

To find the interest of any sum of money, for any number of years and parts of a year.

1. Find the interest for 1 year, and multiply this by the given number of years.

2. If there be months and days, work for the months, by the aliquot parts of a year, and for the days, by simple proportion.

EXAMPLES.

1. What is the interest of 64 dols. 58 cts. for 3 years 5 months and 10 days, at 5 pr. cent? Ans. $ 11 12 cts. 1$\frac{9}{10}$m.

	64 58	
	5	
	32290	Interest for one year in cents.
	3	
	96870	for 3 years.
4 mo. = ⅓	10763	for 4 months.
1 mo. = ¼	2690	for 1 month.
10 days = ⅓	896	for 10 days.

Ans. $ 11,12,19 = 1112 cts. or $ 11,12 c. 1$\frac{9}{10}$m.

2. What is the interest of $ 325 41 cts. for 3 years, and 4 months, at 5 pr. cent? Ans. $ 54 23 cts. 5 m.

3. What will 3000*l.* amount to in 12 years and 10 months, at 6 pr. cent? Ans. 5310*l.*

4. What will $ 730 amount to at 6 pr. cent, in 5 years 7 months and 12 days? Ans. $ 975 99 cts.

INSURANCE, COMMISSION, AND BROKERAGE.

Are allowances to Insurers, Factors, and Brokers, at a stipulated rate pr. cent, as a premium for their services.

The same rules used in simple interest, apply to each of these cases.

1. What is the commission on £287 10 s. at 3½ pr. cent?
Ans. £10 1 s. 3 d.

2. A Broker sells goods for me to the amount of £2575 17 s. 6 d. what is the brokerage at 4 s. pr. cent? Ans. £5 3 s. 0½.

3. What is the insurance of a house, valued at $ 1853, at 75 cts. pr. cent? Ans. $ 13 89¾ cts.

DISCOUNT.

DISCOUNT is an allowance made for the payment of any sum of money before it becomes due; and is the difference between that sum due some time hence, and its present worth.

The *present worth* of any sum, or debt, due some time hence, is such a sum, as, if put to interest, would in that time and at that rate pr. cent, for which the discount is to be made, amount to the sum, or debt then due.

What remains after the discount is deducted, is the *present worth.*

RULE.

As the amount of 100*l.* or 100 dols. at the given rate and time: is to the interest of 100 at the same rate and time, so is the given sum to the discount.

Subtract the discount from the whole debt, and the remainder will be the present worth.

Or; as the amount of 100, is to 100, so is the given sum to the present worth.

PROOF.

Find the amount of the present worth for the time and rate proposed, which must equal the given sum, or debt.

EXAMPLE.

What must be discounted for the ready payment of 100 dols. due a year hence, at 6 pr. cent pr. annum?

As 106 : 6 : : 100 : 5 66 Ans.

100,00 years sum.
5,66 discount.

$ 94,34 the present worth.

2. What sum in ready money, will discharge a debt of £925, due 1 year and 8 months hence, at 6 pr. cent ?

£	£	£	£	s.	d.	
As 110 :	100 : :	925 :	840	18	2	Ans.

3. What is the present worth of 600 dols. due 4 years hence, at 5 pr. cent? Ans. $500

4. *What is the present worth of £100, one quarter due in 3 months, and the remaining 3 quarters, in 5 months, discount 7 pr. cent? Ans. £97 8 s. 10 d. +

5. What is the difference between the interest of $1204, at 5 pr. cent pr. annum, for 8 years, and the discount of the same, for the same time and rate ? Ans. $ 137 60 cts.

EQUATION OF PAYMENTS.

Is finding the equated time, to pay at once, several debts due at different times, so that no loss shall be sustained by either party.

RULE.

Multiply each payment by its time, add the several products together, and divide the sum by the whole debt ; the quotient will be the answer.

PROOF.

The interest of the sum, payable at the equated time, will equal the interest of the several payments.

EXAMPLES.

1. A owes B. $ 380, to be paid as follows, viz. 100 in six months, 120 in 7 months, and 160 in 10 months ; What is the equated time for the payment of the whole debt ?

100 X 6 = 600
120 X 7 = 840
160 X 10 = 1600

380)3040(8 months. Ans.

3. The firm of B. & C. owe to the firm of B. & Co. the sum of $ 300 ; payments as follows : 100 in 3 months, 100 in 4 months, and 100 in 6 months ; required the equated time for the payment of the whole debt ? Ans. $4\frac{1}{3}$ months.

Note. When Sundry sums are to be paid at different times, find the rebate, or present worth of each payment separately, then add them into one sum.

3. P. owes C. £420, which will be due 6 months hence, but P. is willing to pay £60 *now*, provided he can have the rest remain unpaid, a longer time than 6 months; when must it be paid? Ans 7 months.

FELLOWSHIP.

Fellowship, is a rule, by which Merchants, &c. trading in company with a joint stock, are enabled to ascertain each persons particular share of the gain, or loss, in proportion to his share in the joint stock.

Fellowship is either single, or compound.

Single Fellowship.

Is when the several stocks in company are considered without regard to time.

RULE.

As the whole stock is to each mans share in stock, so is the whole gain, or loss, to his share of the gain, or loss.

PROOF.

The sum of the several shares must equal the gain, or loss.

EXAMPLES.

1. A. B. and C. put in stock, and gain 800; A's stock was 1200, B. 4800, and C. 2000: What was each mans share of the gain?

A. 1200
B. 4800
C. 2000

8000

As 800 : { 1200 / 4800 / 2000 } : : 800 : { 120 / 480 / 200 } share of A. / —— B. / —— C. Ans.

2. D. E. and F. trading, gained 120*l.*; D's stock was 140*l.* E's 300, and F's 160: What was each mans share of the gain.

Ans. D's, 28*l.* E's, 60*l.* and F's, 32*l.*

3. Four men, trading with a stock of $ 2400, and gain in 2 years, twice as much, and $ 160 more; A's stock was 400, B's 740, C's 820; what was D's stock, and how much did each man gain.

Note. By this rule, also, a bankrupt estate, may be divided among his creditors.

Ans.

	cts.	mills.
D's stock, $ 440		
A. gained $826	66	6
B. gained $1529	33	3
C. gained $1694	66	6
D. gained $909	33	3

COMPOUND FELLOWSHIP.

Is when the respective stocks of several partners in company are considered with time.

RULE.

Multiply each mans stock by its time, and add the several products together, then

As the sum of the products is to each particular product; so is the whole gain, or loss, to its share of the gain, or loss.

EXAMPLES.

1. Three Merchants trade together. A's stock is 120*l.* for 9 months, B's 100*l.* for 16 months, C's 100*l.* for 14 months, and they gain 100*l.*: What is each mans share.

A's stock 120 X 9 = 1080
B's —— 100 X 16 = 1600
C's —— 100 X 14 = 1400

4080 sum.

As 4080 : { 1080 / 1600 / 1400 } : : 100*l.* { 26*l.* 9*s.* 4*d.* $\frac{1}{4}$ A. share. / 39*l.* 4*s.* 3*d.* $\frac{3}{4}$ B. share. / 34*l.* 6*s.* 3*d.* $\frac{1}{4}$ C. share.

2. Three Merchants join in company. H. puts in $ 620 for 8 months, L. 950*l.* for 11 months, and M. $ 730 for 13 months, and they gain 1800: What was each mans share?

Ans.

A's share	358	55	$4\frac{54}{249}$
B's ——	755	42	$1\frac{171}{249}$
C's ——	686	02	$4\frac{24}{249}$

3. A. began trade, January 1, 1818, with a capital of $ 1000, and on the first of March following, took in B. as a partner, with a capital of $ 1500; three months after which, they admit C. as a third partner, who brought into stock $ 2800, and after trading together till the first of the next year, they find their gain to be $ 1776 50: How ought each one share in the profit?

Ans. A's $ 457 $46\frac{364}{700}$ B's $ 571 $83\frac{222}{700}$ C's $ 747 $\frac{136}{700}$.

ALLIGATION.

Teaches how to mix several simples of different qualities, so that the composition shall be of a *mean, or middle* quality. It consists of two parts, *Alligation Medial, and Alligation Alternate.*

ALLIGATION MEDIAL,

Is the method of finding the mean rate, or price of the compound, by having the rates and quantities of the several simples given.

RULE.

Multiply each quantity by its rate ; then divide the sum of the products by the sum of the quantities, the quotient will be the rate of the compound required.

EXAMPLES.

1, Suppose 15 bushels of rye, at 64 cts. pr. bushel, 18 bushels of corn, at 55 cts. pr. bushel, and 21 bushels of oats, at 28 cts. pr. bushel, were mixed, what is the value of the composition pr. bushel ?

bu.	*cts.*	$ *cts.*
15 X	64 =	9 60
18 X	55 =	9 90
21 X	28 =	5 88
54 ÷		25 38 = 47 cts. Ans.

2. If 18 bushels of wheat, worth $ 1 50 pr. bushel, be mixed with 12 bushels of rye, at $ 1 25 pr. bushel, what is a bushel of this mixture worth ? Ans. $ 1 40

3. Suppose a Wine Merchant mixes together 73¼ gallons of wine, at $ 2 16 pr. gallon, 5½ gallons, at $ 2 pr. gallon, and 4½ gallons, at $ 1 80 pr. gallon ; what will a gallon of this mixture be worth, supposing he should accidentally spill a *quart* of water into it ? Ans $1 99$\frac{1}{6}$.

4. A Goldsmith melted together 5 lb. of gold of 22 carats fine, 22 lb. of 21 carats fine, and 1 lb. of alloy ; what is the quality of the mass ? Ans. 19 carats fine.

ALLIGATION ALTERNATE,

Is the method of finding what quantity of any number of ingredients, whose rates are given, will compose a mixture of a given rate : so that it is the reverse of Alligation Medial, and may be proved by it.

RULE.

1. Write the several rates, or prices of the simples, in a column under each other, and the mean rate, or price of the whole mixture, at the left hand.

2. *Connect, or link the price of each simple, or ingredient, which is less than the mean rate, or price of the whole mixture, with one, or any number of those, which are greater than the *mean* rate, and each greater rate, or price with one, or any number of the less.

3. Write the *difference* between the *mean price*, (or mixture rate) and that of each of the simples, opposite to the rates with which they are connected.

4. Then if only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.

EXAMPLES.

1. A Merchant would mix wine at 14s. 19s. 15s. and 22s. pr. gallon, so that the mixture may be worth 18s the gallon; what quantity of each must be taken?

Mean rate 18 { 14 — 4 at 14s. ; 15 — 1 at 15s. ; 19 — 3 at 19s. ; 22 — 4 at 22s. }

Or thus; 18 { 14 — 1+4 | 5 at 14s. ; 15 — 1 | 1 at 15s. ; 19 — 4+3 | 7 at 19s. ; 22 — 4 | 4 at 22s. }

2. How much wine at 6s. pr. gallon, and at 4s. pr. gallon, must be mixed together, that the composition may be worth 5s. pr. gallon? Ans. 12 gallons of each.

3. A Merchant would mix several kinds of spirits together; some at 7s. some at 10s. some at 5s. and some at 13s. pr. gallon; how much of each sort must the mixture contain, so that a gallon of it shall be worth 9s. 2d.?

Ans. 4 gal. at 5s. 1 gal. at 7s. 2 gal. at 10s. and 4 gal. at 13s.

4. How much grain at 2s. 6d. 3s. 8d. 4s. and 4s. 8d. pr. bush. must be mixed together, that the compound may be worth 3s. 10d. pr. bushel?

Ans. 12 at 2s. 6d. 12 at 3s. 8d. 18 at 4s. and 18 at 4s. 8d.

5. How much water at 0 pr. gallon, may be mixed with liquors at 1s. 7s. and 8s. pr. gallon, so that a gallon of the mixture may be sold for 5s. pr. gallon?

Ans. 5 of water, 5 at 1s. 9 at 7s. and 9 at 8s.

---

*Note. By connecting the less rate with the greater, and placing the differences between them, and the mean rate alternately; the quantities resulting, are such, that there must be precisely as much gained by one quantity, as is lost by the other; therefore the gain and loss upon the whole are *equal*, and are exactly the proposed rate. It is also obvious, that questions under this rule, admit of answers differing, *ad infinetum*; for having found one answer, we may find as many more as we please, by only multiplying, or dividing each of the quantities found by 2, 3, 4, &c.

TARE AND TRETT.

TARE and TRETT are allowances made to the buyer, on some particular commodities.

TARE is the weight of the barrel, box, bag, or whatever contains the articles.

TRETT is an allowance of 4 lb. in every 104 lb. for waste, dust, &c.

GROSS, is the weight of the goods together with the barrel, box, bag, or whatever contains them.

When the tare is deducted from the gross, it leaves what is called the SUTTLE.

NEAT, the weight of the goods, after all allowances are made.

CASE I.

When the tare is a certain weight pr. box, barrel, &c.

RULE.

Multiply the number of boxes, or barrels, &c. by the tare, and subtract the product from the gross, the remainder is the neat weight required.

EXAMPLES.

1. In 7 fraits of raisins, each weighing 5 cwt. 2 qr. 5lb. gross, tare 23lb. pr. frait; how much neat?

23 X 7 = 1 cwt. 1 qr. 21lb.

5, 2, 5
7
38, 3, 7 gross.
1, 1, 21 tare.
37, 1, 14 the answer.

What is the neat weight of 14 hogsheads of tobacco; each weighing 5 cwt. 2 qr. 17 lb. gross, tare 100 pr. hogshead. Ans. 66 cwt. 2 qr. 14 lb.

CASE II.

When the tare is a certain weight pr. cwt.

RULE.

Divide the gross weight by the aliquot parts of a cwt. contained in the tare, and subtract the quotient from the gross; the remainder is the neat weight.

EXAMPLES.

1. Gross 173 cwt. 3 qr. 17 lb. tare 16lb. pr. cwt. how mutch neat?

	cwt.	*qr.*	*lb.*	
	173	3	17	gross.
14 lb. = $\frac{1}{8}$	21	2	26	
2 lb. = $\frac{1}{7}$	3	0	11	
	24	3	9	
	149	0	8	Ans.

2. What is the neat weight of 7 barrels of potash, each weighing 201 lb. gross, tare 10lb. pr. cwt.? Ans. 128 lb. 6 oz.

3. In 25 barrels of figs, each 2 cwt. 1 qr. gross, tare 16 pr. cwt.; how much neat? Ans. 48 cwt. 24lb.

CASE III.

When Trett is allowed with Tare.

RULE.

Divide the suttle by 26, and the quotient will be the Trett, which subtract from the suttle, the remainder is the net weight.

EXAMPLES.

1. In 9 cwt. 2 qr. 17 lb. gross, tare 37 lb. and trett as usual; how much neat?

	cwt.	*qr.*	*lb.*	
	9	2	17	gross.
	0	1	9	tare.
26 |	9	1	8	suttle.
		1	11	trett.
	8	3	25	Answer.

2. In 7 casks of primes, each weighing 3 cwt. 1 qr. 5 lb. gross, tara 17½ lb. pr. cwt. and trett as usual; how much neat? Ans. 18 cwt. 3 qr. 25 lb.

CASE IV.

When Tare Trett and Cloff are allowed.

RULE.

Deduct the tare and trett as before, divide the suttle by 168, the quotient will be the cloff, which subtract from the suttle; the remainder is the neat weight.

EXAMPLES.

1. What is the neat weight of a hhd. of tobacco, weighing 15 cwt. 3. qr. 20lb. gross, tare 7 lb. pr. cwt. and trett and cloff as usual ?

	cwt.	*qr.*	*lb.*	
	15	3	20	gross.
7lb. pr.		3	27	tare.
26)	14	3	21	
		2	18	trett
168)	14	1	13	suttle
			9	cloff
	14	1	4	Ans.

2 In 19 chests of sugar, each containing 13 cwt. 1 qr. 17lb. gross, tare 13lb. pr. cwt. and trett and cloff as usual;—how much neat ? and what is the value 5¾d pr. lb. ?
Ans. 215 cwt. 17lb. and value £577 6s. 5¾

COMPOUND PROPORTION, OR DUBLE RULE OF THREE,

Teaches to solve, at once, such questions, as require two, or more statings in simple proportion, whether Direct, or Inverse.

In this rule, their are always five terms given, to find a sixth. The three first terms of which, are a *suppposition*, the two last a *demand*.

RULE.

In stating the question, place the terms of supposition, so that the principal cause of loss, gain, or action, possess the first place ; that which signifies time, distance of place, in the second place, and the remaining term in the third place. Place the terms of demand under those of the *same kind*, in the supposition.

If the blank place, or term sought, fall under the third term, the proportion is direct ; then multiply the first and second

Note. The following method of stating compound proportion is, by some, prefered,

1. Place that number, or term, which is of the same name, or kind with the answer, in the third place.

2. Then take one term from the supposition, and one from the demand, both of the same *name*, or *kind*, and place them with the third term.

3. Then proceed in the same manner with the two remaining terms.

terms together for a divisor, and the other three for a dividend: but if the *blank* fall under the first, or second term, the proportion is Inverse; then multiply the third and fourth term together for a divisor, and the other three for a dividend; and the quotient will be the answer.

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4. Reduce the *similar* terms to the same denomination, if necessary.

5. Multiply the terms in the second and third place together, and divide their product by the product of those in the first place;—the quotient will be the answer.

EXAMPLE.

If 7 men can build 36 rods of wall in 3 days, how many rods can 20 men build in 14 days?

men 7 : 20 : : 36
days. 3 : 14

21 280
36

21)10080(480
84

168
168

00

In compound proportion, there are always five numbers given to find a sixth, which multiplied into the product of the two first, shall be equal to the product of the other three. And it may be shown in compound, as in simple proportion, that the product of the extremes, is equal to the product of the means; thus 2 X 3 : 12 : : 3 X 4 : 24; here the extremes are 2 X 3 and 24, and the means are 12 and 3 X 4; now the product of 2 X 3 X 24 the extremes, is 144; and the product of 12 X 3 X 4 the means, is also 144; whence it is evident that dividing either the product of the three extremes, or *means*, by any *two* of the means gives the other mean; thus, 144 divided by the product of the two means 3 X 4, gives 12 for the other mean; and it is also manifest, that by dividing the product of the three extremes, or *means*, by the product of any two of the extremes, the other extreme is obtained; thus dividing 144 the product of the means, by 2 X 3 the product of two of the extremes, gives 24 for the other extreme: hence the propriety of the rule in multiplying the 3d. 4th. and 5th. terms, or means together, and dividing by the product of the two first terms, or extremes, to obtain the other extreme.

EXAMPLES.

If 7 men can build 36 rods of wall in 3 days, how many rods can 20 men build in 14 days?

7 : 3 : : 36 terms of supposition.
20 14 terms of demand.
36
—
84
42
—
504
20
—

If 150*l.* in 12 months, gain 9*l.* in what time will 450*l.* gain 54*l.*?

150 : 12 : : 9
450 : : 54 Ans. 2 years.

7 X 3 = 21)10080(480 rods. Ans

BY LOGARITHMS.

In compound, as in simple proportion, the term required may be found by logarithms, if we substitute *addition* for multiplication, and *subtraction* for division.

RULE.

Add together the logarithms of those terms which in common arithmetick are to be multiplied together for a dividend, and from the sum, subtract the sum of the logarithms answering to these terms, which in common arithmetick, are multiplied for a divisor; and the remainder will be the logarithm of the answer. Or more fully; find the arithmetical complement of the logarithms to be subtracted, and then add all the terms together; the sum will be the logarithm of the answer.

EXAMPLES.

2. If 4 men in 12 days, mow 48 acres; how many acres can 8 men mow, in 16 days?

Two first terms	4	a. c.	9.	3979400
	12	a. c.	8.	9208188
Third term	8		0.	9030900
Fourth and fifth terms	48		1.	6812412
	16		1.	2041200
* Term required	128 Ans.		2.	1072100

* It must be observed, that each arithmetical compliment increases the index of the logarithm by 10; as often therefore, as the a. c. is used, the index of the sum of the logarithms, must be made less by so many tens.

1. If 10 bushels of oats, be sufficient for 18 horses 20 days, how many bushels will serve 60 horses 36 days ?

Three last terms	60	log.	1.	7781513
	36		1.	5563025
	10		1.	0000000
Sum of Log.			4.	3344538
Log. to be subtracted	18	log.	1.	2552725
	20		1.	3010300
			2.	5563025

Sum of the Log's. three last terms	4.	3344538
do. of the two first	2.	5563025
Term required 60 =	1.	7781513

3. What principal will gain £262 10s. in 7 years, at £5 pr. cent ? Ans. £750

4. If the interest of 365 dollars for 3 years and 9 months, be $ 82 13 cts. what will be the interest of $ 8 940 for two years and 6 months ? Ans.) 340

5. If £100 in 12 months, gain £6 interest ; how much will £75 gain in 9 months ? Ans. £3 7s. 6d.

6. If £16 18s. be the wages of 16 men, for 8 days ; what sum will 32 men earn in 24 days ? Ans. £ 01 8s.

7. If £100 in 12 months, gain £7 interest ; what principal will gain £3 18s. 9d. in 9 months ? Ans. £75

8. If 200lb. be carried 40 miles, for 40 cents ; how far may 200lb. be carried for $ 60 60 cts. Ans. 60 miles.

COMPOUND INTEREST.

In calculating compound interest, the amount for the first year, is made the principal for the second year ; and the amount for the second, the principal for the third, &c.

As the *Logarithmickal method* of computing compound interest, is by much the most expeditious, it is thought unnecessary to subjoin the old one in this place. Therefore,

To calculate Compound Interest by Logarithms.

RULE.

1. Find the amount of 1 dollar for 1 year ; multiply its logarithm by the number of years, and to the product add the loga-

rithm of the principal. The sum will be the logarithm of the *amount* for the given time.

2. From the amount subtract the principal, and the remainder will be the *interest*.

EXAMPLES.

1. What is the amount of 20 dollars, at 6 pr. cent compound interest, for 100 years?

Amount of 1 dollar for 1 year = $ 1 06 log.	0.0253059
Multiply by the time	100
	2.5305900
Add log. of $ 20, given principal	1.3010300
Amount required $ 6786	3.8316200

2. What is the amount of 425 dollars, at 5 pr. cent compound interest, for 4 years?

Amount of 1 dollar for 1 year = $ 1 05 log.	0.0211893
Multiply by the time	4
	0.0847572
Add log. of 425 given principal	2.6283889
Amount $ 516 59	2.7131461

Note 1. If the the interest becomes due *semianually*, or *quarterly*; find the amount of one dollar, for the half-year or quarter, and multiply the logarithm, by the number of half-years or quarters in the given time.

Note 2. As Simple Interest is performed by a rank of numbers, arithmetically proportional, so it may be shown, that Compound Interest is performed by a rank of numbers *geometrically* proportional.

And it is a principle in Mathematicks, that, if three numbers be in geometrical proportion, the product of the two extremes, is equal to the square of the mean. (See Euclid's Eliments, 20th prop. 7th book.) And on the contrary, if the rectangle contained by the extremes of any three numbers, be equal to the square of the mean, then those three numbers are in geometrical proportion.

Now if 3 dollars be the compound interest of $ 100 for ½ year, or 6 months, then these three numbers 100,103, 106, should be in geometrical proportion; but it may be proved by the aforesaid proposition, they are not; for the rectangle of 100 into 106 is but 10600, and the square of the mean 103, is 10609, which is greater than the product of the two extremes. But the square root of 10600 will be found to be 102,956: so that the true proportional interest of $ 100, for ½ year, is but $ 2 95 cts. 6 m.

3. What is the amount of 1000 dollars, at 6 pr. cent compound interest, for 10 years? Ans. $1790 80

4. Required the amount of 100 dollars, at 6 pr. cent compound interest, for 3 years? Ans. $ 119 10

5. What will 1000 dollars amount to at 7 pr. cent, compound interest, in 4 years? Ans. $ 1310 80

6. What is the compound interest of 876 dollars 90 cts. at 6 pr. cent pr. annum, for 3 years and 6 months? Ans. $ 198 83+

7. What will 100 dollars amount to in 3 years, at 6 pr. cent compound interest, allowing that it becomes due *semiannually*? Ans. $ 127 05+

8. What is the amount of 400 dollars, at 5 pr. cent compound interest, for 1 year, payable *quarterly*? Ans. 420 37

9. What is the amount of 1 cent, at 6 pr. cent compound interest, in 500 years?

Amount of 1 dollar for 1 year = $ 1 06 log.	0.0253059
Multiply by the time	500
	12.6529500
Add log. of principal $ 0 01	—2.0000000
Amount $ 44,973,000,000	10.6529500

INVOLUTION.

A Power is a number produced by multiplying any given number continually by itself a certain number of times.

The number denoting the power, is called the *Index*, or *Exponent* of that power.

To raise a given number, we have the following

RULE.

Multiply the given number, or first power, continually by itself, till the number of multiplications be 1 less than the index of the power to be found, and the last product will be the power required.

Note. Powers are commonly denoted by writing their indices above the first power; as follows.

2 X 2 = 4, the 2d power, or square of 2, or 2^2.

2 X 2 X 2 = 8, the 3d power, or cube of 2, or 2^3.

2 X 2 X 2 X 2 = 16, or biquadrate of 2, or 2^4. &c.

EXAMPLES.

1. Let it be required to raise 45 to its cube, or third power?

45
45
225
180
2025
45
10125
8100

Ans. the 3d power, 91125 or cube of 45

2. What is the square of 3758? Ans. 14122564
3. What is the cube of 327? Ans. 34965783
4. What is the biquadrate, or fourth power of 376? Ans. 19987173376
5. What is the fifth power of ·029? Ans. ·000000020511149
6. What is the sixth power of 48? Ans. 12230590464
7. Required the seventh power of 7? Ans. 823543

EVOLUTION.

Is that, by which we extract the roots of numbers; or find a radical quantity, which multiplied into itself a certain number of times will produce the given power.

TO EXTRACT THE SQUARE ROOT.

RULE.

1. Having distinguished the given number in periods of two figures each, beginning at the place of units, find the greatest square number in the first, or left hand period, place the root of it at the right hand of the given number, (in the manner of a quotient figure in division,) for the first figure of the root, the square of which subtract from the first period, and to the remainder bring down the next period for a dividend.

2. Place the double of the root, already found, on the left of the dividend for a divisor.

Note. Roots are sometimes denoted by writing $\surd$ before the power, with the index of the root against it. Thus the third root of 80 is $\sqrt[3]{80}$, and the second root of 80 is $\surd 80$, the index 2, though omitted, is always to be understood, when the radical sign is written without a numeral index.

3. Consider what figure must be annexed to the divisor, so that if the result be multiplied by it, the product may be equal to, or next less than the dividend, and it will be the second figure of the root.

4. Find a divisor as before, by doubling the figures already in the root; and from these find the next figure of the root, as in the last article; and so on through all the periods to the last.

EXAMPLES.

1. What is the square root of 14122564 ?

14·12·25·64 | 3 the root.
9

67 | 512
469

745 | 4325
3725

7508 | 60064
60064

0 remains.

2. What is the square root of 5499025 ?

5·49·90·25 | 2345 root.
4

43 | 149
129

464 | 2090
1856

4685 | 23425
23425

0 remains.

3. What is the square root of 10342656 ? Ans. 3216
4. What is the square root of 43264 ? Ans. 208
5. What is the square root of 451584 ? Ans. 672
6. What is the square root of 2985984 ? Ans. 1728
7. What is the square root of 998001 ? Ans. 999
8. What is the square root of 964,519236 0241 ? Ans. 31,05671
9. What is the square root of 1030892198,4001 ? Ans. 32107,51
10. * What is the square root of 160 ? Ans. 12,64911 +
11. What is the square root of 2 ? Ans. 1,41421356237 +
12. What is the square root of 10 ? Ans. 3.162277 &c.

* Note. When the given number is a surd; that is, when its root cannot be found *exactly*, without a remainder, the evolution may be carried on, until we obtain a root, sufficiently near the truth, by annexing cyphers to the remainder, and proceeding as in *rational* numbers. In the 10th example; although 12.64911, is not the exact root of 160, yet if it be multiplied by itself, the product will be 159,9999837921, which is not two parts, of which 10000 make an unit, wide of the truth.

TO EXTRACT THE CUBE ROOT.

RULE.

1. Having distinguished the given number into periods of three figures, find the nearest less cube in the first period, set its root in the quotient, and subtract the said cube from the first period; to the remainder bring down the second period, and call this the RESOLVEND.

2. To three times the square of the root, just found, add three times the root itself, setting this one place farther to the right than the former, and call this sum the DIVISOR. Then divide the resolvend, excepting the right hand figure, by the divisor, for the next figure of the root, which annex to the former; calling this last figure e, and the part of the root before found, call a.

3. Add together these three products, namely, three times the square of a multiplied by e, three times a multiplied by the square of e, and the cube of e; setting each of them one place farther towards the right than the former, and call the sum the SUBTRAHEND: which must not exceed the resolvend; if it does, then make the last figure e less, and repeat the operation for finding the subtrahend.

4. Subtract the subtrahend from the resolvend, and to the remainder bring down the next period of the given number for a new resolvend; to which form a new divisor from the whole none root found; and thence another figure of the root, as before and thus continue till the whole is finished.

EXAMPLES.

1. Required the cube root of 436036824287 ?

$3X7^2$	= 149	436·036 824·287	7583 the root.
$3X7$	= 21	343	
1st. Divisor.	1511	93036	1st. resolvend.
		78875	subtract subtrahend.
Add $3X7^2X5$	735	14161824	2d. resolvend.
Add $3X7X5^2$	= 525	13644512	sub. 2d. subtrahend.
5^3	= 125	517312287	3d. resolvend.
1st.	78875	517312287	sub. 3d. sub'end.
$3X75^2$	= 17875	0	remains.
$3X75$	= 225		
2d. Divisor	178975		
Add $3X75^2X8$	= 135000		
Add $3X75X8^2$	= 14400		
8^3	= 512		
2d. Subtrahend		13644512	
$3X758^2$	= 1723692		
$3X758$	2274		
3d. Divisor.		17239194	
Add $3X7582X3$	= 5171076		
Add $3X758X32$	= 20466		
3^3	= 27		
3d. Resolvend		517312287	

The laborious operation of extracting the roots of higher powers, is often so tedious, as to render it highly irksome and forbidding to learners. But, as in LOGARITHMS, addition is made to perform the office of multiplication, and subtraction that of division ; the labour of evolving roots is not only shortened to a degree surpassing credence, but the whole is performed with incomparable facility and expedition.

EVOLUTION BY LOGARITHMS.

EVOLUTION is the opposite of involution. And it was shown in the introduction of logarithms, that quantities are involved, by *multiplying their indices*, or *logarithms*. For the same reason, therefore, the roots of quantities may be extracted, by *dividing their indices*, or *logarithms*.

To extract the root of any number by logarithms.

We have therefore, this general

RULE.

Divide the logarithm of the given quantity, by the number expressing the root to be found.

EXAMPLES.

1. Required to find the cube root of the same number 436036824287, by logarithms ?

	Numbers.		Logarithms.
Power	436036824287	3 |	11. 6395233
Root	7583		3. 8798411

2. What is the squre root of 92613

	Numb.		Log.
Power	9261	3 |	3. 9669579
Root	21		1. 3222193

3. What is the square root of 9801 ?

	Numb.		Log.
Power	9801	2 |	3. 9912704
Root	99		1. 9956352

4. What is the square root of 365 ?

	Numb.		Log.
Power	365	2 |	2. 5622929
Root	19.10498		1. 2811465

5. Required the cube root of 12345 ?

	Numb.		Log.
Power	12345	3 |	4. 0914911
Root	23.11162		1. 3638304

6. What is the 10th root of 2?

	Numb.		Log.
Power	2	10 |	0. 3010300
Root	1.000121		0. 0301030

7. Required the 10th root of 6948?

	Numb.		Log.
Power	6948	10 |	3. 8418598
Root	2.422		0. 3841859

8. What is 100th root of 983?

	Numb.		Log.
Power	983	100 |	2. 9925535
Root	1.071		0. 0992553

9. Required the 365th root of 1.045?

	Numb.		Log.
Power	1.045	365 |	0. 0191163
Root	1.00121		0. 0000524

*10. Required to find the 10000th root of 49680000?

	Numb.		Log.
Power	49680000	10000 |	7. 6961816
Root	1.0017899		0. 0006961

The Logarithms of Powers given, to find their roots.

1. Required the square root of 6561

		Powers.	Logarithms.		Roots.
		6561	3. 8169700	Ans.	81
2.	Of	4096	3. 6123599	Ans.	64
3.	Of	15625	4. 1938200	Ans.	125
4.	Of	46656	4. 6689076	Ans.	216
5.	Of	117649	5. 0705882	Ans.	343
6.	Of	262144	5. 4185400	Ans.	512
7.	Of	531441	5. 7254550	Ans.	729
8.	Of	1679616	6. 2048674	Ans.	1266
9.	Of	5764801	6. 7607844	Ans.	2401
10.	Of	43046721	7. 6339400	Ans.	6561

Required the cube root of the following numbers.

		Powers.	Logarithms.		Roots.
1.	Of	1728	3. 2375437	Ans.	12
2.	Of	8000	3. 9030900	Ans.	20
3.	Of	15625	4. 1938200	Ans.	25
4.	Of	19683	4. 2940914	Ans.	27
5.	Of	10077696	7. 0033614	Ans.	216
6.	Of	244140625	8. 3876400	Ans.	625
7.	Of	68719476736	10. 8370797	Ans.	4096

*Note. We have here an instance of the great rapidity with which arithmetical operations are performed by the use of Logarithms.

8. Of 999600029999 11. 9998698 Ans. 9999
9. Of 205884571094649 14. 3156375 Ans. 59049
10. What is the fourth root of 19987173376 ?
Log. = 10. 3007512 = Ans. 376 the root.
11. What is the fifth root of 307628211067156 25 ?
Log. = 17. 4881030 = Ans. 3145
12. What is the sixth root of 435728381009267809889764416?
Log. = 26. 6392164 ÷ 6 = 4. 4398694 = 27534 the Ans.
13. What is the seventh root of
34487717467307513182492153794673?
Log. = 31. 5376642 ÷ 7 = 4. 5053806 = 32017 Ans.
14. What is the eighth root of
1121016281320476236246497942460481?
Log. = 33. 0496120 ÷ 8 = 4. 1312015 = 13527 Ans.

Nothing can be more easy, than to extract the roots of powers, to which the logarithms are *given*, or may easily be found. But a difficulty may sometimes arise, in the learner's *finding* the exact logarithm to a proposed number, that is much greater than any in the TABLE; yet a very superficial attention to the nature of logarithms, will readily suggest a solution of this seeming difficulty; for, as adding the logarithms of several numbers, is equivalent to *multiplying* by the same numbers, and *subtracting* the logarithm of numbers, the same, as dividing by those numbers; therefore,

To find the logarithm to a proposed number greater than any in the TABLE.

RULE.

Resolve the given quantity into such factors, as will constitute it within the limits of the table, add together the logarithms of these factors, and the sum will be the logarithm to the proposed number.

EXAMPLES.

1. Required the logarithm of the natural number, 11042 ? Here it is evident, that dividing the given quantity by two, will constitute it within the limits of the TABLE; as follows,

	Factors.		
2)11042(5521		Log.	3. 7420177
2		Log.	0. 3010300
Given number.	11042	required Log.	4. 0430477

2. Find the logarithm of 15378.

Factors.		Thus,
3)15378(5126	Log.	3. 7097786
3	Log.	0. 4771213
Given number 15378	required Log.	4. 1868999

3. Required to find the logarithm of 17304

Factors.		
4)17304(4326	Log.	3. 6360805
4	Log.	0. 6020600
Given number. 17304	required Log.	4. 2381405

			Logarithms.
4.	What is the logarithm Of 19505 ?	Ans.	4. 2923668
5.	Of 25596 ?	Ans.	4. 4081722
6.	Of 39126 ?	Ans.	4. 5924655
7.	Of 57320 ?	Ans.	4. 7583062
8.	Of 71464 ?	Ans.	4. 8540873
9.	Of 89897 ?	Ans.	4. 9513229
10.	Of 119844 ?	Ans.	5. 0786162
11.	Of 217975 ?	Ans.	5. 3384067
12.	Of 3089725 ?	Ans.	6. 4899199

CASE II.

When there are cyphers on the right hand of the given number.

RULE.

Find a logarithm to the significant figures, as before, and increase the index by as many units, as there are cyphers on the right of the given number.

EXAMPLES.

1. What is the logarithm of 57640 ?

The logarithm of 5764 is 3. 7607240 ; and increasing the index 3, by 1 ; we shall have 4. 7607240 for the logarithm of 57640

2. Required the logarithm

	Of 586400	Ans.	5. 7681940
3.	Of 6495000	Ans.	6. 8125792
4.	Of 72970000	Ans.	7. 8632634
5.	Of 910100000	Ans.	8. 9590891
6.	Of 44.973.000.000	Ans.	10. 6529500

PRACTICAL QUESTIONS IN EVOLUTION AND INVOLUTION.

BROBLEM I.

To find a mean proportional between two numbers.

RULE.

Add together the logarithms of the given numbers, divide the sum by 2 ; the quotient will be the logarithm of the mean proportional required.

1. Required the mean proportional between 45 and 180.

Thus { 45 = 1. 6532125
180 = 2. 2552725

2)3. 9084850

Mean proportional required 90 = 1. 9542425

2. Required a mean proportional to the numbers 64 and 256

Ans. 128

PROBLEM II.

Any number of soldiers being given, to place them in a square Battalia of men.

RULE.

Divide the logarithm of the given number by 2 ; the quotient will be the logarithm of the answer.

3. Let 9216 men be ordered to form a square battalia ; how many must stand in rank and file ?

9216 2)3. 9645425 Log.

Ans. 96 1. 9822712

4. How many must stand in rank and file, so that 5625 men shall compose a square ? Ans. 75

5. Let 8450 men be so formed, as that the number in rank may be double the number in file.

8450 ÷ 2 = 4225 = log. 3. 6258267 ÷ 2 = 1. 8129133 = 65

2)8450

4225 log. 2)3. 6258267

Ans. 65 in file 1. 8129133

Multiply 2

And 130 in rank.

Note. When the question requires double, trible, or quadruple, the number of men to stand in rank, as in file ; divide the logarithm of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ &c. of the given number by 2, the quotient will be the number in file, which double, triple, quadruple &c. and the product will be the number in rank.

6. Required to set out 27648 fruit trees, so that the number in length, shall be to the number in breadth as 3 is to 1 ; how must they be placed ?

Ans. 288 in length, and 96 in breadth.

PROBLEM III.

Any two sides of a rightangled triangle being given, to find the other side. *

Case 1. When the base and perpendicular are given, to find the hypothenuse.

RULE.

Add the squares of the two legs together, and extract the square root of the sum.

7. A triangular piece of ground measures 30 rods on one side, and 40 rods on another ; required the length of the remaining, or longest side.

30 X 30 = 900
40 X 40 = 1600

2500(50 hypothenuse, or
25 longest side

00

8. Required the length of a brace in a building, so that the lower end of it shall be 8 feet, and the upper end, 6 feet from the right angle. Ans. 10 feet.

9. Suppose the lower end of the brace to rest in a post 3 feet, and the upper end framed into a plate 2 feet 3 inches from the right angle : required its length. Ans. 3 feet 9 inches.

10. What will be the length of a brace, when it is required that the distances from the right angle to either end, should be 2 feet 6 inches, and 3 feet 4 inches ? Ans. 4 feet 2 inches.

11. Two stages start from the same place ; one goes directly south at the rate of 9 miles an hour, for 3 hours, the other due west, for 4 hours, at the same rate ; in what time would they now meet, were their course turned directly towards each other, continuing at the same rate per hour ? Ans. 2½ hours.

12. Required the length of a scaling ladder, to reach the top of a wall, whose height is 28 feet, the breadth of the ditch before it being 45 feet ? Ans. 53 feet.

* Note. The square of the hypothenuse, or the longest side of a rightangled triangle (by 47th. Theorem B. 1. Euc.) is equal to the sum of the squares of the other two sides ; and consequently the the difference of the squares of the hypothenuse and either of the other sides is the square of the remaining side.

PROBLEM IV.

CASE 2.

The hypothenuse and one leg. being given, to find the other leg.

RULE.

Subtract the square of the given leg. from the square of the hypothenuse, and extract the square root of the difference.

13. What is the perpendicular of a right angle triangle, whose base is 56 feet, and hypothenuse 65 ? Ans. 33 feet.

14. What is the base of a right angled triangle, the hypothenuse being 159 feet, and the perpendicular 84 feet ?

Ans. 135 feet.

15. A line of 65 yards will reach from the top of a precipice, standing close by the side of a brook, to the opposite bank ; required the breadth of the brook, the heighth of the precipice being 33 yards ? Ans. 56 feet.

16. A ladder of 50 feet long, being placed in a street, reached a window 28 feet from the ground on one side ; and by turning the ladder over, without removing the foot, it touched a moulding 36 feet high on the other side ; required the breadth of the street. Ans. 76. 1233335 feet.

17. Two ships sail from the same port; one, due east 84 leagues, and the other, directly south 135 leagues : how far are they asunder ?* Ans. 159 leagues.

PROBLEM V.

To find the CIRCUMFERENCE of a circle from its diameter.

RULE.

Multiply the diameter by 3.14159

OR

Multiply the diameter by 355, and divide the product by 113.

Ex. 1. If the diameter of the earth be 7930 miles, what is the circumference ? 3. 14159 X 7930 = 24913 miles.

2. How many miles does the earth move, in revolving round the sun ; supposing the orbit to be a circle, whose diameter is 190 million miles ? Ans. 596,902,100.

3. If the diameter of a wheel be $4\frac{1}{2}$ feet ; what is the circumference ? Ans. 14 feet $1\frac{5}{8}$ inches.

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* Note. The square root may in the same manner be applied to navigation ; and when deprived of other means of solving problems of that nature, the following proportion will serve to find the course.

As the sum of the hypothenuse (or distance) and half the greater leg. (whether difference of latitude, or departure) is to the less leg. so is 86, to the sine of the angle opposite the less leg.

4. What is the circumference of a circular island, whose diameter is 45 rods? Ans. 141 rods, $1\frac{3}{7}$ yard.

5. What is the whole distance of space, through which the planet Hershel moves, in revolving round the center of the system, supposing its orbit to be a circle, whose diameter is 1,800 millions miles? Ans. 11,309,724,000 miles.

PROBLEM VI.

To find the DIAMETER of a circle from its circumference.

RULE.

Divide the circumference by 3.14159

OR

Multiply the circumference by 113, and divide the product by 355: Or multiply the circumference by .31831, and the product will be the diameter.

Ex. 1. If the circumference of the earth be 24913 miles, what is the diameter? Ans. 7930 miles.

2. If the periphery of a wheel be 6 feet 6 inches; what is its diameter? Ans. 2 feet $\frac{4}{7}$ inches.

3. If the circumference of the Sun be 2,800,000 miles, what is his diameter? Ans. 891,267 miles.

4. If the circumference of the Moon be 6850 miles, what is her diameter? Ans. 2180 miles.

5. If the whole extent of the earth's orbit be 596,902,100 miles how far are we from the Sun? Ans. 95,000,000miles.

PROBLEM VII.

To find the Area of a circle.

RULE.

Multiply the square of the diameter by .7854

OR

Multiply half the diameter into half the circumference.

Ex. 1. What is the area of a circle whose diameter is 623? Ans. 304836

2. How many acres are there in a circular island, whose diameter is 124 rods? Ans. 75 acres, 76 rods.

3. What is the area of a circle, whose diameter is 7 feet. Ans. 38.4846

4. How many square ards yare in a circle, whose diameter is $3\frac{1}{2}$ feet? Ans. 1. 069

5. What is the area of a circle, whose diameter is 1, and whose circumference is 3. 14159? Ans. .7854

If the diameter of a circle is not given, the area may be found by multiplying the square of the circumference by .07958. (Sup Euc. 8. 1.)

Ex. 1. What is the area of a circle, whose circumference is 136 feet? Ans. 1472 feet.

2. What is the surface of a circular fish pond, which is 10 rods in circumference? Ans. 7. 95800 rods.

PROBLEM VIII.

To find the diameter of the earth, from the known height of a distant mountain, whose summit is just visible in the horizon.

RULE.

*From the square of the height, subtract the height.

Ex. 1. The summit of Mount Chimborazo in South America is about 4 miles above the level of the ocean. If a strait line from this touch the surface of the water at the distance of 178$\frac{1}{2}$ miles; what is the diameter of the earth? Ans. 7940 miles.

2. The White Mountains in New-Hampshire are about 7100 feet high above the level of Connecticut River; and a strait line from the summit of the mountains will touch the surface of the water at the distance of 103$\frac{1}{2}$ miles, what is the diameter of the earth? Ans. 7940 miles.

PROBLEM IX.

To find the greatest distance at which a given object can be seen on the surface of the earth.

RULE.

To the product of the height of the object into the diameter of the earth, add the square of the height, and extract the square root of the sum.

Ex. 1. If the diameter of the earth be 7940 miles, and Mount Ætna 2 miles high, how far can its summit be seen at sea? Ans. 126 miles.

2. Suppose the diameter of the earth as in the first example; at what distance may a steeple be seen on level ground, allowing it to be 165 feet in height? Ans. 21$\frac{1}{8}$ miles.

If a man standing on a level plain, has his eye elevated 5$\frac{1}{2}$ feet above the ground; to what distance can he see the surface of the plain? Ans. 2$\frac{7}{8}$ miles.

4 The top of a ship's mast 132 feet high is just visible in the horizon, to an observer on the deck of another ship 33 feet from the surface of the water; how far are they asunder?

Ans. 21$\frac{1}{8}$ miles.

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*Note 1. See Euclid's Eliments, 36. 3.

Note. 2. The actual distance at which an object can be seen, is increased by the refraction of the rays of light in the air. (See End. Nat. Phil.) But if no allowance be made for this refraction, the distance to which a person can see the plane surface of the ocean, is equal to a *tangent* to the earth drawn from the observer's eye.

PROBLEM X.

To find the Area of a Triangle.

RULE.

Multiply the base of the given triangle into half its perpendicular height ; or half the base into the whole perpendicular, and the product will be the answer.

Ex. 1. Required the area of a triangle whose base, or longest side is 36 inches, and the perpendicular height 16 inches.

Ans. 36 X 8 = 288 inches.

2. Required the area of a triangular garden, whose base, or longest side is 15.6 rods, and the perpendicular opposite the base is 9 rods. Ans. 70.2 rods.

PROBLEM XI.

*To find the convex surface of a Cylinder.**

**Diffinition.* A Cylinder is a round body whose bases are circles, like a round column or stick of timber of equal bigness from end to end.

RULE.

Multiply the length into the circumference of the base.

Ex. 1. How many square feet in the superficial contents of a cylinder which is 42 feet long, and 15 inches in diameter.

Ans. 42 X 1.25 X 3.14159 = 164.933 square feet.

2. Required the convex surface of a cylindrical stick of timber, whose axis is 5 feet, and the diameter 7 inches.

Ans. 1520 inches.

PROBLEM XII.

To find the solidity of a Cylinder.

RULE.

Find the area of the base (by Prob. VII.) which multiply into the length, and the product will be the solid contents.

1. What is the solid contents of a round stick of timber whose diameter is 18 inches, and length 20 feet ?

18 in. = 1.5 ft.
X 1.5

2.25 X .7854 = 1.76715 area of base.
Or 18 inches 20 length.

18 inches. Ans. 35·34300

324 X·7854 = 254·4696 inches, area of the base.
20 length in feet.

144)5089·3920(35·343 solid feet, Ans.

2. What is the solidity of cylinder, whose length is 121, and diameter 45.2 ? Ans. 45.2^2X·7854X121 = 194156.6

PROBLEM XIII.

To find the solidity of a CONE.

Deffinition. A CONE is a solid whose base is a circle, from which it decreases gradually to a point in the top, called the VERTEX.

A line drawn from the vertex, perpendicular to the base, is called the *height* of the cone.

RULE.

Multiply the area of the base by the height, and $\frac{1}{3}$ of the product will be the content.

Ex. 1. What is the solidity of a cone, whose height is 12 feet 6 inches, and the diameter of the base 2 feet 6 inches?

$2{\cdot}5^2 \times {\cdot}7854 \times 12{\cdot}5 \div 3 = 20{\cdot}453125$ feet, Ans.

2. Required the solidity of a conical monument, that is 9 feet high, and the diameter of its base $2\frac{1}{2}$ feet.

Ans. 14·726250 feet.

PROBLEM XIV.

To find the solidity of a Frustrum of a cone.

Deffinition. A FRUSTRUM of a cone is what remains after any portion of the upper end is cut off, by a plane paralell to the base.

RULE.

Add together the areas of the two ends, and the square root of the product of these areas; and multiply the sum by $\frac{1}{3}$ of the perpendicular height, and the result will be the solid content.

OR

2. Divide the difference of the cubes of the diameters of the two ends, by the difference of the diameters, and this quotient, being multiplied by ·7854 and again by $\frac{1}{3}$ of the height, will give the solidity.

EXAMPLES.

1. Required the solidity of a frustrum of a cone, whose altitude, or height is 18 feet, the greatest diameter 8 feet, and the least 4 feet.

BY THE 1st. RULE.

$8^2 \times {\cdot}7854 =$ 50·2656, = area of base.
$4^2 \times {\cdot}7854 =$ 12·5664, = do. of the other end.
$\sqrt{12{\cdot}5664 \times 50{\cdot}2656} =$ 25·1328, = $\sqrt{}$ the pro'd of the 2 areas.
Multiply 37·9648 the sum
by $\frac{1}{3}$ of 18 = 6

Ans. 527·7888 solid inches.

BY THE 2d. RULE.

$8^3 - 4^3 = 448 \div (8-4) = 112 \times {\cdot}7854 \times 6 =$
527·7888 in. Ans.

The latter method, in many cases, will be found preferable to the former in point of expidition.

2. What is the content of the frustrum of a conical block, whose height is 20 inches, and the diameter of its two ends 28 and 20 inches? Ans. 9131·5840

The number of *gallons* or *bushels* which a vessel will contain may be found, by calculating the capacity in *inches*, and then dividing by the number of inches in 1 gallon or bushel; as by the following

TABLE OF SOLID MEASURE.

1728	cubic inches	= 1 cubic foot
27	cubic feet	= 1 cubic yard,
4492$\frac{1}{9}$	cubic feet	= 1 cubic rod,
32768000	cubic rods	= 1 cubic mile,
282	cubic inches	= 1 ale gallon,
231	cubic inches	= 1 wine gallon,
2150·42	cubic inches	= 1 bushel,

1 cubic foot of pure water weighs 1000 ounces, Avoirdupois, or 62$\frac{1}{2}$ pounds.

EXAMPLES.

1. What is the capacity of a conical cistern, which is 9 feet deep, 4 feet in diameter at the bottom, and 3 feet at the top?

Ans. 87·18 cubic feet X 7·4805* = 652·15 wine gallons.

2. How many gallons of ale can be put into a vat in the form of a conic frustrum, if the larger diameter be 7 feet, the smaller diameter 6 feet and the depth 8 feet? Ans. 1886·5458 gallons.

3. There is a cistern in a distillery whose altitude is 10 feet, the greater diameter 14 feet, and the smaller diameter 12 feet; required its capacity in hogsheads.

$14^3 - 12^3 \div 14 - 12 \times .7854 \times \frac{10}{3} \times 7.4805 \div 63 =$

Ans. 157·918193 hhd.

PROBLEM XV.

To find the surface of a Sphere.

Deffinition. A SPHERE, or globe is a round solid body, in the center of which is a point, from which all lines drawn to the surface are equal.

RULE.

Multiply the diameter by the circumference.

Note. In like manner, the convex surface of any zone or segment is found by multiplying its height by the whole circumference of the sphere.

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* Note. When the capacity is in feet, multiply by 7·4805, because $\frac{1728}{231} = 7.4805$ the number of wine gallons in 1 cubic foot: When the *ale* gallon is required, multiply the feet by 6·1276, because $\frac{1728}{282} = 6.1276$; but if the capacity be calculated in inches *divied* by the number of cubic inches, in the gallon.

EXAMPLES.

1. What is the convex surface of a sphere, whose diameter is 7 inches, and circumference 22 inches ?

Ans. 7 X 22 = 154 in.

2 Required the surface of a globe, whose diameter or axis, is 24 inches. 24 X 3·14159 X 24 = 1809·5616 inches, Ans.

3. Considering the earth as a sphere, whose circumference is 25000 miles ; how many square miles are there on its surface ?

Ans. 198945750 sq. miles.

4. The axis of a sphere being 42 inches, what is the convex superficies of the segment, whose height is 9 inches ?

Ans. 42 X 3·14156 X 9 = 1187·5248 inches.

5. If the circumference of the sun be 2800000 miles, what is the surface ? Ans. 2495547600000 sq. miles.

PROBLEM XVI.

To find the solidity of a SPHERE.

RULE.

1. Multiply the cube of the diameter by ·5236.

OR

2. Multiply the square of the diameter by $\frac{1}{6}$ of the circumference.

OR

3. Multiply the surface by $\frac{1}{6}$ of the the diameter.

EXAMPLES.

1. What is the solidity of a sphere, whose diameter is 1 foot ?

12^3 X ·5236 = 904·7808 inches, Ans.

Or 12^2 X 6·28318 = 904·7808 inches.

Or 452·38896 X 2 = 904·7808 inches.

2. What is the solid content of a sphere 4 feet 6 inches in diameter ? Ans. 47.7130500 feet.

3. Required the number of solid miles contained in the earth, supposing its circumference to be 25000 miles.

Ans. 263858149120 miles.

4. How many wine gallons will fill a hollow sphere 2 feet 8 inches in diameter ?

The capacity is 9.9288 feet X 7.4805 = 1 hhd. 11.27 gallons.

5. How many gallons of water may be put into a hollow sphere that is 4 feet in diameter, and what will be the weight of the water ?

Note. The numbers 3·14159, ·7854, ·5236, should be made perfectly familliar. The *first* expresses the ratio of the *circumference* of a circle to the *diameter* ; the *second*, the ratio of the *area* of a circle to the *square* of the diameter ; and the *third*, the ratio of the *solidity* of a sphere to the *cube* of the diameter.

The second is $\frac{1}{4}$, and the third is $\frac{1}{6}$ of the first.

Ans. 205.33832704 gallons, and the weight is 12833.64544 lb.

6. If the diameter of the moon be 2180 miles, what is her solidity? Ans. 5424600000 miles.

When the solidity of a sphere is given, the diameter may be found by dividing the solidity by .5236, and extracting the cube root of the quotient.

7. What is the diameter of a sphere, whose solidity is 65.45 cubic feet? $3\sqrt{\frac{65}{52}\cdot\frac{45}{36}} = 5$ feet Ans.

8. What must be the diameter of a sphere, to contain $105\frac{3}{4}$ gallons of wine? Ans. 3 feet.

9. Required the diameter of a globe, to contain 16755 pounds of water. Ans. 8 feet.

10. How many globes that are 3 inches each in diameter, are equal to another globe whose diameter is 12 inches?

Ans. 64.

Note. The solid contents of similar figures are in proportion to each other, as the cubes of their homologous sides, or diameters. Euc. El.

12. If a cannon ball 6 inches in diameter, weigh 32lb. what will another ball weigh, whose diameter is 3 inches?

$6^3 = 216$ and $3^3 = 27$, then as 216 : 32 : : 27 : 4 lb. Ans.

13. If a metalic globe 8 inches in diameter, weigh 72 lb. what will be the weight of a globe of the same metal, whose diameter shall be 4 inches? Ans. 9 lb.

14. If a globe of silver 3 inches in diameter, be worth $150; how many such globes will be equal in value to $9600?

Ans. 64.

ANNUITIES, OR PENSIONS.

An Annuity, is a sum of money payable every year, for a certain number of years, or forever.

When the debtor keeps the annuity in his own hands beyond the time of payment, it is said to be in Arrears.

The sum of all the annuities for the time they have been forborn together with the interest due upon each, is called the Amount.

If an annuity be bought off, or paid all at once, at the beginning of the first year, the price, which ought to be given for it, is called the Present Worth.

To find the amount of an Annuity at Simple Interest.

RULE.

1. Find the sum of the natural series of numbers, 1, 2, 3, &c. up to the given number of years wanting *one*.

2. Multiply this sum by one year's interest of the annuity, and the product will be the whole interest due upon the annuity.

3. To this product add the product of the annuity multiplied into the time, and the sum will be the amount sought.

EXAMPLES.

1. What is the amount of an annuity of £50 for 7 years, allowing simple interest at 5 pr. cent?

1 + 2 + 3 + 4 + 5 + 6 = 21 = 3 X 7

£2 10s. = 1 years interest of 50*l*.

3

7 10

7

52 10

350 0 = £50 X 7

£402 10 = amount required.

2. If a pension of $600 pr. annum be forborn 5 years, what will it amount to, allowing 6 pr. cent simple interest?

Ans. $3360

3. If a salary of $750 annually, remain unpaid for 4 years, how much must be paid at the end of said term, allowing 4½ pr. cent simple interest. Ans. $3202 50

To find the present worth of an Annuity at Simple Interest.

RULE.

Find the present worth of each year by itself, discounting from the time it becomes due, and the sum of all these will be the present worth required.

EXAMPLES.

1. What is the present worth of an annuity of $100, to continue 5 years, at 6 pr. cent pr. annum simple interest?

106 : 100 = 94.3396 = present worth for 1st year.
112 : 100 = 89.2857 = " 2d year.
118 : 100 = 84.7457 = " 3d year.
124 : 100 = 80.6451 = " 4th year.
130 : 100 = 76.9230 = " 5th year.

425.9391 = $425 93 cts. 9 m. $\frac{1}{10}$ = present worth of the annuity required.

2. What is the present worth of $400 pr. annum, to continue 4 years at 6 pr. cent? Ans. $1396.06503

3. What is the present worth of an annuity, or pension of £300, to continue 4 years, at 5 pr. cent pr. annum, simple interest? Ans. £1782 3s. 8½d.

To find the Amount of an Annuity at Compound interest.

RULE.

1. Make 1 the first term of a geometrical progression, and the amount of £1. or $1 for one year, at the given rate pr. cent, the ratio.

2. Carry the series to as many terms as the number of years, and find its sum.

3. Multiply the sum thus found by the given annuity, the product will be the amount sought.

EXAMPLES.

1. If a salary of $600 be forborn (or remain unpaid) 7 years; what will it amount to at 6 pr. cent pr. annum, compound interest? 1 + 1.060000 + 1.123600 + 1.1910116 + 1.262476 + 1.338225 + 1.418519 + 1.503630 = 8.897466 = sum of the series.*

Multiplied by 600

gives 5338.4796 = $5338 47 cts. $9\frac{5}{10}$ m. the amount sought.

Or, By Table III.

Multiply the Tabular number under the rate, and opposite to the time, by the annuity, and the product will be the amount sought.

2. If a pension of $175 pr. annum, be forborn 20 years, at 6 pr. cent compound interest; what is the amount?

Tabular number = 36.785590

175 Annuity.

Ans. 6437.478250 = $6437.47 c. $8\frac{1}{4}$ m.

3. Suppose $ 50 pr. annum, with compound interest at $5\frac{1}{2}$ pr. cent. be 10 years in arrears; required the amount.

Tabular number = 12·875354 X 50 = $ 643·76c.7m. Ans.

4. What will a pension of £ 120 pr. annum, amount to in 3 years, at 5 pr. cent. compound interest? Ans. £378 6s.

5. The salary of the President of the United States, is $ 25000, supposing the whole be in arrears during the period of his elec-

*The sum of the series thus found, is the amount of £1, or $1 annuity, for the given time, which may be found in Table III, ready calculated. *(The method of constructing these Tables will be shown hereafter.)*

Hence, either the amount, or present worth of annuities, continuing for a term not exceeding 40 years, may readily be found by the Tables for that purpose.

tion, or 4 years; what would then be the amount of his salary, allowing 6 pr. cent, compound interest? Ans. $ 109365,40

To find the present worth of Annuities at Compound Interest.

RULE.

1. Divide the annuity by the ratio, or the amount of $ 1, or £1 for one year, and the quotient will be the present worth of the first year's annuity.

2. Divide the annuity by the square of the ratio, and the quotient will be the present worth of the annuity for the second year.

3. Find, in like manner, the present worth of each year by itself, and the sum of all these will be the present worth of the annuity sought.

EXAMPLES.

1. What is the present worth of an annuity of $ 40 to continue 5 years, discounting at 5 pr. cent. pr. annum, compound interest?

Ratio)^1 = 1·05)40· 00000(38·095 = present worth 1st. year.
Ratio)^2 = 1·1025)40· 00000(36·281 = " 2d. year.
Ratio)^3 = 1·57525)40· 00000(34·556 = " 3d. year.
Ratio)^4 = 1·215506)40· 00000(32 899 = " 4th. year.
Ratio)^5 = 1·276218)40. 00000(31·342 = " 5th. year.

173·173 = $ 173 17c. 3m. whole present worth of the annuity required.

Or, By Table IV.

Multiply the tabular number under the rate and opposite to the time, by the annuity, and the product will be the present worth required.

2. What is the present worth of an annuity of $ 50 to continue 5 years, at 6 pr. cent, pr. annum, compound interest?

Tabular number = 4·21236 X 50 = Ans. $ 210 61 8

3. If the pension of an officer, serving in the Revolutionary War, be 20 dollars a month, or 240 dollars annually; what is its present worth, allowing a discount of 6 pr. cent, pr. annum, compound interest, supposing it to continue 10 years?

Ans. $ 1766 41c. $9\frac{3}{10}$m.

To find the present worth of a FREEHOLD ESTATE, *or an Annuity to continue forever, at Compound Interest.*

RULE.

As the rate pr. cent. is to $ 100, so is the yearly income to the value required.

1. What is the worth of a freehold estate of $ 40 pr. annum, allowing 5 pr. cent, to the purchaser?

As 5 : 100 : : 40 : $ 800 Ans.

2. What is a freehold estate of £75 a year worth, allowing the buyer 6 pr. cent, compound interest for his money ?

Ans. £1250

3. An estate brings in yearly $ 79·20 what would it sell for, allowing the purchaser 4½ pr. cent, compound interest for his money ? Ans. $ 1760.

To find the present worth of an Annuity, or Freehold Estate in Reversion at compound interest.

RULE.

Find the present value of the estate (by the foregoing rule) as though it were to be entered on immediately, and divide said value by that power of the ratio denoted by the time of reversion, and the quotient will be the present worth of the Estate in Reversion.

EXAMPLES.

1. The reversion of a freehold estate of £79.4s. pr. annum, to commence 7 years hence, is to be sold ; what is it worth in ready money, allowing the purchaser 4½ pr. cent, for his money?

As 4·5 : 100 : : 79·2 : 1760 = present worth, if entered on immediately.

And $1{\cdot}045)^7 = 1{\cdot}360862)1760\,000(1293{\cdot}297$ = £1293. 5s. 11¼d. = present worth of 1760l. for 7 years, or the whole present worth.

Or, By Table IV.

Find the present worth af the annuity, or rent, for the time of reversion, which subtract from the value of the immediate possession, and the remainder will be the value of the estate in reversion.

2. What is the present worth of a freehold estate of $ 40 pr. annum, to commence 7 years hence allowing the purchaser 5 pr. cent ?

Tabular number = 5·78637

40 = annuity, or rent.

231·45480 = present worth of rent.

5 : 100 : : 40 : 800·0000 = value of immediate possession.

568·5452 = $ 568·54c. 5⅕m. Ans.

3. Which is the most valuable, an income of £100 pr. annum for 15 years, or the reversion of an equal income forever afterward, computing at the rate of 5 pr. cent, pr. annum, compound interest ?

Ans. The first term of 15 years is better than the reversion forever afterward, by 17*l.* 18*s.* 7½d.

VULGAR FRACTIONS.

The learner will find it convenient, and indeed necessary, as he progresses in the more intricate parts of Arithmetick, to be thoroughly acquainted with *Vulgar Fractions*. They were briefly introduced immediately after *Simple Proportion* (page 56) as a preliminary to the subject of Decimal Arithmitick ; but we shall here consider, more extensively, the intimate relation between Vulgar, and Decimal Fractions, together with their relative and important use in Arithmetick in general.

Vulgar Fractions are either proper, improper, simple, compound, or mixed.

1. A *Simple*, or *proper* fraction is one, whose numerator is less than the denominator ; as $\frac{3}{4}$,$\frac{5}{8}$,$\frac{7}{9}$, &c.

2. An *Improper fraction* is one, whose numerator exceeds the denominator ; as $\frac{8}{3}$,$\frac{15}{12}$, &c.

4. A *Mixed Number* is composed of a whole number and a fraction ; as $8\frac{3}{4}$,$25\frac{6}{13}$, &c,

Note. Any whole number may be expressed like a fraction, by writing 1 under the given number for a denominator ; as $8 = \frac{8}{1}$ and $12 = \frac{12}{1}$, &c.

3. A *Compound fraction* is the fraction of a fraction, coupled by the word *of*, as ; $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}$ of $\frac{9}{1}$, &c.

5. The *Common Measure* of two or more numbers is that number, which will divide each of them without a remainder.

Thus, 4 is the common measure of 12, 16, and 20 ; and the *greatest* number, that will do this, is called the *Greatest Common Measure*.

6. A number, which can be measured by two or more numbers, is called the *Common Multiple* ; and if it be the least number, which can be so measured, is called their *Least Common Multiple :* thus, 24, 36, 48 and 60 are each a common multiple of 3, 4, and 6 ; but their *least* common multiple is 12.

To find the least Common Multiple of two or more numbers.

RULE.

1. Divide by any number that will divide two or more of the given numbers without a remainder, and set the quotient, together with the undivided numbers, in a line beneath.

2. Divide the second lines as before, and so on till there are no *two* numbers that can be divided ; then, the continued product of the divisors and quotients, will give the multiple required.

EXAMPLES.

1. What is the least common multiple of 6, 8, 10, and 12?

Thus.
2)6 8 10 12
3)3 4 5 6
2)1 4 5 2
1 2 5 1

The product of the divisors = 12 and 12 X 2 X 5 120 Ans.

2. What is the least common multiple of 4 and 63 ?
Ans. 12

3. What is the least common multiple of 3,4,8, and 12 ?
Ans. 24

4. What is the least common multiple of 4,5,6, and 10 ?
Ans. 60

5. What is the least number that can be divided by the 9 digits, separately, without a remainder ? Ans. 2520

REDUCTION OF VULGAR FRACTIONS.

Is the bringing them out of one form into another, in order to prepare them for the operations of Addition, Subtraction, &c.

CASE I.

To abbreviate, or reduce fractions to their lowest terms.

RULE.

1. Divide the terms of the given fraction by any number, that will divide them without a remainder, and these quotients again in the same manner ; and so on till it appears, that there is no number greater than 1, which will divide them again, and the fraction will be in its lowest terms.

OR

2. Divide both the terms of the fraction by their greatest common measure, and the quotients will be the lowest terms of the fraction required.

EXAMPLES.

1. Reduce $\frac{144}{240}$ to its lowest terms.

(2) (2) (3) (2) (2)
$\frac{144}{240} = \frac{72}{120} = \frac{36}{60} = \frac{12}{20} = \frac{6}{10} = \frac{3}{5}$ the answer.—(see page 56)

Or thus; 144)240(1
144

96)144(1
96

48)96(2
48

Therefore 48 is the greatest common measure, and $48)\frac{144}{240} = \frac{3}{5}$ the same as before.

2. Reduce $\frac{5184}{6912}$ to its lowest terms. Ans. $\frac{3}{4}$
3. Reduce $\frac{1344}{4536}$ to its lowest terms. Ans. $\frac{7}{8}$
4. Reduce $\frac{252}{364}$ to its least terms. Ans. $\frac{9}{13}$
5. Reduce $\frac{192}{576}$ to its lowest terms. Ans. $\frac{55}{64}$

CASE II.

To reduce a Mixed Number to its equivalent improper fraction.

RULE.

Multiply the whole number by the denominator of the fraction, and add the numerator to the product, then that sum written above the denominator will form the fraction required.

EXAMPLES.

1. Reduce $25\frac{3}{8}$ to its equivalent improper fraction.
   $25 \times 8 + 3 = 193$, then the fraction will become $\frac{193}{8}$
2. Reduce $27\frac{2}{9}$ to its equivalent improper fraction.
   Ans. $\frac{245}{9}$
3. Reduce $45\frac{7}{8}$ to its equivalent improper fraction.
   Ans. $\frac{367}{8}$
4. Reduce $100\frac{52}{71}$ to its equivalent improper fraction.
   Ans. $\frac{7219}{71}$
5. Reduce $15\frac{19}{59}$ to its equivalent improper fraction.
   Ans. $\frac{490}{31}$

CASE III.

To find the value of an improper fraction.

RULE.

Divide the numerator by the denominator, and the quotient will be the whole or mixed number sought.

EXAMPLES.

1. Find the value of $\frac{3848}{21}$ 21)3848(183$\frac{5}{21}$ Ans.
2. Find the value of $\frac{108}{12}$ Ans. 9.
3. Find the value of $\frac{1245}{22}$ Ans. $56\frac{13}{22}$
4. Reduce $\frac{933}{11}$ to its equivalent whole, or mixed number.
   Ans. $84\frac{9}{11}$
5. Reduce $\frac{4679}{27}$ to its equivalent whole, or mixed number.
   Ans. $173\frac{8}{27}$

CASE IV.

To reduce a whole number to an equivalent fraction, having a given denominator.

RULE.

Multiply the whole number by the given denominator, and place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 8 to a fraction, whose denominator shall be 9.
   $8 \times 9 = 72$; and the fraction will become $\frac{72}{9}$ Ans.
2. Reduce 13 to a fraction, whose denominator shall be 12.
   Ans. $\frac{156}{12}$
3. Reduce 100 to a fraction, whose denominator shall be 79.
   Ans. $\frac{7900}{79}$

CASE V.

To reduce a compound fraction to an equivalent simple one.

RULE.

1. Reduce all whole and mixed numbers to their equivalent fractions.

2. Multiply all the numerators together for a new numerator, and all the denominators together for the denominator, and they will form the fraction required.

EXAMPLES.

1. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{8}{11}$ to a simple fraction.

$$\frac{2 \times 3 \times 4 \times 8 = 192}{3 \times 4 \times 5 \times 11 = 660} = \frac{16}{55} \text{ the answer.}$$

2. Reduce $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{4}{7}$ to a simple fraction. Ans. $\frac{5}{21}$
3. Reduce $\frac{3}{4}$ of $\frac{5}{9}$ of 10 to a simple fraction. Ans. $4\frac{1}{6}$
4. Reduce $\frac{3}{7}$ of $\frac{2}{5}$ of $\frac{9}{11}$ of $21\frac{1}{3}$ to a simple fraction.
   Ans. $2\frac{18}{19}\frac{6}{25}\frac{2}{5}$

CASE VI.

To reduce fractions of different denominations to equivalent fractions, having a common denominator.

RULE.

1 Reduce all fractions to simple terms.

2. Multiply each numerator into all the denominators, except its own, for a new numerator ; and all the denominators together, for a common denominator, which written under the several numerators, will give the fractions required.

EXAMPLES.

1. Reduce $\frac{1}{2}$ $\frac{1}{5}$, and $\frac{3}{7}$ to equivalent fractions, having a common denominator.

$1 \times 5 \times 7 = 35$ the new numerator for $\frac{1}{2}$
$3 \times 2 \times 7 = 42$ " for $\frac{1}{5}$
$4 \times 2 \times 5 = 40$ " for $\frac{4}{7}$
$2 \times 5 \times 7 = 70$ the common denominator.

Therefore the new equivalent fractions are $\frac{35}{70}$, $\frac{42}{70}$, and $\frac{40}{70}$

2. Reduce $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{11}{12}$ to a common denominator.
   Ans. $\frac{840}{960}$, $\frac{834}{960}$, $\frac{880}{960}$

3. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $12\frac{4}{3}$ to a common denominator.
Ans. $\frac{54}{72}$, $\frac{60}{72}$, $\frac{838}{72}$

4. Reduce $\frac{1}{3}$, $\frac{3}{4}$ of $\frac{4}{5}$, $5\frac{1}{2}$, and $\frac{2}{19}$ to a common denominator.
Ans. $\frac{120}{570}$, $\frac{342}{570}$, $\frac{31135}{570}$, $\frac{60}{570}$

5. Reduce $\frac{1}{2}$, $\frac{2}{2}$ of $\frac{5}{6}$, $7\frac{1}{4}$, and $\frac{3}{13}$ to a common denominator.
Ans. $\frac{936}{1872}$, $\frac{1040}{1872}$, $\frac{14508}{1872}$, $\frac{432}{1872}$

CASE VII.

To reduce the fraction of one denomination to the fraction of another, retaining the same value.

RULE.

1. Reduce the given fraction to such a compound one, as will express the value of the given fraction, by comparing it with all the denominations between it and that denomination to which it is to be reduced.

2. Reduce the compound fraction, thus made, to a simple one. (See Case V.)

EXAMPLES.

1. Reduce $\frac{3}{8}$ of a penny to the fraction of a pound.
By comparing it, it becomes $\frac{3}{8}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a pound.

$$\frac{3 \times 1 \times 1}{8 \times 12 \times 20} = \frac{3}{1920} \text{ the answer.}$$

2. Reduce $\frac{3}{1920}$ of a pound to the fraction of a penny.
Make a compound fraction of it thus;
$\frac{3}{1920}$ of $\frac{20}{1}$ of $\frac{12}{1} = \frac{720}{1920} = \frac{3}{8}$ the Ans.

3. Reduce $\frac{2}{9}$ of a shilling to the fraction of a pound.
Ans. $\frac{1}{90}$

4. Reduce $\frac{2}{3}$ of a farthing to the fraction of a pound.
Ans. $\frac{1}{1440}$

5. Reduce $\frac{6}{7}$ of a pound avoirdupois to the fraction of a cwt.
Ans. $\frac{3}{392}$

6. Reduce $\frac{9}{63252}$ of a hhd. of wine to the fraction of a pint.
Ans. $\frac{9}{1512}$

7. Reduce $5\frac{1}{2}$ furlongs to the fraction of a mile. Ans. $\frac{11}{160}$

8. Reduce $\frac{1}{2}\frac{1}{4}\frac{1}{4}$ of a week to the fraction of an hour. Ans. $\frac{3}{4}$

9. Reduce 7s. 6d. to the fraction of a pound. Ans. $\frac{3}{8}$

10. Reduce $5\frac{3}{4}$d. to the fraction of a shilling. Ans. $\frac{23}{48}$

ADDITION OF VULGAR FRACTIONS.

RULE.

Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different integers to those of the same; and all of them to a common denominator: then

the sum of the numerators written over the common denominator, will be the sum of the fractions required.

EXAMPLES.

Add $3\frac{5}{8}$, $\frac{7}{8}$, $\frac{4}{5}$ of $\frac{7}{8}$, and 7 together.

First $3\frac{5}{8} = \frac{29}{8}$, $\frac{4}{5}$ of $\frac{7}{8} = \frac{7}{10}$, $7 = \frac{7}{1}$

Then the fractions are $\frac{29}{8}$, $\frac{7}{8}$, $\frac{7}{10}$, and $\frac{7}{1}$. Therefore,

$29 \times 8 \times 10 \times 1 = 2320$
$7 \times 8 \times 10 \times 1 = 560$
$7 \times 8 \times 8 \times 1 = 448$
$7 \times 8 \times 8 \times 10 = 4480$

$8 \times 8 \times 10 \times 1 = 640$

$\frac{7808}{640} = 12\frac{1}{5}$ the answer.

2. Add $\frac{2}{7}$, $\frac{9}{10}$, and $\frac{7}{8}$ of $5\frac{1}{2}$ together. Ans. $6\frac{9}{80}$
3. Add $12\frac{1}{2}$, $3\frac{2}{3}$, and $4\frac{3}{4}$ together. Ans. $20\frac{14}{12}$
4. Add $\frac{1}{7}l.$ $\frac{2}{3}$s. and $\frac{5}{12}$ of a penny together.
Ans. 3s. 1d. $1\frac{10}{21}$qrs.
5. What is the sum of $\frac{2}{7}$ of $15l.$ $3\frac{3}{7}l.$ $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{3}{5}$ of a pound, and $\frac{2}{3}$ of $\frac{3}{7}$ of a shilling? Ans. £1 17s. $5\frac{1}{7}$d.
6. Add $\frac{3}{8}$ of a mile, $\frac{2}{3}$ of a yard, and $\frac{3}{4}$ of a foot together.
Ans. 120 rods, 2 ft. 9 in.
7. Add $\frac{4}{7}$ of a ton, and $\frac{9}{10}$ of cwt. together.
Ans. 12 cwt. 1 qr. 8 lb. $12\frac{8}{10}$ oz.
8. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour together.
Ans. 2d. 14h. 30m.

SUBTRACTION OF VULGAR FRACTIONS.

RULE.

Prepare the fractions as in addition, and the difference of the numerators, written above the common denominator, will give the difference of the fractions required.

EXAMPLES.

1. From $\frac{3}{7}$ take $\frac{2}{9}$ of $\frac{3}{7}$.

$\frac{2}{9}$ of $\frac{3}{7} = \frac{2}{21}$; then the fractions are $\frac{2}{21}$ and $\frac{3}{7}$, therefore

$2 \times 3 = 6$
$21 \times 2 = 42$
and $21 \times 3 = 63$ the common denominator.

and $42 - 6 = 36$
Ans. $\frac{36}{63} = \frac{4}{7}$

2. From $\frac{97}{100}$ take $\frac{3}{7}$. Ans $\frac{379}{700}$
3. From $14\frac{1}{4}$ take $\frac{2}{3}$ of 19. Ans. $1\frac{7}{12}$
4. From $\frac{1}{2}$ $l.$ take $\frac{3}{4}$ $s.$ Ans. 9s. 3d.
5. From $\frac{1}{3}$ oz. take $\frac{7}{8}$ of a pwt Ans. 11 pwt. 3 gr.

6. From $3\frac{2}{3}$ weeks, take $\frac{1}{7}$ of a day, and $\frac{1}{2}$ of $\frac{7}{3}$ of $\frac{1}{4}$ of an hour.
Ans. 3w. 4d. 12h. 19m. $17\frac{1}{7}$ sec.

7. The sum of three numbers is $56\frac{3}{5}$; the *first* number is $12\frac{5}{8}$ and the *second* $21\frac{7}{12}$; required the *third*. Ans. $22\frac{47}{120}$

8. What number added to $11\frac{5}{7}$ will make $36\frac{31}{168}$?
Ans. $24\frac{71}{77}$

MULTIPLICATION OF VULGAR FRACTIONS.

Reduce compound fractions to single ones, and mixed numbers to improper fractions; then multiply all the numerators together for a new numerator, and all the denominators together for the denominator of the product required.

EXAMPLES.

1. Multiply $\frac{1}{2}$ of 7 by $\frac{3}{8}$ Ans. $1\frac{3}{4}$
2. Multiply $\frac{5}{9}$ by $\frac{3}{7}$ Ans. $\frac{15}{63}$
3. Multiply $7\frac{1}{2}$ by $9\frac{1}{4}$ Ans. $69\frac{3}{8}$
4. Multiply $\frac{2}{9}$ of $\frac{3}{5}$ by $\frac{5}{8}$ of $3\frac{2}{7}$ Ans. $\frac{16}{84}$
5. Multiply $4\frac{1}{2}$ $\frac{3}{4}$ of $\frac{4}{7}$ and $18\frac{4}{5}$ continually together.
Ans. $9\frac{90}{140}$

DIVISION OF VULGAR FRACTIONS.

RULE.

Prepare the fractions as in multiplication; then invert the divisor, and proceed exactly as in multiplication: the product will be the quotient required.

EXAMPLES.

1. Divide $\frac{2}{5}$ of 9 by $\frac{3}{8}$ of $7\frac{1}{2}$

$\frac{2}{5}$ of $\frac{9}{1} = \frac{2 \times 9}{5 \times 1} = \frac{18}{5}$ and $\frac{3}{8}$ of $7\frac{1}{2} = \frac{45}{16}$; therefore,

$\frac{18}{5} \div \frac{45}{16} = \frac{18 \times 16}{5 \times 45} = \frac{288}{225}$ $1\frac{63}{225}$ the quotient required.

2. Divide $\frac{5}{7}$ by $\frac{3}{5}$. Ans. $1\frac{4}{21}$
3. Divide $\frac{7}{8}$ by 4. Ans. $\frac{7}{32}$
4. Divide $4\frac{5}{9}$ by $\frac{5}{9}$ of 4. Ans. $2\frac{1}{20}$
5. Divide 7 by $\frac{3}{8}$. Ans. $18\frac{2}{3}$

6. Divide $\frac{1}{5}$ of 19 by $\frac{2}{3}$ of $\frac{3}{4}$. Ans. $7\frac{3}{5}$
7. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$ Ans. $\frac{2}{3}$
8. What number multiplied by $\frac{3}{7}$, will make $11\frac{9}{17}$?
Ans. $26\frac{46}{51}$

SIMPLE PROPORTION IN VULGAR FRACTIONS.

RULE.

1. Prepare the fraction as before; then state the question agreeably to the rules in Simple Proportion of whole numbers.

2. Consider whether the proportion be *Direct* or *Inverse*; if direct, then invert the *first* term of the proportion; but if the proportion be inverse, invert the *third* term.

3. Then multiply all the three terms continually together, and the product will be the answer.

EXAMPLES.

1. If $\frac{2}{3}$ of a yard cost $\frac{5}{6}$ of a pound, what will $\frac{11}{17}$ of a yard cost ?
Thus, $\frac{3}{2} : \frac{5}{6} :: \frac{11}{17} : \frac{165}{252} = 12$s. 1d. $2\frac{6}{17}$ qrs. Ans.
2. If $\frac{3}{4}$ of a yard cost $\frac{29}{80}$ of a pound, what will $9\frac{1}{3}$ yards cost ?
Ans. £4 10s. 2d. $2\frac{2}{3}$ qrs.
3. If 8d. buy $\frac{3}{4}$ of a pound of sugar, how much will $10\frac{2}{3}$d. buy ?
Ans. 1lb.
4. At 7s. 0d. $1\frac{23}{25}$ qrs. pr. bushel, what will be the value of 15 bushels ? £5 5s. 7d. $0\frac{4}{5}$ qr.
5. If $\frac{7}{8}$ of a ship be worth $\frac{3}{9}$ of her cargo valued at 8000*l.* what is the whole ship and cargo worth ? Ans. £10031 14s. $11\frac{1}{21}$d.
6. A. and B. own a ship and cargo worth 16000*l.* A owns $\frac{3}{8}$ of the cargo and $\frac{4}{7}$ of the ship; but by accident at sea, they lose $\frac{1}{8}$ of the cargo, which is $214\frac{2}{7}$*l.* less than B's. share in the ship, required the values of the ship and cargo, and each one's respective share in the same.

Ans. £4000 value of ship, and 12000*l.* do. of cargo.

A's share of cargo = $3937\frac{1}{2}$*l.* } B's share of cargo = $6562\frac{1}{2}$*l.*
A's do. of ship = $2285\frac{5}{7}$*l.* } B's do. of ship = $1714\frac{2}{7}$*l.*

A's do. of the whole £$6223\frac{3}{14}$ B's do. of the whole £$8276\frac{11}{28}$

REDUCTION OF DECIMAL FRACTIONS.

CASE I.

To reduce numbers of different denominations to their equivalent value.

RULE.

Bring the given denominations to a vulgar fraction, and reduce said fraction to its equivalent decimal value. (See Case III, page 57, also Rule 2, page 64.)

EXAMPLES.

1. Reduce 10s. 6d. 2 qrs. to the decimal of a pound.

1*l.* X 20 X 12 X 4 = 960 and 10s X 12 X 6 X 4 ÷ 2 = 506, therefore, 506 ÷ 960 = .527083 Ans.

2. Reduce 13s. 5½d. to the decimal of a pound. Ans. .6729

3. Reduce 3 qrs. 2 na. to the decimal of a yard. Ans. .875

4. Reduce 17 yds. 1 ft. 6 in. to the decimal of a mile. Ans. .00994318

5. Reduce 10 weeks, 2 days, to the decimal of a year. Ans. .1972602, &c.

CASE II.

To find the value of a decimal in the known parts of the integer.

RULE.

1. Multiply the decimal by the number of parts in the next less denomination, and cut off so many places for a remainder, on the right, as there are places in the given decimal.

2. Multiply the remainder by the next inferiour denomination, and cut off a remainder as before; and so on through all the parts of the integer, and the several denominations standing on the left, make the answer.

EXAMPLES.

1. What is the value of .37623 of a pound?

20

7.42460
12

6.29520
4

1.18080 Ans. 7s. 6d. 1 qr.

2. What is the value of .8322916 of a pound? Ans. 16s. 7½d.

3. What is the value of .625 of a shilling? Ans. 7½d.

4. What is the value of .764425 of a pound Troy? Ans. 9 oz. 3 pwt. 11 gr.

5. Find the value of .875 of a yard. Ans. 3 qr. 2 na.

6. Find the value of .61 of a ton of wine.
Ans. 2 hhds. 27 gals. 2 qts. 1 pt.
7. What is the value of .8469 of a degree?
Ans. 58 m. 6 fur. 35 po. 0 ft. 11 in.
8. What is the value of .569 of a year?
Ans. 207 d. 16 h. 26 m. 24 sec.

CASE IV.

To find the decimal of any number of shillings, pence, and farthings, by inspection.

RULE.

1. Write half the greatest even number of shillings for the first decimal figure.
2. Let the farthings in the given pence and farthings, possess the second and third places; observing to increase the second place by 5, if the shilings be odd; and the third place by 1, when the farthings exceed 12, and by 2, when they exceed 36.

EXAMPLES.

1. *Find the decimal expression of 9s. 7½d. by inspection.*

.4 = ½ 8s.
.05 = for the odd shilling
30 = the farthings in 7½d.
1 for excess of 12

£.481 = decimal required.

2. What is the decimal value of 17s. 8½d.? Ans. £.885
3. What is the decimal expression of 7s. 9¼d.? Ans. £.391
4. Find the decimal value of 15l. 3s. 9½d. Ans. £15.19

CASE V.

To find the value of any decimal of a pound by inspection.

RULE.

1. Double the first figure, or place of tenths in the decimal, for so much of the answer in shillings, increasing the sum by 1, if the second figure be 5, or more than 5.
2. After the 5 is deducted, call the remaining figures in the second and third places, so many farthings, for the remainder of the answer, abating 1, if they exceed 12, and 2, if they exceed 36.

Note. When the decimal has but 2 figures, if any thing remain after the shillings are deducted, annex cyphers on the right.

EXAMPLES.

1. Find the value of £.876, by inspection.

.876
16s. = double of 8.
1 { for the 5 in the second place which is to be deducted out of 7.
And 6 ½ = 26 farthings remain to be added.
Deduct ¼ for the excess of 12.
Ans. 17s. 6 ¼d.

2. Find the value of £.679, by inspection. Ans. 13s. 7d.
3. Find the value of £.842, by inspection. Ans. 16s. 10d.
4. Find the value of £.790 by inspection. Ans. 15s. 9¾d.
5. Find the value of £.097 by inspection. Ans. 1s. 11½d.

SIMPLE PROPORTION BY DECIMALS.

RULE.

Reduce fractions to decimals, and state the question as in whole numbers ; multiply the second and third terms together, and divide by the first, and the quotient will be the answer.

EXAMPLES.

1. If $\frac{3}{8}$ of a yard cost $\frac{2}{5}$ of a pound, what will $9\frac{5}{8}$ yard cost ?

$\frac{3}{8} =$ .375 yds.
$\frac{2}{5} =$ .41.
$9\frac{5}{8} =$ 9.625 yds.

Therefore .375 yds. : 41. : : 9.625 yds. : £10.2666 ; or 10l. 5s. 3d. 3 qrs.

2. If $\frac{3}{5}$ of a yard cost $\frac{7}{12}$ of a pound, what will $\frac{6}{15}$ of an English ell cost ? $\frac{3}{5}$ of a yard $= \frac{3}{5}$ of $\frac{4}{1}$ of $\frac{1}{5} = \frac{12}{25}$ of an ell.

Then $\frac{25}{25}$ ell : $\frac{7}{12}$l. : : $\frac{6}{15}$ ell. : $\frac{3}{7}\frac{5}{2}$l. = 9s. 8d. 2 qrs. Ans. or,

$\frac{12}{25} = .48$
$\frac{7}{12} = .5833$ Then .48 : .5833 : : .4 : .486 = 9s. 8d. 2rs.
$\frac{6}{15} = .4$

3. At 7½d. pr. lb. what will be the price of an cwt. of sugar ?

7·5 X 112 = 840 = 3l. 10s. Ans.

4. What is the value of $3\frac{3}{4}$ cwt. of coffee at $23\frac{1}{2}$ cts. pr. lb. ?

Ans. $ 98 70c.

5. What is the value of 2 qrs. 1 na. of velvet at 19s. 8¼d. pr. English ell ? Ans. 8s. 10d. 1qr. $\frac{7}{10}$

6. If $\frac{1}{4}$ of a yard of satin cost 7s. 3d. how many yards can I buy for 13l. 15s. 6d. ? Ans. $28\frac{1}{2}$ yds.

7. What is the value of $\frac{5}{9}$ of a tun of wine, when $\frac{5}{8}$ of a gallon costs $\frac{5}{7}$ of a pound ? Ans. £140

8. At $1\frac{1}{2}$l. pr. cwt. what does $3\frac{1}{2}$ lb. come to ? Ans. $10\frac{5}{7}$d.

9. What is the tax upon 745l. 14s. 8d. at 3s. 6d. on the pound ?

Ans. £130 10s. $3\frac{1}{5}$qrs.

10. A person shares $\frac{3}{5}$ in a certain prize, and sells $\frac{3}{4}$ of it for 171l. what was the whole amount of the prize ? Ans. £380

11. If, when the days are $13\frac{5}{6}$ hours long, a traveller perform his journey in $35\frac{1}{2}$ days ; in how many days will he perform the same journey, when the days are $11\frac{9}{10}$ hours long ?

Ans. $40\frac{6}{9}\frac{1}{5}\frac{5}{2}$ days.

12. A regiment of soldiers, consisting of 976 men, are to be new clothed, each coat to contain $2\frac{1}{2}$ yds. of cloth, that is $1\frac{5}{4}$ yd. wide, and to be lined with shalloon, $\frac{7}{8}$ yd. wide; how many yards of shalloon will line them? Ans. 4531 yds. 1 qr. $2\frac{2}{4}$ na.

SIMPLE INTEREST BY DECIMALS.

A TABLE OF RATIOS.

RATE PER CENT.	RATIO.	RATE PER CENT.	RATIO.
3	·03	$5\frac{1}{2}$	·055
4	.04	6	·06
$4\frac{1}{2}$	·045	$6\frac{1}{2}$	·065
5	·05	7	·07

RATIO is the simple interest of £1, or $1 for one year at the rate per cent. agreed on.

RULE

Multiply the principal, Ratio and Time continually together, and the last product will be the interest required.

EXAMPLES.

1. Required the interest of 537 dolls. 58 cts. for 4 years and 6 months, at 5 per cent. pr. annum, simple interest.

$537.58 principal
.05 ratio.

26.8790 interest for 1 year.
4·5 multiply by the time.

1343950
1075160

$120.95550 Ans. $120 95 cts. 5 m. &c.

2. What is the interest on $268 17 cts. for 3 years and 9 months, at $4\frac{1}{2}$ pr. cent, simple interest?
Ans. 268.17 X .045 X 3.75 = 45.2536875 = $45 25 cts. 3 m. +

3. What is the interest of 118l. 9s. for 1 year and 6 months at 6 pr. cent pr. annum? Ans. 10l. 13s. 2d. 2 qrs.

4 Required the amount of 648 dolls. 50 cts. for 12.75 years, at $5\frac{1}{2}$ pr. cent, simple interest. Ans. $1103 26 cts. +

5. What is the amount of 69l. 8s. for 3 years, $1\frac{1}{4}$ months, at $5\frac{3}{4}$ pr. cent, simple interest? Ans. £81 6s. 6d. 3 qrs.

CASE II.

The AMOUNT, TIME, and RATIO given, to find the PRINCIPAL.

RULE.

Multiply the ratio by the time, add unity to the product for a divisor, by which divide the amount, and the quotient will be the principal.

EXAMPLES.

1. What principal will amount to 264.3125, in 5 years, at 5 pr. cent pr. annum ?

.05 X 5 + 1 = 1.25) 264.3125 (211.45 Ans.

2. What principal will amount to $658.53550, in 4½ years, at 5 pr. cent. ? Ans. $557.58 cts.

3. What principal will amount to $313.4236875, in 3 years and 9 months, at 4½ pr. cent ? Ans. $268 17 cts.

4. What principal will amount to £956 10s. 4.125d. in 8 years and 9 months, at 5½ pr. cent ? Ans. £645 15s.

CASE III.

The AMOUNT, PRINCIPAL, and TIME given, to find the RATIO.

RULE.

Subtract the principal from the amount, divide the remainder by the product of the time and principal, and the quotient will be the ratio.

EXAMPLES.

1. At what rate pr. cent, will $950 75 cts. amount to $1235.9750, in 5 years ?

From the amount = 1235.9750
Take the principal = 950.75

950.75 X 5 = 4753.75)285.2250(.06 = 6 pr. cent. Ans.
285.2250

2. At what rate pr. cent will $715.45 cts. amount to $94.54940, in 2 years and 5 months ? Ans. 6 pr cent.

3. What rate pr. ct. will $268.17 cts. amount to $313.4236875, in 3¾ years ? Ans. 4½ pr. cent.

4. At what rate per cent will £319 5s. amount to £62.253750 in 3 years and 3 months ? Ans. 6 per cent.

CASE IV.

The AMOUNT, PRINCIPAL, and RATE per cent given, to find the TIME.

RULE

Subtract the principal from the amount, divide the remainder by the product of the ratio and principal ; and the quotient will be the time.

EXAMPLES.

1. In what time will $537 58 cts. amount to $658.53550, at 5 pr. cent pr. annum ?

From the amount $658.55550
Subtract the principal 537.58

537.58 X .05 26.8790)120.95550(4½ years answer.
120.95550

2. In what time will $268 17 cents amount to $313.4236875, at 4½ per cent per annum? Ans. 3 years 9 months.

3. In what time will $950 75 cents amount to $1235.9750, at 6 per cent per annum? Ans. 5 years.

4. In what time will £319 5s. amount to £381.503750, at 6 per cent? Ans. 3¼ years.

TO CALCULATE INTEREST FOR DAYS.

RULE.

Multiply the principal by the given number of days, and that product by the ratio; divide the last product by 365, and the quotient will be the interest required.

EXAMPLES.

1. What is the interest of 178l. 15s. for 87 days, at 6 per cent per annum? 178.75 X .87 X .06 = $36.5500 ÷ 365 = 2.2891 = 2l. 5s. 9d. 1 qr. Ans.

As the process of division is generally more tedious than that of multiplication, it is often convenient to substitute one for the other.

Now if we substitute, in the place of 365, the *reciprocal* of that quality; that is, the quotient arising from dividing 1 by 365, we shall have a number which *multiplied* into the principal, will give the same result, as *dividing* by its corellative quantity; Thus, 1 ÷ 365 = ·00274, therefore to multiply a given number by ·00274,* is equivalent to *dividing* the same number by 365.

2. What is the interest of $ 100, for 75 days, at 6 pr. cent. pr. annum? 100 X 75 X ·06 X ·00274 = $ 1 23c. 3m. Ans.

3. What is the interest of $ 148 50 cts. for 96 days, at 5 pr. cent pr. annum? Ans. $ 1 95c. 3m.

4. What is the interest of $ 312 for 25 days, at 7½ pr. cent? Ans. $ 1 60c. 3m.

5. What will £5·5 amount to in 256 days, at 4½ pr. cent pr. annum? Ans. £5 12s 11d. ·|-

* Note. The cyphers on the left may be always neglected, by observing to point off 5 figures of the product, on the right for decimal parts.

TABLE I.

Shewing the Amount of 1 Dollar, or 1 Pound for 31 *Years*, at 5, and 6 pr. Cent. *Simple Interest.*

Years.	5	6
1	1·05	1·06
2	1·10	1·12
3	1·15	1·18
4	1·20	1·24
5	1·25	1·30
6	1·30	1·36
7	1·35	1·42
8	1·40	1·48
9	1·45	1·54
10	1·50	1·60
11	1·55	1·66
12	1·60	1·72
13	1·65	1·78
14	1·70	1·84
15	1·75	1·90
16	1·80	1·96
17	1·85	2·02
18	1·90	2·08
19	1·95	2·14
20	2·00	2·20
21	2·05	2·26
22	2·10	2·32
23	2·15	2·38
24	2·20	2·44
25	2.25	2·50
26	2·30	2·56
27	2·35	2·62
28	2·40	2·68
29	2·45	2·74
30	2·50	2·80
31	2.55	2·86

TABLE II.

Shewing the Rebate of 1 Dollar, or 1 Pound for 31 *Years*, at 5 and 6 pr. Cent. *Simple Interest.*

5	6
·952380	·943396
·909091	·892857
·869565	·847457
·833333	·806451
·800000	·769230
·769230	·735294
·740740	·704225
714286	·675675
·689655	·649350
·666666	·625000
·645161	·602409
·625000	·581395
·606060	·561797
·588235	·543478
·571448	·526315
·555555	·510204
.540540	·495049
526315	·480769
·512820	·467289
·500000	·454545
487804	·442477
476190	·431034
·465116	·420168
454545	·409836
·444444	·400000
·434781	·390625
·425532	·381679
·416666	·373134
·408163	·364965
400000	·357143
·393157	·349650

TABLE III.

Shewing the Amo of 1 Dollar, or 1 Pou for *Months*, at 5, and 6 pr. Cent. *Simple Interest.*

5	6	Mon.
1·00416	1·005	1
1·00833	1·010	2
1 01249	1.015	3
1·01666	1·020	4
1·02083	1·025	5
1 02499	1·030	6
1 02916	1 035	7
1·03333	1·040	8
1·03749	1·045	9
1·04166	1·050	10
1·04583	1·055	11
1·05000	1·060	12

TABLE IV.

Shewing the Rebat or Present worth of 1 Dollar, or 1 Pound, fo *Months* discounting at 5, and 6 per Cent. *Simple Interest.*

5	6	Months
99585	99502	1
99173	.99009	2
98766	·98522	3
98361	·98039	4
97959	·97560	5
97561	·97087	6
·97263	·96628	7
·96772	·96153	8
·96387	·95690	9
·96006	·95238	10
95617	·94786	11
·95238	94339	12

CONSTUCTION OF TABLES. &c.

The two first give the AMOUNT, and REBATE, or PRESENT WORTH of $ 1, or £ 1, from 1 to 31 years inclusively, at 5 and 6 pr. cent. Simple interest. They are calculated by making 1 dollar, or 1 pound the PRINCIPAL in the first, and 1 dollar or 1 pound the AMOUNT in the second; or dividing *unity* by the srveral numbers in the first TABLE, gives the numbers in the second TABLE. The third and fourth Tables are of the same nature with the first two, and are therefore subject to the same principle of construction.

APPLICATION AND USE OF THE PRECEEDING TABLES.

CASE I.

To find the AMOUNT *of any given sum for years and months, at 5, and 6 pr. cent, Simple Interest.*

RULE.

To the Tabular number found in Table I, under the given rate and opposite the time in years, add the number found on the *right of the decimal point* in Table III, under the given rate and opposite the months, and multiply this sum by the principal; and the product will be the amount sought.

EXAMPLES.

1. What will $ 100 amount to in 7 years and 8 months, at 6 pr. cent pr. annum, Simple Interest?

Tabular number = 1.42 = amount of $1 for 7 years.
Do. .040 = do. for 8 months.

1.460
Multiply by .100 = principal.

$ 146.000 = Ans.

2. Required the amount of $318 50 cents, for 5 years, at 6 per cent per annum. Ans. 414 05cts.

3. What will $753 25 cents amount to in 4 years and 7 months, at 5 per cent per annum? Ans. $925 86c. $3\frac{77}{100}$m.

4. Required the amount of 112l. 10s. for 3 years and 5 months, at 6 per cent. Ans. 135l. 11s. 4d.

5. Required the amount of 180l. 8s. for 11 months, at 6 per cent per annum. Ans. 190l. 6s. 5d. 1 qr.

CASE II.

To find the Interest of any given sum for years and months, at 5 and 6 per cent.

RULE.

1. Find the *amonnt* as before, from which subtract the principal, and the remainder will be the interest.

OR

2. Multiply the number found on the *right of the decimal point* in the table, by the principal, and the product is the interest required.

EXAMPLES.

1. What is the interest of $400 forborn 3¼ years, at 5 per cent per annum?

Tabular number for 3 years = 1.20
Do. for 6 months. .02499

1.22499
Multiply by 400

489.99600 Amount.
Subtract 400. Principal.

Answer. $89.99600 = $90. *ferc.*

2. What will be the interest of $210 35 cents, for 9½ years, at 6 per cent per annum? Ans. $123 05c. 4.75 m.

3. What will be the interest on £45 10s. for 1 year and 7 months, at 6 per cent? Ans. £4 6s. 5d. 1 qr.

4. Required the interest of £896 15s. for 7 months, at 5 per cent per annum. Ans. £27 2s. 11d. 3 qrs.

CASE III.

To find the Rebate or Present worth of any given sum for years and months.

RULE.

Multiply the Tabular number under the given rate and opposite the time by the principal, and the product will be the *present worth.*

EXAMPLES.

1. What is the rebate, or present worth of $ 100 due 1 year hence, discounting at 6 per cent per annum?

Tabular number = .943396
100

$94.33.9$\frac{6}{10}$ Ans.

2. What is the present worth of $180 50 cents, due 5 years hence, at 5 per cent per annum? Ans. $138 84c. 6m.

3. How much ready money will pay a debt of £112 10s. due 3 years hence, discounting at 6 per cent? £95 6s. 11d.

4. How much ready money is equal in value to £315 8s. due 7 months hence, allowing 5 per cent discount?

Ans. £306 15s. 4d.

Note. When the discount is required, subtract the present worth from the principal, and the remainder is the discount.

5. What is the discount on £500, due 7 years hence, at 6 per cent per annum? £Ans. 147 17s. 9d.

COMPOUND INTEREST BY DECIMALS.

RULE.

1. Find the amount of $1, or £1, for one year at the given rate per cent.

2. Involve the amount, thus found, to such a power, as is denoted by the number of years, and multiply this power by the principal, or given sum, and the product will be the amount required.

3. Subtract the principal from the amount, and the remainder will be the *interest*.

EXAMPLES.

1. What is the compound interest of £500 for 4 years, at 5 per cent per annum?

The amount of 1l. for 1 year = 1.05 and

$1.05^4 \times 500 = 607.753125$ = the amount.

500

107.753125 = £107 15s. 0¾d. interest required.

2. What is the amount of £760 10s. for 4 years, at 4 per cent? Ans. 889l. 13s. 6½d.

A Table of the amount of $1. *or* 1*l. at* 6 *per cent per annum, for months.*

Mon.	Dec. parts.	Mo.	Dec. parts.	Mo.	Dec. parts.
1	1,00487	5	1,02457	9	1,04462
2	1,00976	6	1,02956	10	1,04975
3	1,01467	7	1,03457	11	1,05489
4	1,01961	8	1,03961	12	1,06

When the given time consists of years and months, seek the amount of $1 &c. in the table for years, and the amount of $1 &c. in the foregoing table for the months, and the continual product of these tabular numbers into the principal, will give the amount required.

Note. Subtract the principal from the amount, and the remainder is the compound interest.

EXAMPLES

1. Required the amonnt of £480 for 5 years and 6 months, at 6 per cent per annum compound interest.

Tabular number of 1*l.* for 5 years = 1.338225
Do. " for 6 months = 1.029560

1.77782931
480 = prin'l.

Ans. £661.2341 &c.

2. What will $100 amount to, forborn 7 years and 10 months, at 6 per cent per annum? Ans. $157 82c. 3m.

3. What is the compound interest of $210 50 cents, for 8 years, at 6 per cent? Ans. $29 48c. 7m.+

4. What is the compound interest of 80l. 4s. for 9 years and 4 months, at 6 per cent per annum? Ans. 47l. 19s. 0½d.

Another method of computing Compound Interest for years, months, and days.

RULE.

To the Logarithm of the principal, add the several logarithms answering to the number of years, months, and days, found in the following tables, and their sum will be the logarithm of the amount required.

Logarithmick Tables, at 6 per cent per annum, for years, months and days.

Y.	*dec. pts.*	*Y.*	*dec. pts.*	*Y.*	*dec. pts.*	*Y.*	*dec. pts.*
1	,025306	11	,278366	21	,531426	31	,784586
2	,050612	12	,303672	22	,556732	32	,809792
3	,075918	13	,328978	23	,582038	33	,835098
4	,101224	14	,354284	24	,607344	34	,860404
5	,126530	15	,379690	25	,632650	35	,885710
6	,151836	16	,404896	26	,657956	36	,911016
7	,177142	17	,430202	27	,683262	37	,936322
8	,202448	18	,455058	28	,708568	38	,961628
9	,227754	19	,480814	29	,733974	39	,986934
10	,253060	20	,506120	30	,759380	40	1.01224

M.	dec pts.	M	dec. pts.	M.	dec pts.	. .	dec pts
1	,002160	4	,008600	7	,014940	10	,021189
2	,004321	5	,010724	8	,017033	11	,023252
3	,006466	6	,012337	9	,010116		

D		D.		D.		D.	
1	,000071	9	,000642	17	,001212	25	,001781
2	,000143	10	,000713	18	,001284	26	,001852
3	,000215	11	,000785	19	,001355	27	,001923
4	,000287	12	,000857	20	,001426	28	,001994
5	,000358	13	,000928	21	,001497	29	,002065
6	,000429	14	,000999	22	,001568	30	,002136
7	,000500	15	,000107	23	,001639	31	,002207
8	,000571	16	,001142	24	,001710		

1. Required the amount of $436 50 cents, for 3 years, 8 months and 12 days.

Log. of principal = 2. 6399842
Add Log. of years = 0. 0759180
Do. of months = 0. 0170330
Do. of days = 0. 0008570

Ans. $541 75 2. 7337922 Amount requir'd

2. What will $175 amount to, in 10 years and 5 months, at 6 per cent per annum? Ans. $321 28 cts.

CASE II.

The amount, rate, and time given, to find the principal.

RULE.

1. Divide the amount of the given sum by the amount of $1, or £1, for the given time, and the quotient will be the principal:

OR

2. Multiply the PRESENT WORTH of $1, or £1, for the given number of years, at the given rate, by the amount; the product will be the principal.

EXAMPLES.

1. What principal at 6 per cent per annum, will amount to $757.4856, in 4 years?

By Table I.

Divide by the tabular amount of $ 1, for 4 years } = 1·262476)757·485600(=$ 600

By Table II.

Multiply by the present worth of $ 1 for 4 years, at 6 per cent. } 757·4856
·792093

Ans. $ 600 principal.

2. What principal at 6 per cent. per annum, will amount to £757 9s. 8½d. in 4 years? Ans. 600*l.*

ARITHMETICAL PROGRESSION.

Any rank of numbers, increasing by a common excess, and decreasing by a common difference, is said to be in ARITHMETICAL PROGRESSION.

Thus $\left\{\begin{matrix}1.2.3.4.5.6.7\\2.4.6.8.10.12.14\end{matrix}\right\}$ Ascending series.

$\left\{\begin{matrix}14.12.10.8.6 \text{ \&c.}\\7.\ 6.\ 5.4\ 3 \text{ \&c.}\end{matrix}\right\}$ Desending series.

The numbers which form the series, are called the TERMS of the progression; the first and last terms of which are called the EXTREMES.

Any *three* of the *five* following terms being given, the other two may be readily found.

1. The first term.
2. The last term.
3. The number of terms.
4. The common difference.
5. The sum of all the terms.

PROBLEM I.

The first term, the last term, and the number of terms being given, to find the sum of all the terms.

RULE.

Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

EXAMPLES.

1. The first term of an arithmetical progression is 5, the last term, 60, and the number of terms 12, required the sum of the series. $5 + 60 \times 12 \div 2 = 380$ Ans.

2. The first term of an arithmetical progression is 3, the last term 112, and the number of terms 18; required the sum of the series. Ans. 1035

3. How many strokes do the clocks of Venice, (which go to 24 o'clock,) strike in the compass of a day? Ans. 300

4. Suppose a man lay up 1 cent the first day of the year, 2 cents the second, and 3 the third day, and so on in arithmetical progression, every day increasing 1 cent; how much will he have saved at the year's end? Ans. $667,95 cents.

5. A merchant bought 100 yards of cloth in arithmetical progression; he gave 5 cents for the first yard, and 1 dollar for the last, what did the cloth amount to? Ans. $52,50 cts.

6. If 100 stones be placed in a right line, exactly a yard asunder, and the first a yard from a basket, what length of ground will that man go, who gathers them up singly, returning with them one by one to the basket?

Ans. 5 miles, 233 rods, 2 yards.

PROBLEM II.

The first term, the last term, and the number of terms given, to find the COMMON DIFFERENCE.

RULE.

Divide the difference of the extremes by the number of terms less by 1, and the quotient will be the common difference required.

EXAMPLES.

1. If the extremes be 3 and 19, and the number of terms 9, what is the common difference, and the sum of the whole series?

Extremes { 19, 3 } — 19 − 3 = 16

9 − 1 = 8) 16 (2 common difference

16

—

19
3
—
22
9
—
½) 198 (99 sum of series.

2. A man had 10 sons, whose several ages differed alike, the youngest was 4 years old, and the oldest 40; what was the common difference of their ages? Ans. 4 years.

3. A man travels from Manchester to London in 6 days; every days journey was greater than the preceeding one, by a common excess; he traveled 20 miles the first day, and 40 miles the last; what was the common increase of each succeeding day's, journey and the distance from Manchester to London?

Ans. daily increase 4 miles, and the distance of journey 180 miles.

PROBLEM III.

The two extremes and the common difference given, to find the number of terms.

RULE.

Divide the difference of the extremes by the common difference, and the quotient, increased by 1, is the number of terms required.

EXAMPLES.

1. If the extremes be 3 and 19, and the common difference 2, what is the number of terms?

$19 - 3 \div 2 = 8 + 1 = 9$ Ans.

2. A man, going a journey, travelled the first day 3 miles and the last day 43, and increased his journey every day 5 miles; how many days did he travel? Ans. 9 days.

GEOMETRICAL PROGRESSION.

Any series of numbers are in GEOMETRICAL PROGRESSION, when the several terms increase by a common multiplier, or decrease by a common divisor;—Thus, 3, 6, 12, 24, 48 &c. is a series in geometrical progression, increasing by the common *Multiplier* 2; and 81, 27, 9, 3, 1, &c. is a series in geometrical progression, decreasing by the common *divisor* 3.

The number, by which the series is constantly increased, or diminished, is called the RATIO.

PROBLEM I.

Given the first term, the last term, (or *extremes*) and the ratio, to find the sum of the series.

RULE.

Multiply the last term by the ratio, and from the product subtract the first term, and the remainder, divided by the ratio less 1, will give the sum of all the terms of the series.

EXAMPLES.

1. The first term of a series in geometrical progression is 3, the last term 531441, and the ratio 3; required the sum of all the terms.

The series is 3,9,27,81,243,729,2187,6561,19683,59049, 177147,531441. Then,

$$531441 \times 3) - 3 (\div 3 - 1 = 797160 \text{ Ans.}$$

2. The extremes of a geometrical progression are 1 and 65536, and the ratio 4; what is the sum of the series?

Ans. 87381

PROBLEM II.

Given the first term and the ratio, to find any other term assigned.

CASE I.

*When the first term of the series and the ratio are equal.**

1. Write down a few of the leading terms of the series, and place their indices over them, beginning with an unit, or 1.

2. Add together such of the most convenient indices, as will make up the entire index to the sum required.

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* Note. When the first term of the series is equal to the ratio, the indices must begin with an unit, and the indices added must make the entire index of the term required; but if the first term be *greater*, or *less* than the ratio, the indices must begin with a cypher, and the indices added must make an index less by 1 than the number, expressing the place of the term sought.

3. Multiply the terms of the geometrical series belonging to those indices, continually together, and the product will be the term sought.

EXAMPLES

* If the first term be 2 and the ratio 2, what is the 15th term ?

Thus, { 1,2,3, 4, 5, 6 Indices.
{ 2,4,8,16,32,64 Leading terms.

Then 4+5+6 = 15 the index of 15th. term, and product of 16, X 32 X 64, = 32768 term required.

2. A merchant bought 22 hhd. of wine for 2 mills for the first hhd. 4 for the second, 8 for the third, and so on in duplicate proportion geometrically ; what did the whole amount to at that rate ?

The 22d. or last term is 4194304.

Then $\frac{2 \text{ X } 4194304 - 2}{2 - 1} = 8398606$, the sum of all the terms

(by PROB. I.) = £8398,60c. 6m. Ans.

3. A labourer agreed to work one whole year for a rich miser, to receive no other reward than 3 farthings for the 1st. month, 2¼d. for the 2d. month, 6¾d. for the 3d. month, and so on, in triple proportion geometrically ; what did his wages amount to in one year, and what was the average price of each day's labour ? Ans. His wages for one year amounted to £830 7s. 6d. and the average price of each day was £2 7s. 6d.

CASE II.

When the first term of the series and the ratio are different, that is, when the first term is either greater or less than the ratio.

RULE.

1. Write a few of the leading terms as before, and begin their indices with a cypher.

2. Add together the most convenient indices to make an index, less by 1 than the number expressing the term sought.

3. Multiply the terms of the geometrical series together, belonging to those indices, and make the product a dividend.

4. Raise the first term to a power whose index is one less than the number of terms multiplied, and make the result a divisor.

5. Divide the dividend by the divisor, and the quotient will be the term sought.

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* Note. See this principle explained by logarithms,—page 44

EXAMPLES.

1. If the first term of a geometrical series be 5, and the ratio 3, what is the 11th term ?

Thus, { 0, 1, 2, 3, 4, Indices.
{ 5,15,45,135,405, Leading terms.

And $1+2+3+4 = 10$ the index of the 11th. term.

$15X45X135X405 = 282943125$

$\frac{282943125}{125} = 2263545$ the 11th. term required.

$5^3 = 125$ the divisor.

Here the number of terms multipled are 4 ; therefore the 1st. term raised to the 4th. power less by 1 is the 3d. power, or cube of 5 = 125 the divisor.

2. What debt can be discharged in a year, by paying 2 cents for the first month, 8 cents for the second, 32 cents for the third month, and so on in quadruple proportion, for each month ? Ans. $ 111848 10 cts.

3. An ignorant horse jockey being employed to purchase a number of horses for shipping, very readily agreed with a gentleman, well skilled in numbers, for 28, upon condition that he should give 1 cent for the first horse, 5 for the second, 25 for the third horse, and so on in quintuple proportion to the last horse ; what did they come to at that rate, and how much did they cost per head ?

Ans. the horses came to $ 3077087286529541 01,56 cts. and the average price was, $ 10989597451855503,62 cts. 7$\frac{1}{7}$m. per head.

4. What will a horse cost, computing his worth in geometrical progression by the nails in his shoes, at a farthing for the first nail, 3 farthings for the second, and so on in triple proportion to the last, or 32d. nail ? Ans. £965114681693 13s. 4d.

5. A young man skilled in numbers, agreed with a farmer to work for him 11 years, without any other reward than the produce of one wheat corn for the first year, and that produce to be sowed the second year, and so on from year to year, till the end of the time, allowing the increase to be in a tenfold proportion ; what quantity of wheat is due for such service, and to what does it amount, at $,150 per bushel ?

Ans. 226056$\frac{1}{8}$ bushels, allowing 768 wheat corns to make a pint ; and the amount is $339084 18c. 2$\frac{1}{2}$m.

6. What will be the value of a 64 gun-ship, reckoning 1 penny for the first gun, 2 pence for the second, 4 pence for the third gun, and so on to the last, in duplicate proportion ?

Ans. £78614336404564653 1s. 3d.

7. Suppose America should agree to build 144 ships of the line for Great Britain, at the rate of but 1 farthing for the first ship, 2 for the second, 4 for the third, and so to increase, in a duplicate proportion to the last; what would they all amount to at that rate, and how many globes of standard gold, equal in magnitude to the earth we inhabit, could be formed from the mass, allowing a cubic inch of gold to be worth £53 2s 8d.?

Ship 1 =	1	the 9th Ship =	256	the 18th Ship =	131072
2 =	2		256		131072
3 =	4				
4 =	8		1536		262144
5 =	16		1280		917504
6 =	32		512		131072
7 =	64				393216
8 =	128		65536		131073
9 =	256	18th Ship.	131072		
					17179869184
			The 36 Ship		34359738368
					34359738368
					274877906944
					206158430208
					103079215104
					274877906944
					103079215104
					240518168576
					309237645312
					171798691840
					103079215104
					137438953472
					103079215104
					1180591620717411303424
		The 72d. Ship =			2361183241434822606848

As multiplying the index of any term in a geometrick series, by a given number, gives the index for that *power* of the term, denoted by the multiplying number; therefore, as 72 X 2 = 144, raise the 72d term to the second power, that is, multiply it by itself, and that product by the ratio, the last product will be the amount of the last term, which by (PROB. 1.) will give the sum of all the series: as follows.

The 72d. Ship = 2361183241434822606848
2361183241434822606848

18889465931478580854784
94447329657392904273 92
188894659314785808547 84
1416709944860893564108 80
472236648286964521369 6
472236648286964521369 0
188894659314785808547 84
944473296573929042739 2
708354972430446782054 4
444732965739290427392
236118324143482260684 8
944473296573929042739 2
472236648286964521369 6
708354972430446782054 4
188894659314785808547 84
236118324143482260684 8
236118324143482260684 8
141670994486089356410 88
708354972430446782054 4
472236648286964521369 6

557518629963265578538392956816209037649510 4 Ship.
111503725992653115707678591363241807529902 08 = 144

4)223007451985306231415357182726483615059804 15 T. Sum.

12)55751862996326557853839295681620903764951 03

20)4645971916360546487819941306801741980412 58 — 7d.

£232298595818027324390997065340087099020 12 — 18s. 7d.

Which is the exact amount of the whole number of ships; and is computed according to the conditions of the question.

Now to reduce this to solid gold, divide the amount by 53l. 2s. 8d. (or 12752d.) and it will give 437200417474321287158037616173435830 2 the number of solid inches the mass would contain. And in order to compare this mass with the solidity of the earth, this also must be reduced to inches. The number of solid miles contained in the earth (see page 99) is 263858149120, which multiplied by 254358388736000 the number of cubic inches in 1 mile, gives 66646680917786616312320000 solid inches; by which divide the number of solid inches in the whole mass, and the quotient will be the number of globes, (equal in magnitude to the earth,) contained in the mass, which is the Answer required: as follows.

Solid inches in the earth.	Solid inches in the whole mass.	Number of Globes.
6664668091778-)		Ans.

6616312320000)437200417474321287158037616173435830 2(65599728517
3998800855067196978739200 00

373203319676015892841176161
333233404588935081561600000

399699150870828112795761617
333233404588933081561600000

666574628189503123416161 73
599820128260079546810880000

648373345588707755307361734
599820128260079546810880000

485532173286282084964817343
466526766424506314186240000

190054068616757707795773435
133293361835573232624640000

567607067811844751711334358
533173447342292930498560000

344736204695518212127743583
333233404588933081561600000

115028001065851305661435830
666466809177866163123200 00

483813201480646893491158302
466526766424506314186240000

And something over.

Thus we have proved, what at first might not seem to be easy of belief, that the whole amount in Sterling Money is twenty three thousand two hundred and twenty nine millions of millions of millions of millions of millions of millions, eight hundred fifty nine thousand five hundred and eighty one millions of millions of millions of millions of millions, eight hundred two thousand seven hundred and thirty two millions of millions of millions of millions, four hundred thirty nine thousand and ninety nine millions of millions of millions of millions, seven hundred six thousand five hundred and thirty four millions of millions, eighty seven thousand and ninety nine millions, nine hundred two thousand and twelve pounds, eighteen shillings and seven pence.

And the number of solid globes of gold, equal to the earth we inhabit, is sixty five thousand five hundred and ninety nine millions, seven hundred twenty eight thousand five hundred and seventeen.

POSITION.

Position is a rule, which by false, or supposed numbers taken at pleasure, discovers the true one required.

It is divided into two parts, Single and Double.

SINGLE POSITION.

Single Position teaches to resolve such questions, whose results are proportional to their supposition; and is when the proportions of the *required number* are implied in the conditions of the question.

RULE.

1. Take any number and perform the same operations with it, as are described to be performed in the question.

2. Then say; as the result of the operation : is to the given sum in the question : : so is the supposed number : to the true one required.

EXAMPLES.

1. A Schoolmaster being asked how many schollars he had, said, if I had as many more as I now have, half as many, and one fourth as many, I should then have 99; how many schollars had he? As 110 : 99 : : 40 : 36 Ans.

Suppose he had	40	36
as many =	40	18
½ as many =	20	9
¼ as many =	10	—
	—	Proof 99
Result	110	

Or,—As 110 : 40 : : 99 : 36 the Ans.

2. A person after spending one third and one fourth of his money, had $ 60 left; what had he at first? Ans. $ 144.

3. What number is that, a sixth part of which exceeds an eighth part of it by 20? Ans. 480

4. What sum of money is that, whose third part, fourth part, and fifth part, added together, amount to 94 dollars?

Ans. $ 120

5. In a mixture of corn and oats, ½ of the whole plus 25 bushels was corn, ⅓ part minus 5 bushels was oats; how many bushels were there of each? Ans. 85 of corn, and 35 of oats.

6. What number is that, from which if 5 be subtracted, $\frac{2}{3}$ of the remainder will be 40? Ans. 65.

7. Two travellers, A. and B. 360 miles apart, travel towards each other till they meet. A's progress is 10 miles in an hour, and B's 8; how far does each travel before they meet?

Ans. A. goes 200 miles, and B. 160.

8. If a certain number be divided by 12, the quotient, dividend and divisor added together will amount to 64; what is the number? Ans 64.

9. A. man spent one third of his life in England, one fourth of it in Scotland, and the remainder of it, which was 20 years, in the United States; to what age did he live?

Ans. to the age of 48.

DOUBLE POSITION.

Double Position teaches to resolve questions by making two suppositions of false numbers.

RULE.

1. Take any two convenient numbers, and proceed with each according to the conditions of the question.

2. Find how much the results are different from the result in the question.

3. Multiply the first position by the last errour, and the last position by the first errour.

4. If the errours are alike, divide the difference of the products by the difference of the errours, and the quotient will be the answer.

5. If the errours are unlike, divide the sum of the products by the sum of the errours, and the quotient will be the answer.

Note. The errours are said to be alike, when they are both too great, or both too small; and unlike, when one is too great and the other too small.

EXAMPLES.

1. The ages of 4 persons amount together, to 109 years, A is 7 years older than B, and C is 10 years younger than A, and D is $\frac{3}{5}$ as old as A; required the age of each.

1st. Suppose		2d. Suppose	
A's age	= 40	A.	= 30
B's "	= 33	B.	= 23
C's "	= 30	C.	= 20
D's "	= 24	D.	= 18
	127		91
	109		109
1st. errour	18	2d. errour	18

The errours being *unlike* or one too great, and the other too small.

	Pos.	Err.
	40	30
Therefore	X	
	18	18
	30	40
	540	720

A's = 35
B's = 28
C's = 25
D's = 21

Proof 109

$18 + 18 = 36)1260(35 =$ A's age.

2. Three merchants enter into partnership with a stock of $ 1140, A put in a certain sum, B put in one third as much as A and $ 50 more, and C put in twice as much as B, together with a fifth of what A put in; what was each one's respective share in the stock?

Ans. A put in $ 450, B $ 200, and C $ 490

3. The ages of two persons A and B are such, that 7 years ago, A was three times as old as B; and 7 years hence, A will be twice as old as B; what are their respective ages?

Ans. A's age is 49, and B's 21 years

4. Three persons A, B, and C, purchase a horse for 100 dollars, but neither is able to pay for the whole: the payment would require

The whole of A's money, together with half of B's; or
The whole of B's, with one third of C's; or
The whole of C's, with one fourth of A's;
How much money had each?

Ans. A, had $ 64, B $ 72, and C $ 84

5. The sum of the distances which 3 persons travelled, is 62 miles; A travelled 4 times as far as C, added to twice the dis-

tance that B travelled, and had C travelled 17 times as far as he did, he would then have travelled 3 times as far as B, added to twice the distance that A travelled; required their respective distances? Ans. A travelled 46 miles, B 9, and C 7 miles.

PERMUTATION OF QUANTITIES.

THE PERMUTATION OR VARIATION OF QUANTITIES is the showing how many different ways the order or position of any given number of things may be changed.

To find the number of permutations or changes, that can be made of any given number of things, all different from one another.

RULE.

Multiply all the terms of the natural series of numbers, from one up to the given number, continually together, and the last product will be the answer required.

EXAMPLES.

1. How many changes can be made of the letters in the word *and?*

Proof

1	a	n d
2	a	d n
3	n	a d
4	n	d a
5	d	a n
6	d	n a

1 X 2 X 3 = 6 Ans.

2. How many changes can be rung on 12 bells?
Ans. 479001600

3. How long could a family of 9 persons vary their position at dinner? Ans. 994 years 80 days.

4. How many changes can be made (in position) of the 8 notes in musick? Ans. 40320

5. How many variations may be made of the letters in the English alphabet? Ans. 403291461126605635584000000

CONSTRUCTION OF THE FOLLOWING TABLES BELONGING TO COMPOUND INTEREST.

THE CONSTRUCTION OF THESE TABLES by logarithms, will be best understood by the following proposition. Viz.

Between two numbers given, to find any number of mean proportionals required.

RULE.

1. From the logarithm of the greater number subtract the logarithm of the less, and divide the remainder by the *number of means* increased by 1

2. Add the quotient to the logarithm of the less number, and the sum will be the logarithm of the 1st mean proportional required.

3. To the logarithm last found, add the said quotient, and the sum will be the logarithm of the second mean proportional; and thus proceed, always adding the said quotient to the logarithm of the last proportional found, as far as the question requires.

EXAMPLES.

Required to find, between 16 and 64, 5 mean proportionals.

Logarithm of 64	=	1·8061800
Do. of 16	=	1·2041200
The difference	=	0·6020600
To $\frac{1}{6}$ part for 5 means	=	0·1003433
Add logarithm of 16	=	1·2041200
1st. Mean proportional = 20·158	=	1·3044633
To which add said quotient	=	0·1003433
2d. Mean proportional = 25·398	=	1·4048066
Add quotient		0·1003433
3d. Mean proportional = 32	=	1·5051499
Add quotient		0·1003433
4th. Mean proportional = 40·317	=	1.6054932
Add quotient		0·1003433
5th Mean proportional = 50·796	=	1.7058365

For the construction of the first two following tables are several methods used. We shall mention only that which is most easy and expeditious; which is by logarithms.

For the first Table thus: Find the amount (as already taught) of 1 dollar or 1 pound for 40 years at the given rate per cent, and between the logarithm of the amount and the logarithm of the rate find 40 geometrical mean proportionals, by the last proposition; and these will be the logarithms of the numbers in the first Table. Or,

If we add the logarithm of the rate continually to itself, it will give the same result: Thus, adding the logarithm of the rate to itself, gives the logarithm belonging to the second year, and to this sum adding again the logarithm of the rate, gives the logarithm of the number belonging to the third year, &c. Or, if you multiply the logarithm of the rate, by the numbers 1, 2, 3, 4, 5, 6, &c. gives the logarithm of the numbers belonging to those respective years:

And for the numbers in the second Table find the Arithmetical Complements of the logarithms of the numbers in the first Table, and you will have the logarithms of the numbers in the second Table.

The Logarithmical differences of 045, or 05, or 06, (being the rates here used, *minus unity*,) and the numbers in Table I, are the logarithms of the numbers in Table III.

If, from the logarithms of the numbers in Table III. you subtract the logarithms of the numbers in Table I. you will have the logarithms of the numbers in Table IV.

And their Arithmetical Complements are the logarithms of the numbers in Table V.

TABLE I. *Shewing the amount of* £ 1, *or* £1 *from* 1 *year to* 40.

Y.	4½ *per. ct.*	5 *per. ct.*	6 *per. ct.*
1	1·045000	1·050000	1·060000
2	1·092025	1·102500	1·123600
3	1·141166	1·157625	1·191016
4	1·192518	1·215506	1·262476
5	1·246181	1·276281	1·338225
6	1·302260	1·340095	1·418519
7	1·360861	1·407100	1·503630
8	1·422100	1·477455	1·593848
9	1·486095	1·551328	1·689478
10	1·552969	1·628894	1·790847
11	1·622853	1·710339	1·898298
12	1·695881	1·795856	2·012196
13	1·772196	1·885649	2·132928
14	1·851944	1·979931	2·260903
15	1·935282	2·078928	2·396558
16	2·022370	2·182874	2·547271
17	2·113376	2·292018	2·692772
18	2·208478	2·406619	2·854339
19	2·307860	2·526950	3·025599
20	2·411714	2·653297	3·207135
21	2·520241	2·785962	3·399563
22	2·633652	2·925260	3.603537
23	2·752166	3·071523	3.819749
24	2·876013	3·225099	4.048934
25	3·005434	3·386354	4.291870
26	3·140679	3·555672	4.549382
27	3·282009	3 733456	4 822345
28	3·429699	3·920129	5.111686
29	3·584036	4·116135	5.418387
30	3·745318	4·321942	5.743491
31	3·915857	4 538039	6.088100
32	4 089981	4·764941	6.453386
33	4·274030	5·003188	6.840589
34	4·466561	5·253347	7.251025
35	4·667347	5·516015	7.686086
36	4·877378	5 791810	8.147252
37	5·096860	6 081406	8.636087
38	5 326219	6·385477	9.154252
39	5·565899	6 704751	9 703507
40	5·816464	7 039988	10 285717

TABLE II. *Shewing the present value of* £ 1, *or* £1 *due at the end of any number of years, from* 1 *to* 40

4½ *per. ct.*	5 *per. ct.*	6 *per. ct.*
·956937	·952381	943396
·915730	·907030	889996
·876297	·863838	·839619
·83856	·822702	·792093
·802451	·783526	·747258
·767896	·746215	·704960
·734828	·710681	·665057
·703185	·676839	·627412
·672904	·644609	·591898
·643928	·613913	·558394
·616199	·584679	·562787
·589664	·556837	·496969
·564271	·530321	·468839
·539973	·505068	·442300
·516720	·481017	·417265
·494469	·458111	·393647
·473176	·436297	·371364
·452800	·415521	350343
·433302	·395734	·330513
·414643	·376889	·311804
·396787	·358942	·294155
·379701	·341850	·277505
·363350	·325571	·261797
·347703	·310068	·246978
·332731	·305303	·232998
·318402	·281241	·219810
·304691	·267848	·207368
·291571	·255094	·175630
·279015	·242946	·184555
·267000	·231377	·174110
·255532	·220359	·164255
·244500	·209866	·154957
·233971	·199872	·146186
·223896	·190355	·137912
·214251	·181290	·130105
·205028	·172057	·122741
·196299	·164430	·115793
·187750	·156605	·109182
·179659	·149148	·103002
·171929	·142046	·097170

TABLE III. *Shewing the amount of* £1 *or* £1 *annuity for any number of years, from* 1 *to* 40.

Y.	4½ *pr. cent.*	5 *pr. cent.*	6 *pr. cent*
1	1·000000	1·000000	1·000000
2	2·045000	2·050000	2·060000
3	3·137025	3·152505	3·183600
4	4·278191	4·310125	4·374616
5	5·470710	5·525631	5·637093
6	6·716892	6·801913	6·975318
7	8·019152	8·142008	8·393837
8	9·380014	9·549109	9·897467
9	10·802114	11·026564	11·491315
10	12·288200	12·577892	13·180794
11	13·841179	14·206787	14·971642
12	15.494032	15·917126	16·869940
13	17·159913	17·712985	18·882132
14	18.932109	19·598632	21·015064
15	20·784054	21·578563	23·275968
16	22·719337	23·657492	25·672527
17	24·741707	25·840366	28·212879
18	26·855084	28·132385	30·905652
19	29·063562	30·529003	33·759991
20	31·371423	33·065954	36·785590
21	33·783137	35·719252	39·992725
22	36·303378	38·505214	43·392289
23	38·937030	41·430475	46·995826
24	41·689196	44·501999	50·815576
25	44.565210	47·727099	54·864510
26	47·570645	51·114454	59·156381
27	50·711324	54·669126	63 705763
28	53·993333	58·402583	68·528109
29	57·423033	62·322712	73·639796
30	61·007067	66·438847	79·058183
31	64·752388	70·760790	84 801674
32	68·666245	75·298829	90.889775
33	72·756226	80·063771	97·343161
34	77·030256	85 066959	104·183751
35	81·496618	90·320307	111·434776
36	86·163966	95 836323	119·120863
37	91·041344	101·628139	127·268114
38	96·138205	107·709546	435·904201
39	101·464424	114·095025	145·058458
40	107·030323	120·799774	154 76196[illegible]

TABLE IV. *Shewing the present worth of* £1 *or* £1 *annuity, for any number of years, from* 1 *to* 40.

4½ *pr. ct.*	5 *pr. ct.*	6 *pr. ct.*
0·95694	0·95238	0·94339
1·87267	1 85941	1.83339
2·74896	2·72325	2·673o1
3·58752	3·54595	3·46510
4·38997	4·32948	4·21236
5·15789	5·07569	4·91732
5·89270	5·78637	5·58238
6.59589	6·46321	6·20976
7·26879	7·10782	6·80169
7·91272	7·72173	7·36008
8·52892	8·30641	7·88687
9·11858	8·86325	8 38384
9·68285	9·39357	8·85268
10·22282	9·89864	9·29498
10·73954	10·37966	9·71225
11·23401	10·83777	10 10589
11·70719	11·27407	10·47726
12·15099	11·68958	10 82760
12·59329	12·08532	11·15811
13 00793	12·46221	11 46992
13·40793	12·82115	11·76407
13·78442	13·16300	12·04158
14 14777	13·48807	12·30338
14·49548	13·79864	12·55035
14·82821	14·09394	12·78335
15·14661	14 37518	13 00316
15·45130	14·64303	13·21053
15 74287	14·89813	13·40616
16·02189	15·14107	13·59072
16 28889	15·37245	13·76483
16·54439	15·59281	13·92908
16 78889	15 80268	14 03398
17 02286	16·00255	14·22917
17·24676	16 19290	14 36613
17·46101	16·37419	14 49533
17·66604	16·54685	14·61722
17·86224	16·71129	14 73211
18 04999	16·86789	14 84048
18·22965	17·01704	14·94270
18·40158	12.15900	15·05913

The annuity which $1, *or* £1 *will purchase for any number of years to come, from* 1 *to* 40.

Yrs.	4½ per cent.	5 per cent.	6 per cent.	Yrs.
1	1·04500	1·05000	1·06000	1
2	·53400	·53780	·54544	2
3	·36377	·36721	·37411	3
4	·27874	·28201	·28859	4
5	·22779	·23097	·23739	5
6	·19388	·19702	·20336	6
7	·16970	·17282	·17913	7
8	·15161	·15473	·16103	8
9	·13757	·14069	·14702	9
10	·12638	·12950	·13587	10
11	·11725	·12039	·12679	11
12	·10967	·11282	·11927	12
13	·10327	·10645	·11296	13
14	·09782	·10102	·10758	14
15	·09311	·09624	·10296	15
16	·08901	·09227	·09895	16
17	·08542	·08870	·09544	17
18	·08224	·08555	·09235	18
19	·07941	·08274	·08962	19
20	·07688	·08024	·08718	20
21	·07460	·07800	·08500	21
22	·07254	·07597	·08303	22
23	·07068	·07414	·08128	23
24	·06899	·07247	·07968	24
25	·06744	·07095	·07823	25
26	·06602	·06956	·07690	26
27	·06472	·06829	·07570	27
28	·06352	·06712	·07459	28
29	·06241	·06604	·07358	29
30	·06139	·06505	·07272	30
31	·06044	·06413	·07179	31
32	·05956	·06328	·07100	32
33	·05874	·06249	·07027	33
34	·05798	·06175	·06959	34
35	·05727	·06107	·06899	35
36	·05660	·06043	·06839	36
37	·05224	·05984	·06785	37
38	·05540	·05928	·06735	38
39	·05485	·05876	·06689	39
40	·05434	·05828	·0664	40

PRACTICAL ASTRONOMY.

Containing a number of Astronomical Tables, and an easy method of calculating the times of NEW AND FULL MOONS, *and* ECLIPSES *by them.*

OF ASTRONOMICAL TABLES AND THEIR CONSTRUCTION.

IN constructing tables for computing, at any given instant, the places of the Sun, Moon, and Planets, the first step is to determine, from a series of accurate observations, the time in which those bodies describe a space of 360 degrees, or perform a complete revolution round the Sun, or primary Planet.

When this important element is exactly ascertained, we can easily find, by simple proportion, the space which any Planet describes in any number of years, months, days, hours, minutes, and seconds, upon the supposition that it moves uniformly, or describes equal spaces in equal time, in the circumference of a circle.

But as it has been found from a long series of observations, that all the bodies of the solar system move in eliptical orbits round the Sun, or their primary Planet, placed in one of the foci, we must next determine the form of their orbits, or the nature of the ellipse.* which they describe.

The diameters of the Sun and Moon therefore, subtend different angles at different times, as they are nearer, or more remote from the observer's eye. This proves that the Sun and Moon are constantly changing their distances from the Earth; and they are once at their greatest, and once at their least distance from it, in little more than a complete revolution.

The gradual differences of these angles are not what they would be, if the luminaries moved in circular orbits, the Earth

* An ellipse is a curvilinear figure of an oblate oval form, having two centres called FOCI, or FOUCUSES: The Sun is in the focus of the Earths orbit, and the Earth is in or near that of the Moon's orbit.

being supposed to be placed at some distance from the centre of the orbit, and the centre of the Earth to be in the lower focus of each orbit.

The fartherest point of each orbit from the Earth's centre is called the APOGEE, and the nearest point is called the PERIGEE. These points are diametrically opposite to each other.

Astronomers divide each orbit into 12 equal parts called SIGNS; each sign into 30 equal parts called DEGREES; each degree into 60 equal parts called MINUETS; each minuet into 60 equal parts called SECONDS.

The distance of the Sun or Moon from any given point of its orbit, is reckoned in signs, degrees, minuets, and seconds. Herein is meant the distance that the luminary has moved through from any given point: and not the space it is short of it in coming round again, though it be ever so little.

The distance of the Sun or Moon from its apogee at any given time, is called its MEAN ANOMALY: therefore when the body is in its *apogee*, its anomaly is 0, and in its *perigee*, it is 6 signs.

The motion of the Sun and Moon are observed to be continually accellerated from their apogee to their perigee, and as gradually retarded from their perigee to their apogee; moving with the greatest velocity when the anomaly is 0, and with the least, when the anomaly is 6 signs.

When the luminary is in its apogee or its perigee, its place is the same as it would be, if its motion were equable in all parts of its orbit. The supposed equable motions are called MEAN; the unequable motions are justly called the True.

The mean place of the Sun or Moon is always forwarder than the true place,* while the luminary is moving from its apogee to its perigee; and the true place is always forwarder than the mean, while the luminary is moving from its perigee to its apogee. In the former case the anomaly is always less than 6 signs; and in the latter case, more.

The Moon's orbit crosses the ecliptick in two opposite points, which are called her Nodes; and the time in which she revolves from the Sun to the Sun again, (or from change to change) is called a LUNATION, and would always consist of 29 days, 12 hours, 44 minuets, 3 seconds, 2 thirds, and 58 fourths, if the motions of the Sun and Moon were always equable. Hence, 12 mean lunations contain 354 days, 8 hours, 48 minuets, 36 seconds, 35 thirds, and 40 fourths, which is 10 days, 21 hours, 11 minuets, 23 seconds, 24 thirds, and 20 fourths, less than the length of a common JULIAN YEAR, consisting of 365 days 6 hours; and 13

*The point of the ecliptick in which the Sun or Moon is at any moment of time is called the PLACE of the Sun or Moon at that time.

mean lunations contain 383 days, 21 hours, 32 minuets, 39 seconds, 38 thirds, and 38 fourths, which exceeds the length of a common JULIAN YEAR, by 18 days, 15 hours, 32 minuets, 39 seconds, 38 thirds, and 38 fourths.

The mean time of New Moon being found for any given year and month, as suppose for March 1850, *New Style*, if this mean New Moon happens later than the 11th day of March, then 12 mean lunations, added to the time of this mean New Moon, will give the time of the mean New Moon in March 1851, after abating 365 days. But when the mean New Moon happens to be *before* the 11th of March, we must add 13 mean lunations, in order to have the time of mean New Moon in March the year following; observing always to subtract 365 days in common years, and 366 days in leap-years, from the sum of this addition.

Thus, A. D. 1850, *New Style*, the time of mean New Moon in March, was the 12th day, at 22 hours and 11 seconds past the noon of that day (viz. at 11 seconds past X in the morning of the 13th day, according to common reckoning.) To this we must add 12 mean lunations, or 354 days, 8 hours, 48 minuets, 36 seconds, 35 thirds, and 40 fourths, and the sum will be 367 days, 6 hours, 48 minuets, 47 seconds, 35 thirds and 40 fourths; from which subtract 365 days, because the year 1851 is a common year, and there will remain 2 days, 6 hours, 48 minuets, 47 seconds, 35 thirds and 40 fourths, for the time of mean New Moon in March, A. D. 1851.

Now to find the mean time of New Moon in March A. D. 1852, we must add 13 mean lunations to the mean time of New Moon in the next preceeding year, (because it happened before the 11th day) and the sum will be 386 days, 4 hours 21 minuets 27 seconds 13 thirds and 18 fourths; from which subtract 366 days, because the year 1852 is a leap-year, and there will remain 20 days 4 hours 21 minuets 27 seconds 13 thirds and 18 fourths, to be set down for the time of mean New Moon, in March, A. D. 1852.

In this manner was the first two of the following tables constructed to seconds, thirds, fourths; and then written out to the nearest second. The reason why Astronomers choose to begin the year with *March*, is to avoid the inconvenience of adding a day to the tabular time in leap-years after *February*, or subtracting a day therefrom in *January* and *February* in those years; to which all tables of this kind are subject, which begin the year with January, in calculating the times of New or Full Moons.

The mean anomalies of the Sun and Moon, and the Sun's mean distance from the ascending node of the Moon's orbit, are set down in Table III, from one to 13 mean lunations.

The numbers, for 12 lunations, being added to the radical anomalies of the Sun and Moon, and to the Sun's mean distance

from the Moon's ascending node, at the mean time of New Moon in March 1850, (Table II.) will give their mean anomalies, and the Sun's mean distance from the node, at the time of New Moon in *March* 1851; and being added for 13 lunations to those for 1851, will give them for the time of mean New Moon in *March* 1852. And so on as far as you please to continue the table, (which is here carried on from 1752, to the year 1900,) always rejecting 12 signs when their sum exceeds 12, and setting down the remainder as the proper quantity.

If the number of years belonging to A. D. 1700 (in Table I.) be subtracted from those belonging to 1800, we shall have their whole differences in 100 complete Julian years; which accordingly we find to be 4 days 8 hours 10 minuets 52 seconds 15 thirds 40 fourths, with respect to the time of mean New Moon. These being added together 60 times, (always taking care to throw off a whole lunation when the days exceed 29½) making up 60 centuries, or 6000 years, as in Table VI. which was carried on to seconds, thirds, and fourths; and then written out to the nearest second. In the same manner were the respective anomalies and the Sun's distance from the node found, for these centural years; and then (for want of room) written out only to the nearest minuet, which is sufficient in whole centuries. By means of these two tables, we may find the time of any mean New Moon in *March*, together with the anomalies of the Sun and Moon, and the Sun's distance from the node, at these times, within the limits of 6000 years, either before or after any given year in the 18th. century; and the mean time of any New or Full Moon in any month of the year after *March*, by means of the third and fourth tables, within the same limits, as shown in the precepts for calculation.

Thus it would be a very easy matter to calculate the time of any New or Full Moon, if the Sun and Moon moved equably in all parts of their orbits. But we have already observed that their places are never the same as they would be by equable motions, except when their mean anomalies are either 0, or 6 signs; and that their mean places are always forwarder than than their true places, while their anomalies are less than 6 signs; and their true places are forwarder than the mean, while the anomaly is more.

Hence it is evident, that while the Sun's anomaly is less than 6 signs, the Moon will overtake him, or be opposite to him, sooner than she could if his motion were equable; and later while his anomaly is more than 6 signs. The greatest difference that can possibly happen between the mean and true time of New or Full Moon, on account of the inequality of the Sun's motion, is 3 hours 48 minuets 28 seconds; and this is when the Sun's anomaly is either 3 signs 1 degree, or 8 signs 29 degrees; sooner in the first case, and later in the last. In all

other signs and degrees of anomaly, the difference is gradually less, and vanishes when the anomaly is either 0, or 6 signs.

The Sun is in his apogee on the 30th. of *June*, and in his perigee on the 30th. of *December*, in the present age ; so that he is nearer the earth in our winter than in our summer. The proportional difference of the Sun's apparent diameter at these times, is as 983 to 1017.

The Moon's orbit is dilated in winter, and contracted in summer. The greatest difference is found to be 22 minuets 39 seconds ; the lunations increasing gradually in length while the Sun is moving from his apogee, and decreasing in length while he is moving from his perigee to his apogee. On this account the Moon will be later in coming to her conjunction with the Sun, or being in opposition to him,* from *December* till *June*, and sooner from *June* to *December*, than if her orbit had continued of the same size all the year round.

As both these differences depend on the Sun's anomaly, they may be fitly put together into one table and called *The annual, or first equation for reducing the mean to the true syzygy*.† (See Table VII.)

This equational difference is to be subtracted from the time of mean New or Full Moon when the Sun's anomaly is less than 6 signs, and added when the anomaly is more.

At the greatest, it is 4 hours 10 minuets 57 seconds, *Viz.* 3 hours 48 minuets 28 seconds, on account of the Sun's unequal motion, and 22 minuets 29 seconus, on account of the dilation of the Moon's orbit.

This compound equation would be snfficient for reducing the mean time of New or Full Moon to the true time, if the Moon's orbit were of a circular form, and her motion exactly equable in it. But the Moon's orbit is more elliptical than the Sun's, and her motion in it is so much the more unequal. The difference is so great, that she is sometimes in conjunction with the Sun, or in opposition to him, sooner by 9 hours 47 minuets 54 seconds, than she would be if her motion were equable ; and at other times as much later. The former happens when her mean anomaly is 9 signs 4 degrees, and the latter when it is 2 signs 26 degrees. See Table IX.

At different distances of the Sun from the Moon's apogee, the figure of the Moon's orbit becomes different.

It is longest of all, or most excentrick, when the Sun is in

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* The term *conjunction*, when it respects the relation of the Moon to the Sun, signifies New Moon, or Change ; and opposition is used to signify the place of the Moon at her full.

† The word *syzygy* siguifies both the conjunction and opposition of the Sun and Moon.

the same sign and degree either with the Moon's apogee or perigee ; shortest of all, or least excentrick, when the Sun's distance from the Moon's apogee is either 3 signs or 9 signs ; and at a mean state when the distance is either 1 sign 15 degrees, 4 signs 15 degrees, 7 signs 15 degrees, or 10 signs 15 degrees. When the Moon's orbit is at its greatest excentricity, her apogeal distance from the Earth's centre is to her perigeal distance from it, as 1067 is to 933 ; when least excentrick, as 1043 is to 457, and when at the mean state, as 1055 is to 945.

But the Sun's distance from the Moon's apogee is equal to the quantity of the Moon's mean anomaly at the time of New Moon, and by the addition of 6 signs, it becomes equal in quantity to the Moon's mean anomaly at the time of Full Moon. Therefore, a table may be constructed so as to answer all the various irregualities depending on the different excentricities of the Moon's orbit in the syzygies ; and called *The second equation for reducing the mean to the true syzygy*, (See Table IX.) and the Moon's anomaly, when equated by Table VIII. may be made the proper argument for taking out this second equation of time, which must be added to the former equated time, when the Moon's anomaly is less than 6 signs, and subtracted when the anomaly is more.

There are several other inequalities in the Moon's motion, which sometimes bring the true syzygy a litle sooner, and at other times keep it back a little later than it would otherwise be ; but they are so small, that they may be all omitted except two ; the former of which (See Table X.) depends on the difference between the anomalies of the Sun and Moon in the syzygies and, the latter, (see Table XI.) depends on the Sun's distance from the Moon's nodes at these times. The greatest difference arising from the former, is 4 minuets 58 seconds ; and from the latter, 1 minuet 34 seconds.

Besides the tables already mentioned, there are various others annexed in the following, to facilitate the labour of astronomical calculations, and will be treated of in their proper place.

TABLES.

TABLES

FOR CALCULATING THE TRUE TIME OF NEW AND FULL MOON'S AND ECLIPSES.

TABLE I. *The mean Time of New Moon in* March, *Old Style, with the mean Anomalies of the Sun and Moon, and the Sun's Mean Distance from the Moon's Ascending Node, from* A. D. 1700 *to* A. D. 1800 *inclusive.*

Y. of Chr.	Mean New Moon in March.				Sun's mean Anomaly.				Moon's mean Anomaly.				Sun's mean distance from the Node.			
	D.	H.	M.	S.	s	0	′	″	s	0	′	″	s	0	′	″
1700	8	16	11	25	8	19	58	48	1	22	30	37	6	14	31	7
1701	27	13	44	5	9	8	20	59	0	28	7	42	7	23	14	8
1702	16	22	32	41	8	27	36	51	11	7	55	47	8	1	16	55
1703	6	7	21	18	8	16	52	43	9	17	43	52	8	9	19	42
1704	24	4	53	57	9	5	14	54	8	23	20	57	9	18	2	43
1705	13	13	42	34	8	24	30	47	7	3	9	2	9	26	5	30
1706	2	22	31	11	8	13	46	39	5	12	57	7	10	4	8	17
1707	31	20	3	50	9	2	8	50	4	18	34	13	11	12	51	18
1708	10	4	52	27	8	21	24	48	2	28	22	18	11	20	54	5
1709	29	2	25	7	9	9	46	54	2	3	59	24	0	29	37	6
1710	18	11	13	43	8	29	2	47	0	13	47	30	1	7	39	54
1711	7	20	2	20	8	18	18	39	10	23	35	36	1	15	42	41
1712	25	17	34	59	9	6	40	51	9	29	12	42	2	14	25	43
1713	15	2	23	36	8	25	56	43	8	9	0	47	3	2	28	30
1714	4	11	12	13	8	15	12	35	6	18	48	52	3	10	31	17
1715	23	8	44	52	9	3	34	47	5	24	25	57	4	19	14	18
1716	11	17	33	29	8	22	50	39	4	4	14	2	4	27	17	5
1717	1	2	22	5	8	12	6	32	2	14	2	8	5	5	19	52
1718	19	23	54	45	9	0	28	44	1	19	39	13	6	14	2	54
1719	9	8	43	22	8	19	44	37	11	29	27	18	6	22	5	41
1720	27	6	16	1	9	8	6	49	11	5	4	24	8	0	48	43
1721	16	15	4	38	8	27	22	41	9	14	52	29	8	8	51	29
1722	5	23	53	14	8	16	38	33	7	24	40	34	8	16	54	16
1723	24	21	25	54	9	5	0	45	7	0	17	40	9	25	37	18
1724	13	6	14	31	8	24	16	37	5	10	5	45	10	3	40	5
1725	2	15	3	7	8	13	32	29	3	19	53	50	10	11	42	52
1726	21	12	35	47	9	1	54	41	2	25	30	56	11	20	25	54
1727	10	21	24	23	8	21	10	34	1	5	19	1	11	28	28	41
1728	28	18	57	3	9	9	52	46	0	10	50	7	1	7	11	42
1629	18	3	45	40	8	28	48	39	10	20	44	12	1	15	14	29
1730	7	12	34	16	8	18	4	31	9	0	32	17	1	23	17	16
1731	26	10	6	56	9	6	26	42	8	6	9	23	3	2	0	17
1732	14	18	55	33	8	25	42	34	6	15	57	28	3	10	3	4
1733	4	3	44	9	8	14	58	26	4	25	45	33	3	18	5	51
1734	23	1	16	49	9	3	20	38	4	1	22	39	4	26	48	53
1735	12	10	3	25	8	22	36	30	2	11	10	44	5	4	51	40
1736	0	18	54	2	8	11	52	22	0	20	58	49	5	12	54	27
1737	19	16	26	42	9	0	14	34	11	26	35	55	6	21	37	29
1738	9	1	15	18	8	19	30	26	10	6	24	0	6	29	40	16
1739	27	22	47	56	9	7	52	38	9	12	1	6	8	8	23	18

TABLE I. *continued.* *Old Style.*

A. of Christ	Mean New Moon in March.				Sun's mean Anomaly.				Moon's mean Anomaly.				Sun's mean distance from the Node.			
	D.	H.	M.	S.	s	0	′	″	s	0	′	″	s	0	′	″
1740	16	7	35	34	8	27	8	30	7	21	49	11	8	16	26	5
1741	5	16	25	11	8	16	24	22	6	1	37	16	8	24	28	52
1742	24	13	57	52	9	4	46	34	5	7	14	22	10	3	11	54
1743	13	22	46	27	8	24	2	27	3	17	2	27	10	1	14	41
1744	2	7	35	4	8	13	18	20	1	26	50	32	10	19	17	28
1745	21	5	7	44	9	1	40	32	1	2	27	38	11	28	0	30
1746	10	13	56	20	8	20	56	24	11	12	15	43	0	6	3	17
1747	29	11	29	0	9	9	18	36	10	17	52	49	1	14	46	19
1748	17	20	17	36	8	28	34	28	8	21	40	54	1	23	49	5
1749	7	5	6	13	8	17	50	20	7	7	28	59	2	0	51	52
1750	26	2	38	53	9	6	12	32	6	13	6	5	3	9	54	53
1751	15	11	27	29	8	25	28	24	4	22	54	10	3	17	37	40
1752	3	20	16	6	8	14	44	16	3	2	42	15	3	35	49	27
1753	22	17	48	45	9	3	6	2	2	8	19	21	5	4	23	28
1754	12	2	37	22	8	22	22	20	0	18	7	26	5	12	26	15
1755	1	11	25	59	8	11	38	12	10	27	55	31	5	20	29	2
1756	19	8	58	38	9	0	0	24	10	3	32	37	6	29	12	3
1757	8	17	47	15	8	19	16	16	8	13	20	42	7	7	14	50
1758	27	15	19	54	9	7	38	28	7	23	57	48	8	15	57	52
1759	17	0	8	31	8	26	54	20	5	28	45	54	8	24	0	39
1760	5	8	57	8	8	16	10	12	4	8	34	6	9	2	3	26
1761	24	6	29	47	9	4	32	24	3	14	11	6	10	10	46	27
1762	13	15	13	24	8	23	48	16	1	23	59	11	10	18	49	14
1763	3	0	7	1	8	13	4	8	0	3	47	16	10	26	52	1
1764	20	21	39	40	9	1	26	20	11	9	24	21	0	5	35	2
1765	10	6	28	17	8	20	42	13	9	19	12	26	0	13	37	49
1766	29	4	0	56	9	9	4	20	8	24	49	32	1	22	20	51
1767	18	12	49	33	8	28	20	17	7	4	37	37	2	0	23	38
1768	6	21	38	10	8	17	36	9	5	14	25	42	2	8	26	25
1769	25	19	10	40	9	5	58	21	4	20	2	48	3	17	9	27
1770	15	3	59	26	8	25	14	13	2	29	50	53	3	25	12	14
1771	4	12	48	2	8	14	30	5	1	9	38	58	4	3	15	1
1772	22	10	20	43	9	2	52	17	0	15	16	4	5	11	58	3
1773	11	19	9	19	8	22	8	9	10	25	4	9	5	20	0	50
1774	1	8	57	55	8	11	24	1	9	4	52	14	5	28	3	37
1775	20	1	30	25	8	29	46	13	8	10	29	20	7	6	49	38
1776	8	10	19	12	8	19	2	5	6	20	17	25	7	14	49	25
1777	27	7	51	51	9	7	24	17	5	25	54	31	8	23	32	26
1778	16	16	40	28	8	26	40	9	4	5	42	36	9	1	35	13
1779	6	1	29	4	8	15	56	1	2	15	30	41	9	9	38	0
1780	23	23	1	44	9	4	18	13	1	21	7	47	10	18	21	1
1781	13	7	50	21	8	23	34	5	0	0	55	52	10	26	23	48
1782	2	16	38	57	8	12	49	58	10	10	43	57	11	4	26	35
1783	21	14	11	37	9	1	12	10	9	16	21	3	0	13	9	36
1784	9	23	0	13	8	20	28	3	7	26	9	8	0	21	12	23
1785	28	20	52	56	9	9	50	15	7	1	46	14	1	29	55	25
1786	18	5	21	30	8	28	6	7	5	11	34	19	2	7	58	12

TABLE I. *concluded. Old Style.*

Y. of Christ.	Mean New Moon in March.				Sun's mean Anomaly.				Moon's mean Anomaly.				Sun's mean distance from the Node.			
	D.	H.	M.	S.	s	0	′	″	s	0	′	″	s	0	′	″
1787	7	14	10	6	8	17	21	59	3	21	22	24	2	16	0	59
1788	25	11	42	46	9	5	44	11	2	26	59	36	3	24	44	1
1789	14	20	31	23	8	25	0	3	1	6	47	35	4	2	46	48
1790	4	5	19	59	8	14	15	55	11	16	35	40	4	10	49	35
1791	23	2	52	39	9	2	38	7	10	22	12	46	5	19	52	37
1792	11	11	41	15	8	21	53	59	9	2	0	52	5	27	55	24
1793	30	9	13	55	9	10	16	11	8	7	37	58	7	6	18	26
1794	19	18	2	32	8	29	32	3	6	17	26	4	7	14	21	13
1795	9	2	51	8	8	18	47	55	4	27	14	9	7	22	24	0
1796	27	0	23	48	9	7	10	7	4	2	51	14	9	1	7	1
1797	16	9	12	24	8	26	25	59	2	12	39	19	9	9	9	48
1798	5	18	1	1	8	15	41	51	0	22	27	25	9	17	12	35
1799	24	15	23	41	9	4	4	3	11	28	4	31	10	25	55	37
1800	13	0	22	17	8	23	19	55	10	7	52	36	11	3	58	22

TABLE II. *Mean New Moon, &c. in* March, *New Style, from* A. D. 1752 *to* A. D. 1900.

Y. of Christ.	Mean New Moon in March.				Sun's mean Anomaly.				Moon's mean Anomaly.				Sun's mean distance from the Node.			
	D.	H.	M.	S	s	0	′	″	s	0	′	″	s	0	′	″
1752	14	20	16	6	8	14	44	16	3	2	42	15	3	25	40	27
1753	4	5	4	42	8	4	0	8	1	12	30	20	4	3	43	14
1754	23	2	37	22	8	22	22	20	0	18	7	26	5	12	26	15
1755	12	11	25	59	8	11	38	12	10	27	55	31	5	20	29	2
1756	30	8	58	38	9	0	0	24	10	3	32	37	6	29	12	3
1757	19	17	47	15	8	19	16	16	8	13	20	42	7	7	14	50
1758	9	2	35	51	8	8	32	8	9	23	8	47	7	15	17	38
1759	28	0	8	31	8	26	54	20	5	28	45	54	8	24	0	39
1760	16	8	57	8	8	16	10	12	4	8	34	0	9	2	3	26
1761	5	17	45	44	8	5	26	4	2	18	22	5	9	10	6	13
1762	24	15	18	24	8	23	48	16	1	23	59	11	10	18	49	14
1763	14	0	7	1	8	13	4	8	0	3	47	16	10	26	52	1
1764	2	8	55	36	8	2	20	0	10	13	35	21	11	4	54	48
1765	21	6	28	17	8	20	42	13	9	19	12	26	0	13	57	49
1766	10	15	16	53	8	9	58	5	7	29	0	31	0	21	40	37
1767	29	12	49	33	8	28	20	17	7	4	37	37	2	0	23	38
1768	17	21	38	9	8	17	36	9	5	14	25	42	2	8	26	25
1769	7	6	26	46	8	6	52	1	3	24	13	47	2	16	29	13
1770	26	3	59	26	8	25	14	13	2	29	50	53	3	25	12	14
1771	15	12	48	2	8	14	30	5	1	9	38	58	4	3	15	1
1772	3	21	36	39	8	3	45	57	11	19	27	3	4	11	17	48
1773	22	19	9	19	8	22	8	9	10	25	4	9	5	20	0	50
1774	12	3	57	55	8	11	24	1	9	4	52	14	5	28	3	37
1775	1	12	46	31	8	0	39	53	7	14	40	19	6	6	6	24
1776	19	10	19	12	8	19	2	6	6	20	17	25	7	14	49	25

TABLE II. *continued.* *New Style.*

Y. of Christ.	Mean New Moon. in March.				Sun's mean Anomaly.				Moon's mean Anomaly.				Sun's mean distance from the Node.			
	D.	H.	M.	S.	s	0	′	″	s	0	′	″	s	0	′	″
1777	8	19	7	48	8	8	17	57	5	0	5	30	7	22	52	12
1778	27	16	40	28	8	26	40	9	4	5	42	36	9	1	35	13
1379	17	1	29	4	8	15	56	1	2	15	30	41	9	9	38	0
1780	5	10	15	3	8	5	5	54	0	25	22	58	9	17	36	12
1781	24	7	47	40	8	23	28	4	0	1	0	9	10	26	19	14
1782	13	16	36	14	8	12	43	55	10	10	48	18	11	4	22	1
1783	3	1	24	48	8	1	59	47	8	20	36	28	11	12	24	49
1784	20	22	37	25	8	20	21	57	7	26	13	39	0	21	7	50
1785	10	7	45	59	8	9	37	48	6	6	1	49	0	29	10	38
1786	29	5	18	36	8	27	59	58	5	11	38	59	2	7	53	39
1787	18	14	7	10	8	17	15	50	3	21	27	9	2	15	56	26
1788	6	22	55	45	8	6	31	40	2	1	15	19	2	23	59	14
1789	25	20	28	22	8	24	53	51	1	6	52	30	4	2	42	15
1790	15	5	16	56	8	14	9	42	11	16	40	39	4	10	45	3
1791	4	14	5	30	8	3	25	33	9	26	28	49	4	18	47	50
1792	22	11	38	7	8	21	47	44	9	2	6	0	5	27	30	52
1793	11	20	26	41	8	11	3	35	7	11	54	10	6	5	33	39
1294	30	17	59	18	8	29	25	45	6	17	31	20	7	14	16	41
1795	20	2	47	53	8	18	41	36	4	27	19	30	7	22	19	28
1796	8	11	36	27	8	7	57	28	3	7	7	40	8	0	22	16
1797	27	9	9	4	8	26	19	38	2	12	44	51	9	9	5	17
1798	16	17	57	38	8	15	35	29	0	22	33	0	9	17	8	5
1799	6	2	46	12	8	4	51	20	11	2	21	10	6	25	10	52
1800	25	0	18	49	8	23	13	30	10	7	58	21	11	3	53	54
1801	14	9	7	23	8	12	29	22	8	17	46	31	11	11	56	41
1802	3	17	55	58	8	1	45	13	6	27	34	41	11	19	59	29
1803	22	15	28	35	8	20	7	23	6	3	11	51	0	28	42	30
1804	11	0	17	9	8	9	23	14	4	13	0	1	1	6	45	18
1405	0	9	5	43	8	28	39	5	2	22	48	11	1	14	48	5
1806	19	6	38	20	8	17	1	16	1	28	25	21	2	23	31	7
1807	8	15	26	55	8	6	17	7	0	8	13	31	3	1	33	54
1808	26	12	59	31	8	24	39	17	11	13	50	42	4	10	16	56
1809	15	21	48	5	8	13	55	8	9	23	38	52	4	18	19	43
1810	5	6	36	40	8	3	10	59	8	3	27	2	4	26	22	31
1811	24	4	9	17	8	21	33	10	7	9	4	12	6	5	5	32
1812	12	12	57	51	8	10	49	1	5	18	52	22	6	13	8	20
1813	1	21	46	25	8	0	4	52	3	28	40	32	6	21	11	7
1814	20	19	19	2	8	18	27	3	3	4	17	43	7	29	54	9
1815	10	4	7	37	8	7	42	54	1	14	5	52	8	7	56	56
1816	28	1	40	14	8	26	5	4	0	19	43	3	9	16	39	58
1817	17	10	28	48	8	15	20	55	10	29	31	13	9	24	42	45
1818	6	19	17	22	8	4	36	46	9	9	19	23	10	2	45	33
1819	25	16	54	14	8	23	5	44	8	14	49	17	11	11	29	55
1820	14	1	38	33	8	12	14	48	6	24	44	43	11	19	31	22
1821	3	10	27	7	8	1	30	39	5	4	32	53	11	27	34	9
1822	22	7	49	45	8	19	52	50	4	10	10	34	1	6	17	10
1823	11	15	48	19	8	9	8	41	2	19	58	13	1	14	19	58

TABLE II. *continued. New Style.*

Y. of Christ	Mean New Moon in March.				Sun's Mean Anomaly.				Moon's mean Anomaly.				Sun's mean distance from the Node.			
	D.	H.	M.	S	s	0	′	″	s	0	′	″	s	0	′	″
1824	29	14	20	56	8	27	30	51	1	25	35	24	2	23	2	59
1825	18	23	9	30	8	16	46	42	0	5	33	34	3	1	5	47
1826	8	7	58	4	8	6	2	33	10	15	11	44	3	9	8	34
1827	27	5	30	41	8	24	24	44	9	20	48	54	4	17	51	36
1828	15	14	19	15	8	13	40	35	8	0	37	4	4	25	54	23
1829	4	23	7	50	8	2	56	26	6	10	25	15	5	3	7	11
1830	23	20	40	27	8	21	18	36	5	16	2	55	6	12	40	12
1831	13	5	29	1	8	10	34	27	3	25	50	34	6	20	3	0
1832	1	14	17	35	8	29	50	19	2	5	38	44	6	28	45	47
1833	20	11	50	12	8	18	12	29	1	11	15	55	8	7	28	49
1834	9	20	38	46	8	7	28	20	11	21	4	4	8	15	31	36
1835	28	18	11	23	8	25	50	31	10	26	41	15	9	24	14	38
1836	17	2	59	58	8	15	6	22	9	6	29	25	10	2	17	25
1837	6	11	48	32	8	4	22	13	7	16	17	35	10	10	20	13
1838	25	9	21	9	8	22	44	23	6	21	54	46	11	19	3	14
1839	14	18	9	43	8	12	0	14	5	1	42	55	11	27	6	2
1840	3	2	58	17	8	1	16	5	3	11	31	5	0	5	8	49
1841	22	0	30	54	8	19	38	16	2	17	8	16	1	13	51	51
1842	11	9	19	28	8	8	54	7	0	26	56	26	1	21	54	38
1843	30	6	52	6	8	27	16	17	0	2	33	36	3	0	37	40
1844	18	15	40	40	8	16	32	8	10	12	21	46	3	8	40	27
1845	8	0	29	14	8	5	47	60	8	22	9	56	3	16	43	15
1846	26	22	1	51	8	24	10	10	7	27	47	7	4	25	26	16
1847	16	6	50	25	8	13	26	1	6	7	35	16	5	3	29	4
1848	4	15	38	59	8	2	41	5	4	17	23	26	5	11	31	51
1849	23	13	11	36	8	21	4	3	3	23	0	37	6	20	14	53
1850	12	22	0	11	8	10	19	54	2	2	48	47	6	28	17	40
1851	2	6	48	48	7	29	35	46	1	12	36	52	7	6	20	27
1852	20	4	21	27	8	17	57	58	11	18	13	58	8	15	3	28
1853	9	13	10	4	8	7	13	50	9	28	2	3	8	23	6	15
1854	28	10	42	43	8	25	36	2	9	3	39	9	10	1	49	16
1855	17	19	31	20	8	14	51	54	7	13	27	14	10	9	52	3
1856	5	4	19	57	8	4	7	46	5	23	15	19	10	17	54	50
1857	24	1	52	36	8	22	29	58	4	28	52	25	11	26	37	51
1858	13	10	41	13	8	11	43	50	3	8	40	30	0	4	40	38
1859	2	19	29	49	8	1	1	42	1	18	28	35	0	12	43	25
1860	20	17	2	29	8	19	23	54	0	24	5	41	1	21	46	26
1861	10	1	51	6	8	8	39	46	11	3	53	46	1	29	49	13
1862	28	22	23	45	8	27	1	58	10	9	30	52	3	8	32	14
1863	17	7	12	22	8	16	17	50	8	19	18	57	3	16	35	1
1864	5	16	0	59	8	6	33	42	6	29	7	2	3	24	37	48
1865	24	13	33	38	8	24	55	54	6	4	14	8	5	3	20	49
1866	13	22	22	15	8	14	11	46	4	14	2	13	5	11	23	36
1867	3	7	10	51	8	3	27	38	2	23	50	18	5	19	26	23
1868	21	4	43	31	8	21	49	50	1	29	27	24	6	28	9	24
1869	10	13	31	8	8	11	5	42	0	9	15	29	7	6	12	11
1870	28	11	4	47	8	29	27	54	11	14	52	35	8	14	55	12

TABLE II. *concluded. New Style.*

Y. of Christ.	Mean New Moon in March.				Sun's Mean Anomaly.				Moon's mean Anomaly.				Sun's mean distance from the Node.			
	D.	H.	M.	S.	s	0	′	″	s	0	′	″	s	0	′	″
1871	17	12	53	24	8	18	45	46	9	24	40	40	8	22	57	59
1872	5	4	42	0	8	7	59	38	8	4	28	45	9	1	0	46
1873	24	2	15	40	8	26	21	50	7	10	5	51	10	9	43	47
1874	13	11	4	17	8	15	37	42	5	19	53	56	10	17	46	34
1875	2	19	52	53	8	4	53	34	3	29	42	1	10	25	49	21
1876	20	11	25	33	8	23	15	46	3	5	19	7	0	4	32	22
1877	10	2	14	10	8	12	31	38	1	15	7	12	0	12	35	9
1878	28	23	46	49	9	0	53	50	0	20	44	18	0	21	18	10
1879	18	8	35	26	8	20	9	42	11	0	32	23	1	29	20	57
1880	6	17	24	2	8	9	25	34	9	10	20	28	2	7	23	44
1881	25	14	56	42	8	27	47	46	8	15	57	34	3	16	6	45
1882	14	23	45	19	8	17	3	38	6	25	45	39	3	24	9	32
1883	4	8	33	55	8	6	19	30	5	5	33	44	4	2	12	19
1884	22	6	6	35	8	24	41	42	4	11	10	50	5	10	55	20
1885	11	14	55	11	8	13	57	34	2	20	58	55	5	18	58	7
1886	0	23	43	48	8	3	13	26	1	0	47	0	5	27	0	54
1887	18	21	16	28	8	21	35	38	0	6	24	6	7	5	43	55
1888	7	6	5	4	8	10	51	30	10	16	12	11	7	13	46	42
1889	26	3	37	44	8	29	13	42	9	21	49	17	8	22	29	43
1890	15	12	26	21	8	18	29	34	8	1	37	22	9	0	32	30
1891	4	21	14	57	8	7	45	26	6	11	25	27	9	8	35	17
1892	22	18	47	37	8	26	7	38	4	17	2	33	10	17	18	18
1893	12	3	36	13	8	15	23	30	2	25	50	38	10	25	21	5
1894	1	12	24	50	8	4	39	22	1	6	38	43	11	3	23	52
1895	20	9	57	30	8	23	1	34	0	12	15	49	0	12	6	53
1896	8	18	46	6	8	12	17	26	10	22	3	54	0	29	9	40
1897	27	15	18	47	9	0	39	38	9	21	41	0	1	28	52	41
1898	17	0	7	23	8	19	35	30	8	7	29	5	2	6	55	28
1899	6	8	56	0	8	9	11	22	6	17	17	10	2	14	58	15
1900	24	6	28	40	8	27	33	34	4	22	54	16	3	23	41	16

TABLE III. *Mean Anomalies, and Sun's mean Distance from the Node, for* 13½ *mean Lunations.*

No.	Mean Lunations.				Sun's mean Anomaly.				Moon's mean Anomaly				Sun's mean distance from the Node.			
	D.	H.	M.	S	s	0	′	″	s	0	′	″	s	0	′	″
1	29	12	44	3	0	29	6	19	0	25	49	0	1	0	40	14
2	59	1	28	6	1	28	12	39	1	21	38	1	2	1	20	28
3	88	14	12	9	2	27	18	58	2	17	27	1	3	2	0	42
4	118	2	56	12	3	26	25	17	3	13	16	2	4	2	40	56
5	147	15	40	15	4	25	31	37	4	9	5	2	5	3	21	10
6	177	4	14	18	5	24	37	56	5	4	54	3	6	4	1	24
7	206	17	8	21	6	23	44	15	6	0	43	3	7	4	41	38
8	236	5	52	24	7	22	50	35	6	26	32	3	8	5	21	52
9	265	18	36	27	8	21	56	54	7	22	21	4	9	6	2	6
10	295	7	20	30	9	21	3	14	8	18	10	4	10	6	42	20
11	324	20	4	33	10	20	9	33	9	13	59	5	11	7	22	34
12	354	8	48	30	11	19	15	52	10	9	48	5	0	8	2	47
13	383	21	32	40	0	18	22	12	11	5	37	6	1	8	43	1
½	14	18	22	2	0	14	33	10	6	12	54	30	0	15	20	7

TABLE IV. *The Days of the Year, reckoned from the beginning of* March.

Days.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	January.	February.
1	1	32	62	93	123	154	185	215	246	276	307	338
2	2	33	63	94	124	155	186	216	247	277	308	359
3	3	34	64	95	125	156	187	217	248	278	309	340
4	4	35	65	96	126	157	188	218	249	279	310	341
5	5	36	66	97	127	158	189	219	250	280	311	342
6	6	37	67	98	128	159	190	220	251	281	312	343
7	7	38	68	99	129	160	191	221	252	282	313	344
8	8	39	69	100	130	161	192	222	253	283	314	345
9	9	40	70	101	131	162	193	223	254	284	315	346
10	10	41	71	102	132	163	194	224	255	285	316	347
11	11	42	72	103	133	164	195	225	256	286	317	448
12	12	43	73	104	134	165	196	226	257	287	318	349
13	13	44	74	105	135	166	197	227	258	288	319	350
14	14	45	75	106	136	167	198	228	259	289	320	351
15	15	46	76	107	137	168	199	229	260	290	321	352
16	16	47	77	108	138	169	200	230	261	291	322	353
17	17	48	78	109	139	170	201	231	262	292	323	354
18	18	49	79	110	140	171	202	232	263	293	324	355
19	19	50	80	111	141	172	203	233	264	294	325	356
20	20	51	81	112	142	173	204	234	265	295	326	357
21	21	52	82	113	143	174	205	235	266	296	327	358
22	22	53	83	114	144	175	206	236	267	297	328	359
23	23	54	84	115	145	176	207	237	268	298	329	360
24	24	55	85	116	146	177	208	238	269	299	330	361
25	25	56	86	117	147	178	209	239	270	300	331	362
26	26	57	87	118	148	179	210	240	271	301	332	363
27	27	58	88	119	149	180	211	241	272	302	333	364
28	28	59	89	120	150	181	212	242	273	303	334	365
29	29	60	90	121	151	182	213	243	274	304	335	366
30	30	61	91	122	152	183	214	244	275	305	336	
31	31		92		153	184		245		306	337	

TABLE V. *Mean Lunations from* 1 *to* 100000.

Lunar.	Days Deci. parts.	Days	Hou.	M.	S.	Th.	Fo.
1	29·530590851080	= 29	12	44	3	2	58
2	59·061181702160	59	1	28	6	5	57
3	88·591772553240	88	14	12	9	8	55
4	118·122363404320	118	2	56	12	11	53
5	147·652954255401	147	15	40	15	14	52
6	177·183545106481	177	4	24	18	17	50
7	206·714135957561	206	17	8	21	20	48
8	236·244726808641	236	5	52	24	23	47
9	265·775317659722	265	18	36	27	26	45
10	295·30590851080	295	7	20	30	29	43
20	590·61181702160	590	14	41	0	59	26
30	885·91772553240	885	22	1	31	29	10
40	1181·22363404320	1181	5	22	1	58	53
50	1476·52954255401	1476	12	42	32	28	36
60	1771·83545106481	1771	20	3	2	58	19
70	2067·14135957561	2067	3	23	33	28	2
80	2362·44726808641	2362	10	44	3	57	46
90	2657·75317659722	2657	18	4	34	27	29
100	2953·0590851080	2953	1	25	4	57	12
200	5906.1181702160	5906	2	50	9	54	24
300	8859·1772553240	8859	4	15	14	51	36
400	11812·2363404320	11812	5	40	19	48	48
500	14765·2954255401	14765	7	5	24	46	0
600	17718·3545106481	17718	8	30	29	43	12
700	20671·4135957561	20671	9	55	34	40	24
800	23624·4726808641	23624	11	20	39	37	36
900	26577·5317659722	26577	12	45	44	34	48
1000	29530·590851080	29530	14	10	49	32	0
2000	59061·181702160	59061	4	21	39	4	0
3000	88591·772553140	88591	18	32	28	36	0
4000	118122·363404320	118122	8	43	18	8	0
5000	147652·954255401	147652	22	54	7	40	0
6000	177183·545106481	177183	13	4	57	12	0
7000	206714·135957561	206714	3	15	46	44	0
8000	236244·726801641	236244	17	26	36	16	0
9000	265775·317659722	265775	7	37	25	48	0
10000	295305·90851080	295305	21	48	15	20	0
20000	590611·81702160	590611	19	36	30	40	0
30000	885917 72553240	885917	17	24	46	0	0
40000	1188223·63404320	1188223	15	13	1	20	0
50000	1476529·54255401	1476529	13	1	16	40	0
60000	1771835·45106481	1771835	10	49	32	0	0
70000	2067141·35957561	2067141	8	37	47	20	0
80000	2362447·26808641	2362447	6	25	2	40	0
90000	2657753·17659722	2657753	4	14	18	0	0
100000	2953959·0851080	2953959	2	2	33	20	0

TABLE VI. *The first mean New Moon, with the mean Anomalies of the Sun and Moon, and the Sun's mean Distance from the Ascending Node, next after complete Centuries of Julian Years.*

Lunations.	Julian years	First New Moon.	Sun's mean Anomaly.			Mon's mean Anomaly.			Sun from Node.		
		D. H. M. S.	s	o	′	s	o	′	s	o	′
1237	100	4 8 10 52	0	3	21	8	15	22	4	19	27
2474	200	8 16 21 44	0	6	42	5	0	44	9	8	55
3711	300	13 0 32 37	0	10	3	1	16	6	1	28	22
4948	400	17 8 43 29	0	13	24	10	1	28	6	17	49
6185	500	21 16 54 21	0	16	46	6	16	50	11	7	16
7422	600	26 1 5 14	0	20	7	3	2	12	3	26	44
8658	700	0 20 32 3	11	24	22	10	21	45	7	15	31
9895	800	5 4 42 55	11	27	34	7	7	7	0	4	58
11132	900	9 12 53 47	0	1	4	3	22	29	4	24	25
12369	1000	13 21 4 40	0	4	25	0	7	51	9	13	53
13606	1100	18 5 15 32	0	7	46	8	23	13	2	3	20
14843	1200	22 13 26 24	0	11	7	5	8	35	6	22	47
16080	1300	26 21 37 16	0	14	28	1	23	57	11	12	15
17316	1400	1 17 4 6	11	18	43	9	13	30	3	1	2
18553	1500	6 1 14 58	11	22	4	5	28	52	7	20	29
19790	1606	10 9 25 50	11	25	25	2	14	14	0	9	56
21027	1700	14 17 36 42	11	28	46	10	29	36	4	29	23
22264	1800	19 1 47 35	0	2	8	7	14	58	9	18	51
23501	1900	23 9 58 27	0	5	29	4	0	20	2	8	18
24738	2000	27 18 9 19	0	8	50	0	15	42	6	27	45
25974	2100	2 13 36 8	11	13	5	8	5	15	10	16	32
27211	2200	6 21 47 1	11	16	26	4	20	37	3	6	0
28448	2300	11 5 57 53	11	19	47	1	5	59	7	25	27
29685	2400	15 14 8 45	11	23	8	9	21	21	0	14	54
30922	2500	19 22 19 38	11	26	29	6	6	43	5	4	22
32159	2600	24 6 30 30	11	29	50	2	22	4	9	23	49
33396	2700	28 14 41 22	0	3	11	11	7	26	2	13	16
34632	2800	3 10 8 11	11	7	76	6	26	59	6	2	3
35869	2900	7 18 19 3	11	10	47	3	12	21	10	21	30
37106	3000	12 2 29 56	11	14	8	11	27	43	3	10	58
38343	3100	16 10 40 48	11	17	30	8	13	5	8	0	25
39580	3200	20 18 51 40	11	20	51	4	28	27	0	19	52

TABLE VI. *concluded.*

Lunations.	Julian years.	First New Moon. D	H.	M.	S	Sun's mean Anomaly. s	o	′	Mon's mean Anomaly. s	o	′	Sun from Node. s	o	′
40817	3300	25	3	2	33	11	24	12	1	13	49	5	9	20
42054	3400	29	11	13	25	11	27	33	9	29	11	9	28	47
43290	3500	4	6	40	14	11	1	48	5	18	44	1	17	34
44527	3600	8	14	51	6	11	5	9	2	4	6	6	7	1
45764	3700	12	23	1	59	11	8	30	10	19	28	10	26	29
47001	3800	17	7	12	51	11	11	51	7	4	50	3	15	56
48238	3900	21	15	23	43	11	15	12	3	20	12	8	5	23
49475	4000	25	23	34	35	11	18	33	0	5	34	0	24	50
50711	4100	0	19	1	27	10	22	48	7	25	7	4	13	37
51948	4200	5	3	12	17	10	26	9	4	10	29	9	3	5
53185	4300	6	11	23	9	10	29	31	0	25	51	1	22	32
54422	4400	13	19	34	1	11	2	52	9	11	13	6	11	59
55659	4500	18	3	44	54	11	6	13	5	26	35	11	1	27
56896	4600	22	11	55	46	11	9	34	2	11	57	3	20	54
58133	4700	26	20	6	38	11	12	55	10	27	19	8	10	21
59369	4800	1	15	33	27	10	17	9	6	16	52	11	29	8
60606	4900	5	23	44	20	10	20	21	3	2	14	4	18	36
61843	5000	10	7	55	12	10	23	52	11	17	30	9	8	3
63080	5100	14	16	6	4	10	27	13	8	2	58	1	27	30
64317	5200	19	0	16	56	11	0	34	4	18	20	6	19	57
65354	5300	23	8	27	49	11	3	55	1	3	42	11	6	25
66791	5400	27	16	38	41	11	7	16	9	19	4	2	25	52
68028	5500	2	12	5	30	10	11	31	5	8	37	7	14	39
69265	5600	6	20	16	22	10	14	52	1	23	59	0	4	6
70502	5700	11	4	27	15	10	18	14	10	9	21	4	23	34
71739	5803	15	12	38	7	10	21	35	6	24	43	9	13	1
72976	5900	19	20	48	59	10	24	56	3	10	5	2	2	28
74212	6000	24	4	59	52	10	28	17	11	25	27	6	21	56

If Dr. *Pound* mean Lunation (which we have kept by in making these tables) be added 74212 times to itself, the sum will amount to 6000 Julian years, 24 days 4 hours 59 minutes 51 seconds 40 thirds ; agreeing with the first part of the last line of this table, within half a second.

TABLE VII. *The annual, or first Equation of the mean to the true Syzygy.*

Argument. Sun's mean Anomaly.

Subtract.

Degrees.	0 Signs.			1 Sign.			2 Signs.			3 Signs.			4 Signs.			5 Signs.			Degrees
	H.	M.	S	H.	M.	S	H.	M.	S.	H.	M.	S	H.	M.	S.	H.	M.	S	
0	0	0	0	2	3	12	3	35	0	4	10	53	3	39	30	2	7	45	30
1	0	4	18	2	6	55	3	37	10	4	10	57	3	37	19	2	3	55	29
2	0	8	35	2	10	36	3	39	18	4	10	54	3	35	6	2	0	1	28
3	0	12	51	2	14	14	3	41	23	4	10	49	3	32	50	1	56	5	27
4	0	17	8	2	17	52	3	43	26	4	10	39	3	30	30	1	52	6	26
5	0	21	24	2	21	27	3	45	25	4	10	24	3	28	5	1	48	4	25
6	0	25	39	2	25	9	3	47	19	4	10	4	3	25	35	1	41	1	24
7	0	28	55	2	28	29	3	49	7	4	9	39	3	23	0	1	39	56	23
8	0	34	11	2	31	57	3	50	50	4	9	10	3	20	20	1	35	49	22
9	0	38	26	2	35	22	3	52	29	4	8	37	3	17	35	1	31	41	21
10	0	42	39	2	38	44	3	54	4	4	7	59	3	14	49	1	27	31	20
11	0	46	52	2	42	3	3	55	35	4	7	16	3	11	59	1	23	19	19
12	0	51	4	2	45	18	3	57	2	4	6	29	3	9	6	1	19	5	18
13	0	55	17	2	48	30	3	58	27	4	5	37	3	6	10	1	14	49	17
14	0	59	27	2	51	40	3	59	49	4	4	41	3	3	10	1	10	33	15
15	1	3	36	2	54	48	3	1	7	4	3	40	3	0	7	1	6	15	16
16	1	7	45	2	57	53	4	2	18	4	2	35	2	57	0	1	1	56	14
17	1	11	53	3	0	54	4	3	23	4	1	26	2	53	49	0	57	36	13
18	1	16	0	3	3	51	4	4	22	4	0	12	2	50	36	0	53	15	12
19	1	20	6	3	6	45	4	5	18	3	58	52	2	47	18	0	48	52	11
20	1	24	10	3	9	36	4	6	10	3	57	27	2	43	57	0	44	28	10
21	1	28	12	3	12	24	4	6	58	3	55	59	2	40	33	0	40	2	9
22	1	32	12	3	15	9	4	7	41	3	54	26	2	37	6	0	35	36	8
23	1	36	10	3	17	51	4	8	21	3	52	49	2	33	35	0	31	10	7
24	1	40	6	3	20	30	4	8	57	3	51	9	2	30	2	0	26	44	6
25	1	44	1	3	23	5	4	9	29	3	49	26	2	26	26	0	22	17	5
26	1	47	54	3	25	36	4	9	55	3	47	38	2	22	47	0	17	50	4
27	1	51	46	3	28	3	4	10	16	3	45	44	2	19	5	0	13	23	3
28	1	55	37	3	30	26	4	10	53	3	43	45	2	15	20	0	8	56	2
29	1	59	26	3	32	45	4	10	45	3	41	40	2	11	35	0	4	29	1
30	2	3	12	3	35	0	4	10	53	3	39	30	2	7	45	0	0	0	0
Deg	11 Signs.			10 Signs.			9 Signs.			8 Signs.			7 Signs.			6 Signs.			Deg

Add

TABLE VIII. *Equation of the Moon's mean Anomaly.*

Argument. Sun's mean Anomaly.

Subtract.

Degrees.	0 Signs.			1 Signs.			2 Signs.			3 Signs.			4 Signs.			5 Signs.			Degrees.
	o	′	″	o	′	″	o	′	″	o	′	″	o	′	″	o	′	″	
0	0	0	0	0	46	45	1	21	32	1	35	1	1	23	4	0	48	19	30
1	0	1	37	0	48	10	1	22	21	1	35	2	1	22	14	0	46	51	29
2	0	3	13	0	49	34	1	23	10	1	35	1	1	21	24	0	45	23	28
3	0	4	52	0	50	53	1	23	57	1	35	0	1	20	32	0	43	54	27
4	0	6	28	0	52	19	1	24	41	1	34	57	1	19	38	0	42	24	26
5	0	8	6	0	53	40	1	25	24	1	34	50	1	18	42	0	40	53	25
6	0	9	42	0	55	0	1	26	6	1	34	43	1	17	45	0	39	21	24
7	0	11	20	0	56	21	1	26	48	1	34	33	1	16	48	0	37	49	23
8	0	12	56	0	57	38	1	27	28	1	34	22	1	15	47	0	36	15	22
9	0	14	33	0	58	56	1	28	6	1	34	9	1	14	44	0	34	40	21
10	0	16	10	1	0	13	1	28	43	1	33	53	1	13	41	0	33	5	20
11	0	17	47	1	1	29	1	29	17	1	33	37	1	12	37	0	31	31	19
12	0	19	23	1	2	43	1	29	51	1	33	20	1	11	33	0	29	54	18
13	0	20	59	1	3	56	1	30	22	1	33	0	1	10	26	0	28	18	17
14	0	22	35	1	5	8	1	30	50	1	32	38	1	9	17	0	26	40	16
15	0	24	10	1	6	18	1	31	19	1	32	14	1	8	8	0	25	3	15
16	0	25	45	1	7	27	1	31	45	1	31	50	1	6	58	0	23	23	14
17	0	27	19	1	8	36	1	32	12	1	31	23	1	5	46	0	21	45	13
18	0	28	52	1	9	42	1	32	34	1	30	55	1	4	32	0	20	7	12
19	0	30	25	1	10	49	1	32	57	1	30	25	1	3	19	0	18	28	11
20	0	31	57	1	11	54	1	33	17	1	29	54	1	2	1	0	16	48	10
21	0	33	29	1	12	58	1	33	36	1	29	20	1	0	45	0	15	8	9
22	0	35	2	1	14	1	1	33	52	1	28	45	0	59	26	0	13	28	8
23	0	36	32	1	15	1	1	34	6	1	28	9	0	58	7	0	11	48	7
24	0	38	1	1	16	0	1	34	18	1	27	30	0	56	45	0	10	7	6
25	0	39	29	1	16	59	1	34	30	1	26	50	0	55	23	0	8	20	5
26	0	40	59	1	17	57	1	34	40	1	26	27	0	54	1	0	6	44	4
27	0	42	26	1	18	52	1	34	48	1	25	5	0	52	37	0	5	3	3
28	0	43	54	1	19	47	1	34	54	1	24	39	0	51	12	0	3	21	2
29	0	45	19	1	20	40	1	34	58	2	23	52	0	49	45	0	1	40	1
30	0	47	45	1	21	32	1	35	1	1	23	4	0	48	19		0	0	0
Deg.	11 Signs.			1 Signs.			9 Signs.			8 Signs.			7 Signs.			6 Signs.			Deg.

Add

TABLE IX. *The second Equation of the mean to the true Syzygy.*

Argument. Moon's Equated Anomaly.

Add

Degrees	0 Signs.			1 Sign.			2 Signs.			3 Signs.			4 Signs.			5 Signs.			Degrees
	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	H.	M.	S.	
0	0	0	0	5	12	48	8	47	8	9	46	44	8	8	59	4	34	33	30
1	0	10	58	5	21	56	8	51	45	9	45	3	8	3	12	4	26	1	29
2	0	21	56	5	30	57	8	56	10	9	45	12	7	57	23	4	17	25	28
3	0	32	54	5	39	51	9	0	25	9	44	11	7	51	33	4	8	47	27
4	0	42	52	5	48	37	9	4	31	9	42	59	7	45	46	4	0	7	26
5	0	54	50	5	57	17	9	8	25	9	41	36	7	39	46	3	1	23	25
6	1	5	48	6	5	51	9	12	9	9	40	3	7	33	36	3	42	32	24
7	1	16	46	6	14	19	9	15	43	9	38	19	7	27	22	3	33	38	23
8	1	27	44	6	22	41	9	19	5	9	36	24	7	21	2	2	24	42	22
9	1	38	40	6	30	57	9	22	14	9	34	18	7	14	30	3	15	44	21
10	1	49	33	6	39	4	9	25	12	9	32	1	7	7	50	3	6	45	20
11	2	0	25	6	47	0	9	27	58	9	29	33	7	1	2	2	57	43	19
12	2	11	10	6	54	46	9	30	32	9	26	54	6	54	8	2	48	39	18
13	2	21	54	7	2	24	9	32	58	9	24	4	6	47	9	2	39	34	17
14	2	32	34	7	9	52	9	35	12	9	21	3	6	40	6	2	30	28	16
15	2	43	9	7	17	9	9	37	14	9	17	51	6	32	56	2	21	19	15
16	2	53	38	7	24	10	9	39	8	9	14	28	6	25	40	2	12	8	14
17	3	4	3	7	31	18	9	40	51	9	10	54	6	18	18	2	2	53	13
18	3	14	24	7	38	9	9	42	21	9	7	9	6	10	49	1	53	36	12
19	3	24	42	7	44	51	9	43	42	9	3	13	6	3	16	1	44	16	11
20	3	34	58	7	51	24	9	44	53	8	58	6	5	55	38	1	34	54	10
21	3	45	11	7	57	45	9	45	52	8	54	50	5	47	54	1	25	31	9
22	3	55	21	8	3	56	9	46	38	8	50	24	5	40	4	1	16	7	8
23	4	5	26	8	9	57	9	47	13	8	45	48	5	32	9	1	6	41	7
24	4	25	26	8	15	46	9	47	36	8	41	2	5	24	9	0	57	13	6
25	4	25	20	8	21	24	9	47	49	8	36	6	5	16	5	0	47	44	5
26	4	35	6	8	26	53	9	47	54	8	31	0	5	7	56	0	38	13	4
27	4	44	42	8	32	11	9	47	46	8	25	44	4	59	42	0	28	41	3
28	4	54	11	8	37	19	9	47	33	8	20	18	4	51	15	0	19	8	2
29	5	3	33	8	42	18	9	47	14	8	14	33	4	43	2	0	9	34	1
30	5	12	48	8	47	8	9	46	44	8	8	59	4	34	33	0	0	0	0
Deg.	11 Signs.			10 Signs.			9 Signs.			8 Signs.			7 Signs.			6 Signs.			Deg.

Subtract

TABLE X. *The third Equation of the mean to the true Syzygy.*

Argument. Sun's Anomaly. Moon's Anomaly

Degrees	Signs. 0 Sub. 6 Add		Signs. 1 Sub. 7 Add		Signs. 2 Sub. 8 Add		Degrees.
	M.	S.	M.	S.	M.	S.	
0	0	0	2	22	4	12	30
1	0	5	2	26	4	16	29
2	0	10	2	30	4	18	28
3	0	15	2	34	4	21	27
4	0	20	2	38	4	24	26
5	0	25	2	42	4	27	25
6	0	30	2	46	4	30	24
7	0	35	2	50	4	32	23
8	0	40	2	54	4	34	22
9	0	45	2	58	4	36	21
10	0	50	3	2	4	38	20
11	0	55	3	6	4	40	19
12	1	0	3	10	4	42	18
13	1	5	3	14	4	44	17
14	1	10	3	18	4	46	16
15	1	15	3	22	4	48	15
16	1	20	3	26	4	50	14
17	1	25	3	30	4	51	13
18	1	30	3	34	4	52	12
19	1	35	3	38	4	53	11
20	1	40	3	42	4	54	10
21	1	45	3	45	4	55	9
22	1	49	3	48	4	56	8
23	1	52	3	51	4	57	7
24	1	56	3	54	4	57	6
25	2	0	3	57	4	57	5
26	2	4	4	0	4	58	4
27	2	9	4	3	4	58	3
28	2	13	4	6	4	58	2
29	2	18	4	9	4	58	1
30	2	22	4	12	4	58	0
Degrees	Signs. 5 Sub. 11 Add		Signs. 4 Sub. 10 Add		Signs. 3 Sub. 9 Add		Degrees

TABLE XI. *The fourth Equation of the mean to the true Syzygy.*

Argument. Sun's mean distance from the Node.

	Add						
Deg.	0 6 Sig.		1 7 Sig		2 8 Sig.		Deg.
	M.	S.	M.	S.	M.	S.	
0	0	0	1	22	1	22	30
1	0	4	1	23	1	21	29
2	0	7	1	24	1	20	28
3	0	10	1	25	1	18	27
4	0	13	1	26	1	16	26
5	0	16	1	27	1	14	25
6	0	20	1	28	1	12	24
7	0	23	1	29	1	10	23
8	0	26	1	30	1	8	22
9	0	29	1	31	1	6	21
10	0	32	1	32	1	3	20
11	0	35	1	33	1	0	19
12	0	38	1	33	0	57	18
13	0	41	1	34	0	54	17
14	0	44	1	34	0	51	16
15	0	47	1	34	0	49	15
16	0	50	1	34	0	45	14
17	0	52	1	34	0	41	13
18	0	54	1	34	0	37	12
19	0	57	1	33	0	34	11
20	1	0	1	33	0	31	10
21	1	2	1	32	0	28	9
22	1	5	1	31	0	25	8
23	1	8	1	30	0	22	7
24	1	10	1	29	0	19	6
25	1	12	1	28	0	16	5
26	1	14	1	27	0	13	4
27	1	16	1	26	0	10	3
28	1	18	1	25	0	6	2
29	1	20	1	24	0	3	1
30	1	22	1	22	0	0	0
Degrees	5 11 Si		4 10 Si.		3 9 Sig.		Degrees
	Subtract						

TABLE XII. *The Sun's mean Longitude, Motion, and Anomaly: Old Style.*

Years beginning	Sun's mean Longitude.				Sun's mean Anomaly.			Years complete	Sun's mean Motion.				Suns mean Anomaly.		
	s	o	′	″	s	o	′		s	o	′	″	s	o	′
1	9	7	53	10	6	28	48	19	11	29	24	16	11	29	4
201	9	9	23	50	6	26	57	20	0	0	9	4	11	29	48
301	9	10	9	10	6	26	1	40	0	0	18	8	11	29	37
401	9	10	54	30	6	25	5	60	0	0	27	12	11	29	26
501	9	11	39	50	6	24	9	80	0	0	36	16	11	29	15
1001	9	15	26	30	6	19	32	100	0	0	45	20	11	29	4
1101	9	16	11	50	6	18	36	200	0	1	20	40	11	29	8
1201	9	16	57	10	6	17	40	300	0	2	16	0	11	27	12
1301	9	17	42	30	6	16	44	400	0	3	1	20	11	26	16
1401	9	18	27	50	6	15	49	500	0	3	46	40	11	25	21
1501	9	19	13	10	6	14	53	600	0	4	32	0	11	24	25
1601	9	19	58	30	6	13	57	700	0	5	17	20	11	23	29
1701	9	20	43	50	6	13	1	800	0	6	2	40	11	22	33
1801	9	21	29	10	6	12	6	900	0	6	48	0	11	21	37
								1000	0	7	33	20	11	20	11
								2000	0	15	6	40	11	11	22
								3000	0	22	40	0	11	2	3
								4000	1	0	13	20	10	22	44
								5000	1	7	46	40	10	13	25
								6000	1	15	20	0	10	4	6

Years complete	Sun's Mean Motion.				Sun's mean Anomaly.		
	s	o	′	″	s	o	′
1	11	29	45	40	11	29	45
2	11	29	31	20	11	29	29
3	11	29	17	0	11	29	14
4	0	0	1	49	11	29	58
5	11	29	47	29	11	29	42
6	11	29	33	9	11	29	27
7	11	29	18	49	11	29	11
8	0	0	3	38	11	29	5
9	11	29	49	18	11	29	40
10	11	29	34	58	11	29	24
11	11	29	20	38	11	29	9
12	0	0	5	26	11	29	53
13	11	29	51	7	11	29	37
14	11	29	36	47	11	29	22
15	11	29	22	27	11	29	7
16	0	0	7	15	11	29	50
17	11	29	52	55	11	29	35
18	12	29	38	35	11	29	20

Months.	Sun's mean Motion.				Suns mean Anomaly.		
	s	o	′	″	s	o	′
Jan.	0	0	0	0	0	0	0
Feb.	1	0	33	18	1	0	33
Mar.	1	28	9	11	1	28	9
Apr	2	28	42	30	2	28	42
May.	3	28	16	40	3	28	17
June	4	28	49	58	4	28	50
July	5	28	24	8	5	28	24
Aug	6	29	57	26	6	28	57
Sept	7	29	30	44	7	29	30
Oct.	8	29	4	54	8	29	4
Nov	9	29	38	12	9	29	37
Dec	10	29	12	22	10	29	11

TABLE XII. *concluded.*

Days.	Sun's mean Motion and Anomaly.				Sun's mean Motion and Anomaly.				S. mean Dist. fro. Node.				Sun's mean Motion and Anomaly.				S. mean distance fro. Node.			
	s	o	′	″	H M S	o ′ ″	′ ″ ‴	″ ‴ ⁗	o ′ ″	′ ″ ‴	″ ‴ ⁗	H M S	o ′ ″	′ ″ ‴	″ ‴ ⁗	o ′ ″	′ ″ ‴	″ ‴ ⁗		
1	0	0	59		1 0	2 28	0 2 36		31	1 16	23	1	20	30						
2	0	1	58	17	2 0	4 56	0 5 12		32	1 18	51	1	23	6						
3	0	2	57	25	3 0	7 24	0 7 48		33	1 21	19	1	25	42						
4	0	3	56	33	4 0	9 51	0 10 23		34	1 23	47	1	28	18						
5	0	4	55	42	5 0	12 19	0 12 59		35	1 26	15	1	30	54						
6	0	5	54	50	6 0	14 47	0 15 35		36	1 28	42	1	33	29						
7	0	6	53	58	7 0	17 15	0 18 11		37	1 31	10	1	36	5						
8	0	7	53	7	8 0	19 43	0 20 47		38	1 33	38	1	38	40						
9	0	8	52	15	9 0	22 11	0 23 23		39	1 36	6	1	41	16						
10	0	9	51	23	10 0	24 38	0 25 58		40	1 38	34	1	43	52						
11	0	10	50	32	11 0	27 6	0 28 34		41	1 41	2	1	46	28						
12	0	11	49	40	12 0	29 34	0 31 10		42	1 43	30	1	49	4						
13	0	12	48	48	13 0	32 2	0 33 45		43	1 45	57	1	51	39						
14	0	13	47	57	14 0	34 30	0 36 21		44	1 48	25	1	54	15						
15	0	14	47	5	15 0	36 58	0 38 57		45	1 50	53	1	55	51						
16	0	15	46	13	16 0	39 26	0 41 33		46	1 53	21	1	59	27						
17	0	16	45	22	17 0	41 53	0 44 8		47	1 55	49	2	2	3						
18	0	17	44	30	18 0	44 21	0 46 44		48	1 58	17	2	4	39						
19	0	18	43	38	19 0	46 49	0 49 20		49	2 0	44	2	7	13						
20	0	19	42	47	20 0	49 17	0 51 56		50	2 3	12	2	9	50						
21	0	20	41	55	21 0	51 45	0 54 32		51	2 5	40	2	12	25						
22	0	21	41	3	22 0	54 13	0 57 8		52	2 8	8	2	15	2						
23	0	22	40	12	23 0	56 40	0 59 43		53	2 10	36	2	17	38						
24	0	23	39	20	24 0	59 8	1 2 19		54	2 13	4	2	20	14						
25	0	24	38	28	25 1	1 36	1 4 55		55	2 15	32	2	22	50						
26	0	25	37	37	26 1	4 4	1 7 31		56	2 17	59	2	25	26						
27	0	26	36	45	27 1	6 32	1 10 7		57	3 20	27	2	28	8						
28	0	27	35	53	28 1	9 0	1 12 43		58	2 22	55	2	30	32						
29	0	28	35	2	29 1	11 28	1 15 19		59	2 25	23	2	33	14						
30	0	29	34	10	30 1	13 55	1 17 55		60	2 27	51	2	35	50						
31	1	30	33	18																

In leap years, after *February*, add one day, and o[illegible] mot[illegible].

TABLE XIII. *Equation of the Sun's Centre, or the Difference between his mean and true Place.*

Argument. Sun's mean Anomaly.

Subtract.

Degrees	0 Signs.			1 Sign.			2 Signs.			3 Signs.			4 Signs.			5 Signs.			Degrees
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
0	0	0	0	0	56	47	1	39	6	1	55	37	1	41	12	0	58	53	30
1	0	1	59	0	58	30	1	40	7	1	55	39	1	40	12	0	57	7	29
2	0	3	57	1	0	12	1	41	6	1	55	38	1	39	10	0	55	19	28
3	0	5	56	1	1	53	1	42	3	1	55	36	1	38	6	0	53	30	27
4	0	7	54	1	3	33	1	42	59	1	55	31	1	37	0	0	51	40	26
5	0	9	52	1	5	12	1	43	52	1	55	24	1	35	52	0	49	49	25
6	0	11	50	1	6	50	1	44	44	1	55	15	1	34	43	0	47	57	24
7	0	13	48	1	8	27	1	45	34	1	55	5	1	33	32	0	46	5	23
8	0	15	46	1	10	2	1	46	22	1	54	50	1	32	19	0	44	11	22
9	0	17	43	1	11	36	1	47	8	1	54	35	1	31	4	0	42	16	21
10	0	19	40	1	13	9	1	47	53	1	54	17	1	29	47	0	40	21	20
11	0	21	37	1	14	41	1	48	35	1	53	57	1	28	29	0	38	25	19
12	0	23	33	1	16	11	1	49	15	1	53	36	1	27	9	0	36	28	18
13	0	25	29	1	17	40	1	49	54	1	53	12	1	25	48	0	34	30	17
14	0	27	25	1	19	8	1	50	30	1	52	46	1	24	25	0	32	32	16
15	0	29	20	1	20	34	1	51	5	1	52	18	1	23	0	0	30	33	15
16	0	31	15	1	21	59	1	51	37	1	51	48	1	21	34	0	28	33	14
17	0	33	9	1	23	22	1	52	8	1	51	15	1	20	6	0	26	33	13
18	0	35	2	1	24	44	1	52	36	1	50	41	1	18	36	0	24	33	12
19	0	36	55	1	26	5	1	53	8	1	50	5	1	17	5	0	22	32	11
20	0	38	47	1	27	24	1	53	27	1	49	26	1	15	33	0	20	30	10
21	0	40	39	1	28	41	1	53	50	1	48	46	1	13	59	0	18	28	9
22	0	42	30	1	29	57	1	54	10	1	48	3	1	12	24	0	16	26	8
23	0	44	20	1	31	11	1	54	28	1	47	19	1	10	47	0	14	24	7
24	0	46	9	1	32	25	1	54	44	1	46	32	1	9	9	0	12	21	6
25	0	47	57	1	33	35	1	54	58	1	45	44	1	7	29	0	10	18	5
26	0	49	45	1	34	45	1	55	10	1	44	53	1	5	49	0	8	14	4
27	0	51	32	1	35	55	1	55	20	1	44	1	1	4	7	0	6	11	3
28	0	53	18	1	36	59	1	55	28	1	43	7	1	2	24	0	4	7	2
29	0	55	3	1	38	3	1	55	34	1	42	10	1	0	39	0	2	4	1
30	0	56	47	1	39	6	1	55	37	1	41	12	0	58	53	0	0	0	0
Deg.	11 Signs			10 Signs			9 Signs.			8 Signs.			7 Signs.			6 Signs.			Deg.

Add

TABLE XIV. *The Sun's Declination.*

Argument. Sun's true Place.

Degrees.	Signs 0 ♈ 6 ♎		Signs 1 ♉ 7 ♏		Signs 2 ♊ 8 ♐		Degrees
	0	'	0	'	0	'	
0	0	0	11	30	20	11	30
1	0	24	11	51	20	24	29
2	0	48	12	11	20	36	28
3	1	12	12	32	20	48	27
4	1	36	12	53	20	59	26
5	1	59	13	13	21	10	25
6	2	23	13	33	21	21	24
7	2	47	13	53	21	31	23
8	3	11	14	12	21	41	22
9	3	34	14	31	21	50	21
10	3	58	14	50	21	59	20
11	4	22	15	9	22	8	19
12	4	45	15	28	22	16	18
13	5	9	15	46	22	24	17
14	5	32	16	4	22	31	16
15	5	35	16	22	22	38	15
16	6	18	16	39	22	45	14
17	6	41	16	57	22	51	13
18	7	4	17	14	22	56	12
19	7	27	17	30	23	2	11
20	7	50	17	46	23	6	10
21	8	15	18	2	23	11	9
22	8	35	18	18	23	14	8
23	9	57	18	33	23	18	7
24	9	20	18	48	23	21	6
25	9	42	19	3	23	21	5
26	10	4	19	17	23	25	4
27	10	25	19	31	23	27	3
28	10	47	19	45	23	28	2
29	11	8	19	58	23	29	1
30	11	30	20	11	23	29	0

Degrees	Signs 11 ♓ 5 ♍		Signs 10 ♒ 4 ♌		Signs 9 ♑ 3 ♋		Degrees.
	11	S. N.	10	S. N.	9	S N.	

TABLE XV. *Equation of the Sun's mean Distance from the Node.*

Argument. Sun's mean Anomaly.

Deg.	Subtract. 0 Sig.	1 Sig.	2 Sig.	3 Sig.	4 Sig.	5 Sig.	Deg.
	0 '	0 '	0 '	0 '	0 '	0 '	
0	0 0	1 2	1 47	2 5	1 50	1 4	30
1	0 2	1 4	1 48	2 5	1 48	1 2	29
2	0 4	1 6	1 49	2 5	1 47	1 0	28
3	0 6	1 8	1 50	2 5	1 46	0 58	27
4	0 9	1 10	1 51	2 5	1 45	0 56	26
5	0 11	1 12	1 52	2 5	1 44	0 54	25
6	0 13	1 14	1 53	2 5	1 43	0 52	24
7	0 15	1 16	1 54	2 4	1 41	0 50	23
8	0 17	1 17	1 55	2 4	1 40	0 48	22
9	0 19	1 18	1 56	2 4	1 39	0 46	21
10	0 21	1 19	1 57	2 4	1 37	0 44	20
11	0 23	1 21	1 58	2 3	1 36	0 42	19
12	0 25	1 22	1 58	2 3	1 34	0 40	18
13	0 28	1 24	1 59	2 3	1 33	0 37	17
14	0 30	1 26	2 0	2 2	1 31	0 35	16
15	0 32	1 27	2 0	2 2	1 30	0 33	15
16	0 34	1 28	2 1	2 1	1 28	0 31	14
17	0 36	1 30	2 1	2 1	1 27	0 29	13
18	0 38	1 31	2 2	2 0	1 25	0 27	12
19	0 40	1 34	2 2	2 0	1 24	0 24	11
20	0 42	1 35	2 3	1 59	1 23	0 22	10
21	0 44	1 36	2 3	1 59	1 21	0 20	9
22	0 46	1 37	2 4	1 58	1 19	0 18	8
23	0 48	1 39	2 4	1 57	1 17	0 16	7
24	0 50	1 40	2 4	1 56	1 15	0 13	6
25	0 52	1 41	2 4	1 55	1 13	0 11	5
26	0 54	1 43	2 5	1 54	1 11	0 9	4
27	0 56	1 44	2 5	1 53	1 9	0 7	3
28	0 58	1 45	2 5	1 52	1 8	0 5	2
29	1 0	1 46	2 5	1 51	1 6	0 3	1
30	1 2	1 47	2 5	1 50	1 4	0 0	0

Degrees.	11 Sig.	10 Sig.	9 Sig.	8 Sig.	7 Sig.	6 Sig.	Degrees.
	Add						

TABLE XVI.
The Moon's Latitude in Eclipses.

Argument. Moon's equated Distance from Node.	0 Signs. *North ascend.*			*South descend.*
0	0	′	″	0
1	0	0	0	30
2	0	5	15	29
3	0	10	30	28
4	0	15	45	27
5	0	20	58	26
6	0	26	13	25
7	0	31	26	24
8	0	36	39	23
9	0	41	51	22
10	0	47	22	21
11	0	52	13	20
12	0	57	23	19
13	1	2	31	18
14	1	7	38	17
15	1	12	44	16
16	1	17	49	15
17	1	22	52	14
18	1	27	53	13
19	1	32	52	12
	1	37	49	11
	5 Signs. *North descend*			
	11 Signs. *South Ascend*			

This Table shews the Moon's Latitude a little beyond the utmost limits of Eclipses.

TABLE XVII. *The Moon's horizontal Parallax with the Semidiameters and true Horary Motion of the Sun and Moon, and every sixth Degree of their mean Anomalies, the Quantities for the intermediate Degrees being easily proportioned by sight.*

Anomaly of Sun and Moon.		Moon's horizontal Parallax.		Sun's Semidiameter.		Moon's Semidiameter.		Moon's Horary Motion.		Sun's Horary Motion.		Anomaly of Sun and Moon.	
s	°	′	″	′	″	′	″	′	″	′	″	s	°
0	0	54	29	15	50	14	54	30	10	2	23	12	0
	6	54	31	15	50	14	55	30	12	2	23		24
	12	54	34	15	50	14	56	30	15	2	23		18
	18	54	40	15	51	14	57	30	19	2	23		12
	24	54	47	15	51	14	58	30	26	2	23		6
1	0	54	56	15	52	14	59	30	34	2	24	11	0
	6	55	6	15	53	15	1	30	44	2	24		24
	12	55	17	15	54	15	4	30	55	2	24		11
	18	55	29	15	55	15	8	31	9	2	24		12
	24	55	42	15	9	15	12	31	23	2	25		6
2	0	55	56	15	58	15	17	31	40	2	25	10	0
	6	56	12	15	59	15	22	31	56	2	26		24
	12	56	29	16	1	15	26	32	17	2	27		18
	18	56	48	16	2	15	30	32	39	2	27		12
	24	57	8	16	4	15	36	33	11	6	28		6
3	0	57	30	16	6	15	41	33	23	2	28	9	0
	6	57	52	16	8	15	46	33	47	2	29		24
	12	58	12	16	10	15	52	34	11	2	29		18
	18	58	31	16	11	15	58	34	34	2	29		12
	24	58	49	16	13	16	3	34	58	2	30		6
4	0	59	6	16	14	16	9	35	22	2	30	8	0
	6	59	21	16	15	16	14	35	45	2	31		24
	12	59	35	16	17	16	19	36	0	2	31		18
	18	59	48	16	19	16	24	36	20	2	32		12
	24	60	0	16	20	16	28	36	40	2	32		6
5	0	60	11	16	21	16	31	37	0	2	32	7	0
	6	60	21	16	21	16	32	37	10	2	33		24
	12	60	30	16	22	16	37	37	19	2	33		18
	18	60	38	16	22	16	38	37	28	2	33		12
	24	60	45	16	23	16	39	37	36	2	33		6
6	0	60	45	16	23	16	39	37	40	2	33	6	0

TABLE XVIII. *The Moon's Mean Longitude, and Anomaly for current years.*

	A. D.	Mean Long.				Mean Anom.				Sun from Node			
Years current.		s	o	′	″	s	o	′	″	s	o	′	″
	1761	7	1	8	8	10	12	54	50	2	7	33	33
	1781	11	14	42	54	11	22	19	18	1	10	43	18
	1791	7	14	54	59	6	5	39	35	6	27	19	46
B	1792	0	7	28	40	9	17	26	44	6	7	56	52
	1793	4	16	51	45	0	16	9	59	5	18	37	9
	1794	8	26	14	51	3	14	53	14	4	29	17	26
	1795	1	5	37	57	6	13	36	29	4	9	57	43
B	1796	5	28	11	37	9	25	23	38	3	20	34	49
	1797	10	7	34	43	0	24	6	53	3	1	15	6
	1798	2	16	57	48	3	22	50	8	3	11	55	23
	1799	6	26	20	54	6	21	33	23	1	22	35	40
B	1800	11	5	44	0	9	20	16	38	1	3	15	57
	1801	3	15	7	5	0	18	59	52	0	13	56	14
	1802	7	24	30	11	3	17	43	7	11	24	36	31
	1803	0	3	53	16	6	16	26	22	11	5	16	48
B	1804	4	26	26	57	9	28	13	31	10	15	53	54
	1805	9	5	50	2	0	26	56	46	9	26	34	11
	1806	1	15	13	8	3	25	40	1	9	7	14	28
	1807	5	24	36	14	6	24	23	16	8	17	54	45
B	1808	10	17	9	54	10	6	10	25	7	28	31	51
	1809	2	26	33	0	1	4	53	40	7	9	12	8
	1810	7	5	56	5	4	3	36	55	6	19	52	25
	1811	11	15	19	11	7	2	20	9	6	0	32	42
B	1812	4	7	52	52	10	14	7	18	5	11	9	48
	1813	8	17	15	57	1	12	50	33	4	21	50	5
	1814	0	26	39	3	4	11	33	48	4	2	30	22
	1815	5	6	2	8	7	10	17	3	3	13	10	39
B	1816	9	28	35	49	10	22	4	12	2	23	47	45
	1817	2	7	58	55	1	20	47	27	2	4	28	2
	1818	6	17	22	0	4	19	30	42	1	15	8	19
	1819	10	26	45	6	7	18	13	57	0	25	48	36
B	1820	3	19	18	47	11	0	1	6	0	6	25	42
	1821	7	28	41	54	1	28	44	21	11	17	5	59
	1841	0	12	16	37	3	8	28	51	10	20	15	44

TABLE XIX. *The Sun's Longitude for every day in the year, at noon.*

Days.	January.			February.			March.			April.			May.			June.		
	s	o	′	s	o	′	s	o	′	s	o	′	s	o	′	s	o	′
1	9	11	21	10	12	54	11	11	80	11	55	1	11	12	2	11	2	
2	9	12	23	10	13	55	11	12	80	12	54	1	12	10	2	12	0	
3	9	13	24	10	14	56	11	13	80	13	53	1	13	9	2	12	57	
4	9	14	25	10	15	57	11	14	80	14	52	1	14	7	2	13	54	
5	9	15	26	10	16	57	11	15	80	15	51	1	15	5	2	14	52	
6	9	16	27	10	17	58	11	16	80	16	50	1	16	3	2	15	49	
7	9	17	29	10	18	59	11	17	80	17	49	1	17	1	2	16	46	
8	9	18	30	10	20	0	11	18	80	18	48	1	17	59	2	17	44	
9	9	19	31	10	21	0	11	19	80	19	47	1	18	56	2	18	41	
10	9	20	32	10	22	1	11	20	80	20	45	1	19	54	2	19	39	
11	9	21	33	10	23	1	11	21	70	21	44	1	20	52	2	20	35	
12	9	22	34	10	24	2	11	22	70	22	43	1	21	50	2	21	33	
13	9	23	35	10	25	3	11	23	70	23	41	1	22	48	2	22	30	
14	9	24	36	10	26	3	11	24	60	24	40	1	23	45	2	23	28	
15	9	25	37	10	27	4	11	25	60	25	39	1	24	43	2	24	25	
16	9	26	39	10	28	4	11	26	60	26	37	1	25	41	2	25	22	
17	9	27	39	10	29	4	11	27	50	27	36	1	26	39	2	26	19	
18	9	28	41	11	0	5	11	28	50	28	34	1	27	36	2	27	17	
19	9	29	42	11	1	5	11	29	40	29	33	1	28	34	2	28	14	
20	10	0	43	11	2	6	0	0	41	0	31	1	29	32	2	29	11	
21	10	1	44	11	3	6	0	1	31	1	30	2	0	29	3	0	8	
22	10	2	45	11	4	6	0	2	31	2	28	2	1	27	3	1	6	
23	10	3	46	11	5	7	0	3	21	3	26	2	2	25	3	2	3	
24	10	4	47	11	6	7	0	4	11	4	25	2	3	22	3	3	0	
25	10	5	48	11	7	7	0	5	11	5	23	2	4	20	2	3	57	
26	10	6	49	11	8	7	0	6	0	1	6	21	2	5	17	3	4	55
27	10	7	50	11	9	8	0	6	59	1	7	20	2	6	15	3	5	52
28	10	8	51	11	10	8	0	7	59	1	8	18	2	7	12	3	6	49
29	10	9	52				0	8	58	1	9	16	2	8	10	3	7	46
30	10	10	52				0	9	57	1	10	14	2	9	7	3	8	43
31	11	11	53				0	10	56				2	10	5			

TABLE XIX. *Concluded.*

Days.	July.			August.			Sept.			October.			Nov.			Dec.		
	s	o	′	s	o	′	s	o	′	s	o		s	o	′	s	o	′
1	3	9	41	4	9	17	5	9	7	6	8	26	7	9	15	8	9	32
2	3	10	38	4	10	14	5	10	6	6	9	25	7	10	15	8	10	33
3	3	11	35	4	11	11	5	11	4	6	10	24	7	11	16	8	11	34
4	3	12	32	4	12	9	5	12	2	6	11	24	7	12	16	8	12	35
5	3	13	30	4	13	6	5	13	0	6	12	23	7	13	16	8	13	36
6	3	14	27	4	14	4	5	13	59	6	13	22	7	14	16	8	14	37
7	3	15	28	4	15	2	5	14	57	6	14	21	7	15	17	8	15	38
8	3	16	21	4	15	59	5	15	55	6	15	21	7	16	17	8	16	39
9	3	17	18	4	16	57	5	16	53	6	16	20	7	17	17	8	17	40
10	3	18	15	4	17	54	5	17	52	6	17	19	7	18	18	8	18	41
11	1	19	13	4	18	52	5	18	50	6	18	19	7	19	18	8	19	42
12	3	20	10	4	19	49	5	19	45	6	19	18	7	20	19	8	20	43
13	3	21	7	4	20	47	5	20	47	6	20	18	7	21	19	8	21	44
14	3	22	4	4	21	45	5	21	46	6	21	17	7	22	20	8	22	45
15	3	23	2	4	22	42	5	22	44	6	22	17	7	23	20	9	23	47
16	3	23	59	4	23	40	5	23	43	6	23	17	7	24	21	8	24	48
17	3	24	56	4	24	38	5	24	42	6	24	16	7	25	21	8	25	49
18	3	25	53	4	25	36	5	25	40	6	25	16	7	26	22	8	26	50
19	3	26	51	4	26	33	5	26	39	6	26	16	7	27	23	8	27	51
20	3	27	48	4	27	31	5	27	38	6	27	16	7	28	23	8	28	52
21	3	28	45	4	28	29	5	28	37	6	28	15	7	29	24	8	29	54
22	3	29	43	4	29	27	5	29	35	6	29	15	8	0	25	9	0	55
23	4	0	40	5	0	25	6	0	34	7	0	15	8	1	26	9	1	56
24	4	1	37	5	1	23	6	1	33	7	1	15	8	2	26	9	2	57
25	4	2	35	5	2	21	6	2	32	7	2	15	8	3	27	9	3	58
26	4	3	32	5	3	19	6	3	31	7	3	15	8	4	28	9	4	59
27	4	4	29	5	4	17	6	4	30	7	4	15	8	5	29	9	6	1
28	4	5	27	5	5	15	6	3	29	7	5	15	8	6	30	9	7	2
29	4	6	24	5	6	13	6	6	28	7	6	15	8	7	31	9	8	3
30	4	7	22	5	7	11	6	7	22	7	7	15	8	8	32	9	9	4
31	4	8	19	5	8	9				7	8	15				9	10	5

TABLE XX. *A concise* EQUATION-TABLE, *adapted to the Second Year after Leap-Year, and which will be within a Minute of the Truth for every Year; shewing, to the nearest full Minute, how much a Clock should be faster or slower than the Sun. By* MR. SMEATON.

Months.	Days.	Equ. in Minutes.	Clock faster than the Sun.	Months.	Days.	Equ. in Minutes	Clock faster.	Months.	Days.	Equ. in Minutes	Clock slower than the Sun.	Months.	Days.	Equ. in Minutes	Clock slower than the Sun.
Jan.	1	4		Apr.	1	4	Clock faster.	Aug.	10	5	Clock faster.	Oct.	27	10	
	3	5			4	3			15	4		Nov.	8	16	
	5	6			7	2			20	3			15	15	
	7	7			11	1			24	2			20	14	
	10	8			15	0			28	1			24	13	
	12	9							31	0			27	12	
	15	10			*				*				30	11	
	18	11			19	1		Sept.	3	1		Dec.	2	10	
	21	12			24	2	Clock slower.		6	2	Clock slower than the Sun.		5	9	
	25	13		May	30	3			9	3			7	8	
	31	14			14	4			12	4			9	7	
Feb.	6	15		June	29	3			15	5			11	6	
	21	14			5	2			18	6			13	5	
	27	13			10	1			21	7			15	4	
Mar.	4	12			15	0			24	8			18	3	
	8	11			*				27	9			20	2	
	12	10			20	1	Clock faster.		30	10			22	1	
	15	9			24	2		Oct.	3	11			24	0	
	19	8		July	29	3			6	12			*		
	22	7			4	4			10	13			26	1	Faster
	25	6			11	5			14	14			28	2	
	28	5			26	6			19	15			30	3	

This Table is near enough the truth for regulating common clocks and watches. It may be easily copied by the pen, and being doubled, may be put into a pocket-book.

PRECEPTS RELATIVE TO THE PRECEDING TABLES.

To calculate the true TIME of NEW or FULL MOON.

PRECEPT 1. Write out the mean time of New Moon in *March* for the proposed year, from Table I. old style, or from Table II. in the new; together with the mean anomalies of the Sun and Moon, and the Sun's mean distance from the Moon's ascending node. If you want the time of Full Moon in *March*, add the half lunation at the foot of Table III. with its anomalies, &c. to the former numbers, if the New Moon falls before the 15th of *March*; but if it falls after, subtract the half lunation, with anomalies, &c. belonging to it, from the former numbers, and write down the respective sums or remainders.

2. In these additions or subtractions, observe, that 60 seconds make a minute, 60 minutes make a degree, 30 degrees make a sign, and 12 signs make a circle. When you exceed 12 signs in addition, reject 12, and set down the remainder. When the number of signs to be subtracted is greater than the number you subtract from, add 12 signs to the lesser number, and then you will have a remainder to set down.

In the Tables, signs are marked thus, *S* degrees thus, ° minutes thus, ' and seconds thus, ".

3. When the required New or Full Moon is in any given month after *March*, write out as many lunations, with their anomalies, and the Sun's distance from the node, from Table III. as the given month is after *March*; setting them in order below the numbers taken out for *March*.

4. Add all these together, and they will give the mean time of the required New or Full Moon, with the mean anomalies and Sun's mean distance from the ascending node, which are the arguments for finding the proper equations.

5. With the number of days added together, enter Table IV. under the given month, and against that number you have the day of mean New or Full Moon in the left-hand column, which set before the hours, minutes, and seconds, already found.

But (as it will sometimes happen) if the said number of days fall short of any in the column under the given month, add one lunation and its anomalies, &c. (from Table III.) to the foresaid sums, and then you will have a new sum of days wherewith to enter Table IV. under the given month, where you are sure to find it the second time, if the first falls short.

6. With the signs and degrees of the Sun's anomaly, enter Table VII. and therewith take out the annual or first equation for reducing the mean to the true syzygy; taking care to make proportions in the table for the odd minutes and seconds of the

anomaly, as the table gives the equation only to whole degrees.

Observe, in this and every other case of finding equations, that if the signs are at the head of the table, their degrees are at the left hand, and are reckoned downwards; but if the signs are at the foot of the table, their degrees are at the right hand, and are counted upward; the equation being in the body of the table, under, or over the signs, in a collateral line with the degrees. The titles *Add* or *Subtract* at the head or foot of the Tables where the signs are found, shew whether the equation is to be added to the mean time of New or Full Moon, or subtracted from it. In the table for reducing the mean to the true syzygy, the equation is to be subtracted, if the signs of the Sun's anomaly are found at the head of the table; but it is to be added, if the signs are at the foot.

With the same signs and degrees of the Sun's anomaly, enter Table VIII. and take out the equation of the Moon's mean anomaly; subtract this equation from her mean anomaly, if the signs of the Sun's anomaly be at the head of the table, but add it if they are at the foot; the result will be the Moon's equated anomaly, with which enter Table IX. and take out the second equation for reducing the mean to the true time of New or Full Moon; adding this equation, if the signs of the Moon's anomaly are at the head of the table, but subtracting it if they are at the foot, and the result will give you the mean time of the required New or Full Moon twice equated, which will be sufficiently near for common Almanacks. But when you want to calculate an eclipse, the following equations must be used: thus,

8. Subtract the Moon's equated anomaly from the Sun's mean anomaly, and with the remainder in signs and degrees, enter Table X. and take out the third equation, applying it to the former equated time, as the titles. *Add* or *Subtract* do direct.

9. With the Sun's mean distance from the ascending node, enter Table XI. and take out the equation answering to that argument, adding it to, or subtracting it from the former equated time, as the titles direct, and the result will give the time of New or Full Moon, agreeing with well regulated clocks, or watches, very near the truth. But, to make it agree with the solar, or apparent time, apply the equation of natural days, found in Table XX. and you will have the true time of apparent New or Full Moon required.

The method of calculating the time of any New or Full Moon without the limits of the 19th century, will be shown further on. And a few examples with the precepts, will make the whole work plain.

N. B. The Tables begin the day at noon, and reckon forward from thence to the noon following. Thus,

July the 13th, at 13 hours 14 minutes 32 seconds of tabular time, is July 14th (in common reckoning) at 14 min. 32 sec. past 11 o'clock in the morning.

EXAMPLE I.

Required the true time of New Moon in July, 1748, Old Style

By the Precepts.	New Moon. D. H. M. S.	Sun's Anom. s ° ' "	Moon's Anom. s ° ' "	Sun fro. Node. s ° ' "
March 1748,	17 20 17 36	8 28 54 28	8 21 40 54	1 22 49 5
Add 4 lunations,	118 2 56 12	3 26 25 17	3 13 16 2	4 2 40 56
Mean New Moon,	13 23 13 48	0 24 59 45	0 10 56 56	5 25 30 1
First equation,	— 1 44 1	— 10 17 27	— 39 29	Sun fro. Node, and Arg. for 4th. equation.
Time once equated,	13 21 29 47	0 14 42 18	0 10 17 27	
Second equation,	+ 1 52 42	Arg. 3d. equat.	Arg. 2d. equat.	*Sun Eclipsed.*
Time twice equated,	13 23 21 29			
Third equation,	— 1 13			
Time thrice equated,	13 23 20 16			
Fourth equation,	— 18			
True New Moon,	13 23 19 58			
Equation of days,	— 6 0			
Apparent time.	13 23 13 58			

So the true time of New Moon is 23h 19m. 58 sec. after the noon of the 13th. of July; that is July 14th at 19m. 58 sec. past XI in the morning. But the apparent time is 13m. 58 sec. past XI in the morning.

In this example I look for the year 1748 in Table I. O. S. against which I find the time of mean New Moon in *March*, Sun's anomaly, Moon's anomaly, and Sun's distance from Node, to be as set down in the example. (Agreeable to Precept I.) I then write out as many lunations, with their anomalies, &c. from Table III. as the given month is after *March*; (Precept 3.) which added to the former numbers, gives the mean time of the required New Moon, with their mean anomalies of the Sun and

Moon, and the Sun's mean distance from the Moon's ascending node; which are the arguments for finding the proper equations, (Precept 4.)

With the signs and degrees of the Sun's anomaly, which, in the present case, is 0 signs 24 degrees, I enter Table VII. and look for 0 signs at the top of the Table and 24 degrees in the left hand column, and find in the angle of meeting, 1 h. 40 m. 6 sec. and by making proportions, in the table for the odd 59m.45 sec. (or estimating the Sun's anomaly at 25 deg.) I obtain, for the 1st equation, 1 h. 44 m. 1 sec. which I apply to the time of mean New Moon, as the title *Subtract* at the head of the table directs. (Precept 6.)

With the same argument, (namely, 25 deg.) I enter Table VIII. and take out thence the equation of the Moons mean anomaly, which in the present instance, I find to be 39 m. 29 sec: which I subtract for the Moon's mean anomaly, according to the title on the top of the Table. (Precept 7.)

The result is the argument for finding the 2d equation, with which I enter Table IX. and take out as before the next equation, applying it to the mean New Moon, as the title directs. (precept 7.) This gives the time sufficiently exact for common Almanacks. But when you wish to calculate an eclipse, proceed according to Precept 8. and 9.

EXAMPLE II.

Let it be required to find the true time of the New Moon in April 1764, New Style.

By the Precepts

	New Moon.				Sun's Anom.				Moon's Anom.				Sun fro. Node.			
	D.	H.	M.	S.	s	0	'	"	s	0	'	"	s	0	'	"
March 1764,	2	8	55	36	8	2	20	0	10	13	35	21	11	4	54	48
Add one lunation,	29	12	44	3	0	29	6	19	0	25	49	0	1	0	40	14
Mean New Moon,	31	22	39	39	9	1	26	19	11	9	24	21	0	5	35	2
1st. Equation,	+	4	10	40	11	10	59	18	+	1	34	57	Sun fro. Node and Arg. for 4th. equation.			
Time once equated,	32	1	50	19	9	20	27	1	11	10	59	18				
2d. equation,	—	3	24	49	Arg. 3d equat.				Arg. 2d equat.				*Sun Eclipsed.*			
Time twice equated,	31	22	25	30												
3d. equation,		+	4	37												
Time thrice equated,	31	22	30	7												
4th equation,			+	18												
True New Moon,	31	22	30	25												
Equation of days,		—	3	48												
Apparent time.	31	22	26	37												

Thus the mean time of New Moon is 22 hours, 30 minutes, 25 seconds after the noon of 31st *March* that is *April* 1st at 30 min. 25 sec. after X. in the morning. But the apparent time is 26 min. 37 sec. after X. in the morning.

EXAMPLE III.

Required the true time of Full Moon in April 1819, New Style.

By the Precepts,	New Moon.				Sun's Anom.				Moon's Anom				Sun fro. Node			
	D.	H.	M.	S.	s	0	'	"	s	0	'	"	s	0	'	"
March, 1819,	25	16	54	14	8	23	5	44	8	14	49	17	11	11	29	55
Add 1 lunation,	29	12	44	3	0	29	6	19	0	25	49	0	1	0	40	14
New Moon *April*,	24	5	38	17	9	22	12	3	9	10	38	17	0	12	13	9
Subtract ½ lunatioa,	14	18	22	2	0	14	33	10	6	12	54	30	0	15	20	7
Full Moon *April*,	9	11	16	15	9	7	38	53	2	27	43	47	11	26	53	2
First equation,	+	4	7	55	2	29	17	44	+	1	33	57	Sun fro. Node and Arg. for 4th equation.			
Time once equated,	9	15	24	10	6	8	21	9	2	29	17	44				
Second equation,	+	9	47	6	Arg. 3d equat.				Arg. 2d equat.				*Moon eclipsed*			
Time twice equated,	10	1	11	16	Ans. April 10th, at 11 min. 48 sec. past 1 o'clock, P. M.											
Third equation,				42												
Time thrice equated	10	1	11	58												
Fourth equation,			—	10												
True Full Moon.	10	1	11	48												

EXAMPLE IV.

Required the true time of Full Moon in May 1762, *New Style*.

By the Precepts.	New Moon.				Sun's. Anom.				Moon's Anom.				Sun fro. Node.			
	D.	H.	M.	S.	s	°	'	"	s	°	'	"	s	°	'	"
March 1762,	24	15	18	24	8	23	48	16	1	23	59	11	10	18	49	14
Add 2 lunations,	59	1	28	6	1	28	12	39	1	21	38	1	2	1	20	28
New Moon, May,	22	16	46	30	10	22	0	55	3	15	37	12	0	20	9	42
Subtract ½ lunation,	14	18	22	2	0	14	33	10	6	12	54	30	0	15	20	7
Full Moon, May,	7	22	24	28	10	7	27	45	9	2	42	42	0	4	49	35
First equation,	+	3	16	36	9	3	57	13	+	1	14	33	Sun fro. Node. & Arg. fourth equation.			
Time once equated,	8	1	41	4	1	3	50	30	9	3	57	15				
Second equation,	—	9	47	53	Arg. 3d. equat.				Arg. 2d equat.				*Moon eclipsed,*			
Time twice equated,	7	15	53	11												
Third equation,		—	2	36												
Time thrice equated,	7	15	50	35												
Fourth equation,	.		+	15												
Full Moon required,	7	15	50	50												

Ans. *May* 7th, at 15 h. 50 m. 50 sec. past noon, viz. *May* 8th, at III h. 50 m. 50 sec. in the morning.

To calculate the time of New and Full Moon in a given Year and Month of any particular Century between the Christian Æra and 18th Century.

Precept 1. Find a year of the same number in the 18th Century with that of the year in the century proposed, and take out the mean time of New Moon in March old style, for that year, with the mean nomalies and Sun's mean distance from the node at that time, as already taught.

2. Take as many complete centuries of years from Table VI. as, when subtracted from the aforesaid year in the 18th century, will answer to the given year; and take out the first mean

New Moon and its anomalies, &c. belonging to the said centuries, and set them below those taken out for March in the 18th. century.

3. Subtract the numbers belonging to those centuries, from those of the 18th century, and the remainders will be the mean time and anomalies, &c. of New Moon in March, in the given year of the century proposed.

Then work in all respects for the true time of New or Full Moon, as shown in the above precepts and examples.

4. If the days annexed to these centuries exceed the number of days from the beginning of March taken out in the 18th. century, add a lunation and its anomalies, &c. from Table III. to the time and anomalies of New Moon in March, then proceeed in all respects as above. This circumstance happens in examples 6.

EXAMPLE V.

8. Required the true time of Full Moon in April, Old Style, A. D. 30.
From 1730 subtract 1700 (or 17 centuries) and there remains 30.
By the Precepts,

	New Moon. D. H. M. S.	Sun's. Anom. s ° ' "	Moon's Anom. s ° ' "	Sun fro. Node. s ° ' "
March, 173,	7 12 34 16	8 18 4 31	9 0 32 17	1 23 17 16
Add ½ lunation,	14 18 22 2	0 14 33 10	6 12 54 30	0 15 20 7
Full Moon,	22 6 56 18	9 2 37 41	13 13 26 47	2 8 37 23
1700 years subtract,	14 17 36 42	11 28 46 0	10 29 36 0	4 29 23 0
F.Moon *March*, A.D.30;	7 13 19 36	9 3 51 41	4 13 50 47	9 9 14 23
Add 1 lunation,	29 12 44 3	0 29 6 19	0 25 49 0	1 0 40 14
Full Moon *April*,	6 2 3 39	10 2 58 0	5 9 39 47	10 9 54 37
First equation,	+ 3 28 4	5 10 58 20	+ 1 18 53	Sun fro. Node
Time once equated,	6 5 31 43	4 21 59 20	5 10 58 40	and Arg. 4th equation.
Second equation,	+ 2 57 55	Arg. 3d. equat.	Arg. 2d equat.	
Time twice equated,	6 8 29 38			
Third equation,	— 2 54			
Time thrice equated,	6 8 26 44			
Fourth equation,	— 1 33			
True Full Moon April	6 8 25 11			

Hence it appears, that the true time of Full Moon, in *April*, A. D. 30, old style, was on the 6th day, at 25 m. 11 sec. past VIII. in the evening.

To calculate the true time of New or Full Moon in any given Year and Month before the Christian Æra.

Precept 1. Find a year in the 18th century, which being added to the given number of years before Christ, diminished by 1, shall make a number of complete centuries.

2. Find this number of centuries in Table VI. and subtract the time and anomalies belonging to it from those of the mean New Moon in March, the above found year of the 18th century; and the remainder will denote the time and anomalies, &c. of

the mean New Moon in March, the given year before Christ: Then for the true time of that New Moon, in any month of that year, proceed in the manner taught before.

EXAMPLE VI.

Required the true time of Full Moon in *September*, Old Style, the year before Christ, 201,

The years 200 added to 1800, make 2000, or 20 centuries.

By the Precepts.	New Moon				Sun's Anom.				Moon's Anom.				Sun fro. Node.			
	D.	H.	M.	S.	s	°	'	"	s	°	'	"	s	°	'	"
March 1800,	13	0	22	17	8	23	19	55	10	7	52	36	11	3	58	24
Add 1 lunation,	29	12	44	3	0	29	6	9	0	25	49	0	1	0	40	14
From the sum,	42	13	6	20	9	22	26	14	11	3	41	36	0	4	38	38
Subtract 2000 years,	27	18	9	19	0	8	50	0	0	15	42	0	6	27	45	0
N. M. before Christ 201	14	18	57	1	9	13	36	14	10	17	59	36	5	6	53	38
Add { 6 lunations,	177	4	24	18	5	24	37	56	5	4	54	3	6	4	1	24
Add { half lunation,	14	18	22	2	0	14	33	10	6	12	54	30	0	15	20	7
Full Moon *September*,	22	17	43	21	3	22	47	20	10	5	48	9	11	26	15	9
First equation,	—	3	53	9	10	4	19	52	—	1	28	17	Sun fro. Node,			
Time once equated,	22	13	50	12									and Arg. for			
Second equation,	—	8	25	4	5	18	27	28	10	4	19	52	4th. equation.			
Time twice equated,	22	5	25	8	Arg. 3d. equat.				Arg. 2d. equat.				*Moon eclipsed*			
Third equation,			—	58												
Time thrice equated,	22	5	24	10												
Fourth equation,			—	12												
True Full Moon,	22	5	23	58												

Ans. The true time was *September* 22d day, at 23m. 58 sec. past V in the evening.

These Tables are calculated for the meridian of London; but they will serve for any other place, with equal exactness, by subtracting 4 minutes from the tabular time. for every degree that the meridian of the given place is westward of London, or adding 4 minutes for every degree that the meridian of the given place is eastward of London; as in example VII.

EXAMPLE VII.

Required the true time of the Full Moon at Babylon, Long. 36°, 25', 15'' E. in *October*, old style, the 4008th before the first year of Christ, or 4007th before the year of his birth.

The years 4007 added to 1793, make 5800, or 53 centuries.

By the Precepts	New Moon.				Sun's Anom.				Moon's Anom				Sun fro. Node			
	D.	H.	M.	S	s	°	'	''	s	°	'	''	s	°	'	''
March 1793,	30	9	13	55	9	10	16	11	8	7	37	58	7	6	18	26
Subtract 5800 years,	15	12	38	7	10	21	55	0	6	24	43	0	9	13	1	0
N M. before Christ 4007	14	20	55	48	10	18	41	11	1	12	54	58	9	23	17	20
Add { 7 lunations,	206	17	8	21	6	23	44	15	6	0	43	3	7	4	41	38
Add { ½ lunation,	14	18	22	2	0	14	53	10	6	12	54	30	0	15	20	7
Full Moon, *October*,	22	8	6	11	5	26	58	36	1	26	32	31	5	13	19	11
1st. Equation,		—	13	28	1	26	27	26		—	5	5	Sun fro. Node			
Time once equated,	22	7	52	43	4	0	51	10	1	26	27	26	and Arg. for			
2d. equation,	+	8	29	20	Arg. 3d equat.				Arg. 2d equat.				4th. equation.			
Time twice equated,	22	16	22	3												
3d. equation,		—	4	10												
Time thrice equated,	22	16	17	53												
4th equation,			—	51												
Full Moon at London,	22	16	17	2												
Add for Babylon,		2	25	41												
True time there.	22	18	42	43												

So that, on the meridian of *London* the true time was *October* 23d. at 17 m. 2 sec. past IV. in the morning; but at *Babylon*, the true time was *October* 23d. at 42 m. 43 sec. past VI. in the morning.

This is supposed by some to have been the year of the Creation.

To calculate the true time of New or Full Moon in any given year after the 19th. Century.

Precept 1. Find a year of the same number in the 18th. century with that of the year proposed, and take out the time and anomalies, &c. of New Moon in March, old style, for that year, in Table I.

2. Take so many years from Table VI. as, when added to the above-mentioned year in the 18th. century, will answer to the given year in which the New or Full Moon is required; and take out the first New Moon, with its anomalies, for these complete centuries;

Ans. N. M. 29th day III o'clock 52 m. 28 sec. and Sun's dist. from node 11 *S.* 29 ° 56' 57 ". (Sun eclipsed.*)

3. Required the true time of New Moon, and at that time the Sun's distance from the ascending node, in April, A. D. 1819, N. S. Ans. New Moon 24th day at 46 m. 21 sec. past XI. A. M. and Sun's dist. from node 12 ° 10 ' 9 ". (Sun eclipsed, visible.)

4. Required to find the true time of opposition of the Sun and Moon, and the simultaneous distance of the Sun from the node, in *October*, A. D. 1819, N. S.

Ans. True Full Moon 3d day, at 7m. 48 sec. past III. in the evening, and Sun's dist. from descending node 50 ' 5 " (Moon eclipsed, total.)

5. Required the true time of conjunction of the Sun and Moon, September, A. D. 1820, N. S.

Ans. On the 7th day, at 16 m. 55 sec. past II. P. M. (Sun eclipsed.)

6. Required the true time of Full Moon at Boston, Long. 70 ° 37 ' 15 " W. in May, A. D. 1826, N. S.

Ans. 21st day, at 29 m. 58 sec. past X in the morning. (Moon eclipsed.)

7. Let it be required to find the true time of New Moon in *July*, 1980, O. S. and how far short the Sun will be at that time from the Moon's ascending node.

Ans. 29th day, at 52 m. 28 sec. past III. in the morning. And the Sun will be only 3 ', 3 ", short of the Moon's node, N. ascending. (Consequently, the Sun must suffer a total eclipse.)

To calculate the true Place of the Sun for any given Moment of Time.

Precept 1. In Table XII. find the next lesser year in number to that in which the Sun's place is sought, and write out his mean longitude and anomaly answering thereto: to which add his mean motion and anomaly for the complete residue of years, months, days, hours, minutes, and seconds, down to the given time, and this will be the Sun's mean place and anomaly

* Note. When the Sun is within 17 degrees of either of the Moon's nodes at the time of New Moon, he will be eclipsed at that time: and when he is within 12 degrees of either of the nodes at the time of Full Moon, the Moon will be eclipsed at that time.—See the method of calculating Eclipses.

at that time, in the old style;* provided the said time be in any year after the Christian Æra.

2. Enter Table XIII. with the Sun's mean anomaly; and making proportions for the odd minutes and seconds thereof, take out the equation of the Sun's centre: which being applied to his mean place, as the title *Add or Subtract* directs, will give his true place or longitude from the vernal equinox, at the time for which it was required.

EXAMPLE I.

Required the Sun's true place, *July* 13th 1748, *Old Style*, at 23 hours 19 minutes 58 seconds past noon? In common reckoning, *July* 14th, at 19 minutes 58 seconds past XI. in the forenoon.

		Sun's Long.				Sun's Anom.			
		S	0	'	"	S	0	'	"
To the radical year after Christ 1701		9	20	43	50	6	13	1	0
Add complete years	40	0	0	18	8	11	29	37	0
	7	11	29	18	49	11	29	11	0
	July	5	28	24	8	5	28	24	0
Bissextile, Days	13	0	13	47	57	0	13	47	57
Hours	23	0	0	56	40	0	0	56	40
Minutes	19	0	0	0	46	0	0	0	46
Seconds.	58	0	0	0	2	0	0	0	2
Sun's mean place at the given time.		4	3	30	20	0	24	58	25
Equation of the Sun's centre, subtract			—	47	57	Mean Anom.			
Sun's true place at the same time.		4	2	42	23	or 2 of Cancer.	42	23	

* N. B. Although this Table is constructed according to the *Old Style*, yet it will serve, with equal exactness, for the New, by diminishing the day of the month in this Table by 12, for the present age. Thus, suppose the required time to be on the 28th day of May N. S. Instead of the numbers answering to that day, write out those for the 16th day, &c. But if the required time be *within* the limits of the 18th century, subtract 11 days from the given time. *See Example* II. on the next page.

EXAMPLE II.

Required the Sun's true place, *March* 31st, 1764, *New Style*, at 22 hours 30 minutes 25 seconds, past the noon of that day?

	Sun's Long.				Sun's Anom.			
	S	☌	'	"	S	☌	'	"
To the radical year after Christ 1701	9	20	43	50	6	13	1	0
Add complete years 60	0	0	27	12	11	29	26	0
3	11	29	17	0	11	29	14	0
March	1	28	9	11	1	28	9	0
31st day—11=Bissextile, Days 20		20	41	55		20	41	55
Hours 22			54	13			54	13
Minutes 30			1	14			1	14
Seconds 25				1				1
Sun's mean place at the given time.	0	10	14	36	9	1	27	23
Equations of Sun's centre, add	+	1	55	31	Mean Anom.			
Sun's true place at the same time.	0	12	10	7	or 12 of Aries.	10		7

EX. III. Required the Suns true place and anomaly, July 28th, 15h. 52m. 26 sec. past noon, in the year 1980, ☉. *S.*

Ans. 4 *S.* 18 ° 35 ' 8 " from the vernal equinox, and Suns anomaly, 1 *S.* 7 ° 18 ' 9 ".

To find the Sun's Distance from the Moon's Ascending Node, at the time of any given New or Full Moon; and consequently to know whether there is an Eclipse at that Time, or not.

The Sun's distance from the Moon's ascending node, is the argument for finding the Moon's fourth equation in the syzygies, and therefore it is taken into all the foregoing examples in finding the times of these phenomina.

Thus, at the mean time of New Moon in *July*, 1748, the Sun's mean distance from the ascending node is 5 *S.* 25 ° 30 ' 1 ". *See Example I.* page 175.

The descending node is opposite to the ascending one, and they are, therefore, just six signs distant from each other.

When the Sun is within 17 degrees of either of the nodes at the time of New Moon, he will be eclipsed at that time: and when he is within 12 degrees of either of the nodes at the time

of Full Moon, the Moon will be eclipsed.* Thus we find there will be an eclipse of the Sun at the time of New Moon in *July*, 1748.

But the true time of that New Moon comes out by the equations to be 6 minutes 10 seconds later than the mean time thereof, by comparing these times in the above example: and therefore, (in this, and all similar cases) we must add the Sun's motion from the node during that interval to the above mean distance, 5 *S.* 25 ° 30 ' 1 ", which motion is found in Table XII for 6 minutes, 10 seconds, to be 14 ". And to this we must apply the equation of the Sun's mean distance from the node, in Table XV. found by the Sun's anomaly, which at the mean time of New Moon in example I. we estimated at 25 °, and then we shall have the Sun's true distance from the node, at the true time of New Moon, as follows:

	Sun from Node. s 0 ' "
At the mean time of New Moon in *July*, 1748.	5 25 30 1
Sun's motion from the node for 6 minutes 10 seconds.	14
	0
Sun's mean distance from node at true New Moon	5 25 30 15
Equation of mean distance from node, subtract	52
Sun's true distance from the ascending node; that is,	5 25 29 23
	4 ° 30 ' 37 "

from the descending node; which being far within the above limit of 17 degrees, shows that the Sun must then be eclipsed.

And now we shall shew how to project this, or any other eclipse, either of the Sun or Moon.

* Note. This admits of some variation: for in apogeal eclipses, the solar limit is but 16½ degrees; and in perigeal eclipses, it is 18½. When the Full Moon is in her apogee, she will be eclipsed if she be within 10½ degrees of the node; and when she is full in her perigee, she will be eclipsed if she be within 12$\frac{1}{20}$ degrees of the node.

TO PROJECT AN ECLIPSE OF THE SUN.

In order to this, we must find the ten following Elements by means of the Tables.

1. The true time of conjunction of the Sun and Moon; and at that time,
2. The semidiameter of the Earth's disk* as seen from the Moon, which is always equal to the Moon's horizontal parallax.
3. The Sun's distance from the solstitial colure to which he is then nearest.
4. The Sun's declination.
5. The angle of the Moon's visible path with the ecliptick.
6. The Moon's latitude.
7. The Moon's true horary motion from the Sun.
8. The Sun's semidiameter.
9. The Moon's semidiameter.
10. The semidiameter of the penumbra.

We shall now proceed to find these elements for the Sun's Eclipse in *July*, 1748. O. S.

1. *To find the true time of New Moon.* This, by example I. *page* 175, is found to be on the 14th day of the said month, at 19 minutes 58 seconds past XI. in the morning.

2. *To find the Moon's horizontal parallax, or semidiameter of the Earth's disk, as seen from the Moon.* Enter Table XVII. with the signs and degrees of the Moon's anomaly, (making proportions, because the anomaly is in the table only to every 6th degree,) and thereby take out the Moon's horizontal parallax; which, for the above time answering to the anomaly 10 ° 56 ' 56 ", is 54 ' 33 ".

3. *To find the Sun's distance from the nearest solstice, viz. the beginning of Cancer, which is* 3 *signs, or* 90 *degrees from the beginning of Aries.*

It appears by the example on page 187 (where the Sun's place is calculated to the above time of New Moon) that the

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* Note. The body, or face of the Sun, or Moon, as it appears to a spectator on the Earth; or of the Earth, as it would appear to a spectator at the Sun, or Moon, is called its Disk.

Sun's longitude from the beginning of Aries is there 4*S*. 2° 42' 23", that is 2° 42' 23" from the beginning of Cancer; Thus

	s	°	'	"
From the Sun's Longitude or place	4	2	42	23
Subtract,	3	0	0	0
Remains the Sun's distance from the solstice of Cancer.	1	2	42	23

Or, 32° 42' 23" : each sign containing 30 degrees.

4. *To find the Sun's declination.* Enter Table XIV. with the signs and degrees of the Sun's true place, viz. 4*S*, 2° and making proportion for the 42' 23", take out the Sun's declination answering to his true place, and it will be found to be 19° 38' 8" North.

5. *To find the Moon's latitude.* This depends on her distance from her ascending node, which is the same as the Sun's distance from it at the time of New Moon : and with this the Moon's latitude is found in Table XVI.

Now we have already found that the Sun's equated distance from the ascending node, at the time of New Moon in *July* 1748, is 5*S*. 25° 29' 23". *See the* 189th *page.* Therefore, enter Table XVI. with 5 signs at the bottom, and 25 and 26 degrees at the right hand counted upward, and take out 26° 13', the latitude for 5*S*. 25° ; and 20° 59', the latitude for 5*S*. 26' : and by making proportion between these latitudes for the 29' 23" by which the Moon's distance exceeds the 25th. degree ; her true latitude will be found to be 23 ' 36 " North Ascending.

6. *To find the Moon's true horary motion from the Sun.* With the Moon's anomaly, viz. 0*S*. 10° 56' 56", enter Table XVII. and take out the Moon's horary motion ; which, by making proportion in that table, will be found to be 30' 14". Then, with the Sun's anomaly, 25°, take out his horary motion 2' 23" from the same table ; and subtracting the latter from the former, there will remain 27' 51" for the Moon's true horary motion from the Sun.

7. *To find the angle of the Moon's visible path with the Ecliptick.* This, in the projection of eclipses, may be always rated at 5° 35', without any sensible errour.

8, 9. *To find the semidiameters of the Sun and Moon.* These are found in the same Table, and by the same arguments, as their horary motions.—In the present case the Sun's anomaly gives his semidiameter 15' 51", and the Moon's anomaly gives her diameter 14' 56".

10. *To find the semidiameter of the Penumbra.* Add the Moon's semidiameter to the Sun's, and their sum will be the semidiameter of the penumbra, viz. 30' 47".

Now collect these elements, that they may be found the more readily when they are wanted in the construction of this Eclipse.

	D. H. M. S.
1. True time of New Moon in July, 1748,	14 11 19 58
	° ′ ″
2. Sun's diameter of Earth's disk,	0 54 33
3. Sun's distance from the nearest solstice,	32 42 23
4. Sun's declination, North,	19 38 8
5. Moon's latitude North descending,	0 23 36
6. Moon's horary motion from the Sun,	0 27 51
7. Angle of the Moon's visible path with the ecliptick,	5 35 0
8. Sun's semidiameter,	15 51
9. Moon's semidiameter,	14 56
10. Semidiameter of the penumbra,	30 47

TO PROJECT AN ECLIPSE OF THE SUN GEOMETRICALLY.

Make a scale of any convenient length, as A. C. (Fig. 1.) and divide it into 60 equal parts, reckoning each part to be one minute, or the sixtieth part of a degree.

Then, take the semidiameter of the Earth's disk, 54 minutes, 33 seconds, (or 54½) from the scale, in your compasses; and with that extent, set one foot in the end C of the scale as a centre; and with the other foot describe the semicircle A D B, for the circumference of the northern half of the Earth's illuminated disk, or surface, because we live on the north side of the Equator; continue the line A C to B; so A C B shall be a portion of the Ecliptick, equal to the diameter of the Earth, as seen from the Sun, or Moon at that time.

Upon the centre C, raise the straight line C D H, perpendicular to A C B; and call the line C D H, the axis of the ecliptick.

Being provided with a good sector, open it to the radius C A in the line of chords; and taking from thence the chord of 23½ degrees in your compass, set it off both ways from D to G and to E, in the periphery of the semi-disk. [But, as much the greater number of those into whose hands this work may fall, are not supposed to be thoroughly skilled in the use of Mathematical Instruments, we shall pursue somewhat a different method; which, in point of simplicity and precision, is no less preferable:] Or:—

Divide the quadrants A D and D B, each into 90 equal parts,

for degrees, beginning at D. Then connect the points E and G (which are distant 23½ degrees on each side of D) with the straight line E F G; in which the North pole P of the Earth's disk will always be found.

When the Sun is in Aries, Taurus, Gemini, Cancer, Leo, and Virgo, the North pole of the Earth is enlightened by the Sun: but while the Sun is in the other six signs, the South pole is enlightened, and the North pole is in the dark.

And when the Sun is in Capricorn, Aquarius, Pisces, Aries, Taurus, and Gemini; the northern half of the Earth's axis C XII P lies to the right hand of the axis of the ecliptick, as seen from the Sun; and to the left hand, while the Sun is in the other six signs.

The order, and the names of the *Signs*, the months and days of the year, in which the Sun appears to enter these Signs, are as follows:

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
Aries,	*Taurus*,	*Gemini*,	*Cancer*,	*Leo*,	*Virgo*,
March,	April,	May,	June,	July,	August,
20,	20,	21,	21,	23,	23,

(7.)	(8.)	(9.)	(10,)	(11,)	(12,)
Libra,	*Scorpio*,	*Sagittarius*,	*Capricornus*,	*Aquarius*,	*Pisces*,
Sept.	October,	November,	December,	January,	February,
23,	23,	22,	21	20,	19.

Open the sector, till the radius (or distance of the two 90's) of the signs be equal to the length of D G, and take the sine of the Sun's distance from the solstice (32 ° 42 ' 23 ") as nearly as you can guess, in your compasses, from the line of sines, and set off that distance from F P, in the line E F G, because the Earth's axis lies to the left hand of the axis of the ecliptick, as seen from the Sun in the month of July. Or;

Set one foot of the compasses in the point F, where the line E F G intersects the axis of the ecliptick C D H; and, having extended the other foot from F to E, or from F to G, describe the semicircle E H G, and divide its quadrant H E into 90 equal parts or degrees.—If the Earth's axis had lain to the right hand from the axis of the ecliptick, the quadrant H G must have been divided into 90 degrees, and not the quadrant H E.

As the Sun is 32 degrees 42 minutes 23 seconds, (which may be estimated 32⅔ degrees) from the nearest (or summer) solstice, which is the first point of Cancer, on the noon of the 14th of *July* 1748, draw the right line I P, parallel to H D, from 32⅔ degrees of the quadrant H E till it meets the line E F G at

P, then from P to C, draw the right line P C ; so P C shall be the northern half of the Earth's axis, and P the North pole.

As the Sun is on the North side of the Equator in *July*, and consequently nearer the point of the heaven just over London (or the vertex of London) than the Equator is ; subtract his declination, 19 degrees 38 minutes (neglecting the 8 seconds) from the Latitude of London, 51 degrees 30 minutes, and the remainder will be 31 degrees 52 minutes, for the Sun's distance from the vertex of London on the noon of *July* the 14th.

From the point *k* (in the right hand side of the semicircle A D B) at 31 degrees 52 minutes, counted upward from B, draw *k l*, parallel to C D : and taking the extent *k l* in your compasses, set it from C to XII on the Earth's axis C P. So, the point XII shall be the place of London, at the instant when it is noon at that place on the 14th. of *July* 1748.

Add the Sun's declination 19° 38', to the Latitude of London 51° 30', and the sum will be 71 degrees 8 minutes, for the Sun's distance from the vertex of London on the 14th of *July* at midnight. Therefore,

From 71° 8', counted upward from B to *m* in the right hand side of the semicircle A D B, draw the right line *m n* parallel to C D.

Then, taking the extent *m n* in your compasses, set it from C towards or beyond P on the Earth's axis C P, as it happens to reach short of P or beyond it : but in the present case, it reaches so little above P, that we may reckon C P, to be its whole extent : and so, the point P shall represent the place or situation of London at midnight, beyond the illuminated part of the Earth's disk, as seen from the Sun ; and consequently, in the dark part thereof.

Divide the part of the Earth's axis between XII and P into two equal parts, XII K and P K ; then, through the point K, draw the right line VI K VI (till it meets, on each side, the periphery of the disk) perpendicular to the Earth's axis C XII K P.

Now, to draw the parallel of latitude of any given place, as suppose *London*, or the path of that place on the Earth's enlightened disk, as seen from the Sun, from Sun-rise till Sun-set, proceed as follows·

Subtract the Latitude of London, 51° 30', from 90° 00', and there will remain 38½ for its colatitude, which take in your compasses, from the line of chords, making C A or *C* B radius ; Or,

From 38½ degrees, counted upward from B to *v* in the semicircle A D B, draw the right line *v w* ; and, having taken its length in your compasss, set off that extent both ways from K in the Earth's axis, to VI and VI, in the line VI K VI.

The compasses being opened from K to VI, set one foot in K as a centre, and with the other describe the semicircle VI 7 8 9 10 11 12 1 2 3 4 5 VI, and divide it into 12 equal parts, Then, from these points of division (7 8 9, &c.) draw the dotted lines 7 *a*, 8 *b*, 9 *c*, 10 *d*, &c. all parallel to the Earth's axis C XII P, as in the figure.

With the small extent P K as radius, describe the semicircle P 6 5 4 3 2 1 XII, and divide the lower quadrant into 6 equal parts as in the points 1, 2, 3, 4, 5, 6 ; because the Sun has North declination.

But if the Sun had South declination, the other quadrant must have been so divided.

Through the said division points of the quadrant XII 1 2 3 4 &c. draw the right lines XI 1 XI, X 2 X, IX 3 IX, VIII 4 VIII VII 5 VII, all parallel to the right line VI K VI ; and through the points where these lines meet the former parallel lines 7 *a*, 8 *b*, 9 *c*, 10 *d*, &c. draw the elliptical curve VI VII VIII IX X XI XII I II III IV V VI, which may be done by hand, from point to point ; and set the hour-letters to these points where the right lines meet in the curve, as in the figure.*

This curve shall represent the parallel of Latitude of London, or, the path which London (by the Earth's motion on its axis) appears to describe on the Earth's disk, as seen from the Sun on the 14th of *July*, from VI in the morning untill VI in the evening : and the points VI, VII, VIII, IX, &c. in the curve shall be the point of the disk where London would be at each of these hours respectively, as seen from the Sun.

If the Sun's declination had been as far South as it was North, the diurnal path of London would have been on the upper side of the line VI K VI ; that is the ellipse, of which the curve VI VII, VIII, IX, X, &c. is a part, would have been *complete*, and must have been regulated by dividing the upper quadrant P 6 (of the small semicircle) into 6 equal parts, and drawing lines parallel to VI K VI, as before, till they meet the intercepting lines drawn through the division points of the quadrant P C. The points in which this elliptical curve would touch the periphery of Earth's disk, would denote the instant of the Sun's rising, and of setting at the given place.

Make C A or C B the radius of a line of chords on the sector, and take therefrom the chord of 5 ° 35 ', the angle of the Moon's visible path with the Ecliptick : Or,

From the point M, viz. at 5 degrees 35 minutes, to the right

* N. B. The hour letters on the right hand side of XII, towards VI (in the *Figure)* viz. XI X IX VIII VII, is an errour in sculpture ; it ought to be I II III IV V &c. The reader is therefore required, to correct this trivial mistake in projecting Eclipses.

hand of the axis of the Ecliptick C D, draw the right line M C, for the axis of the Moon's orbit as seen from the Sun, because the Moon's Latitude is *North descending*, on the 14th *July* 1748. If her Latitude had been *North ascending*, the axis of her orbit must have been drawn 5 degrees 35 minutes on the left hand from the axis of the Ecliptick.

N. B. The axis of the Moon's orbit lies the same way when her Latitude is South-ascending, as when it is North-ascending; and the same way when South descending, as when North descending.

Take the Moon's Latitude 23' 36", from the scale C A in your compasses, and set that extent from C to *q* on the axis (C D) of the Ecliptick. Then, through the point *q*, draw the right line IX X XI *q z* XII I &c. perpendicular to the axis of the Moon's orbit C *z* M; and this line shall be the path of the centre of the Moon's shadow over the Earth: and will represent as much of the Moon's orbit, seen from the Sun, as she moves through, during the time that her shadow or penumbra is passing over the Earth.

From C, on the scale A C, take the Moon's horary motion from the Sun, 27' 51", in your compasses; and make the small scale A B (Fig. 2.) equal in length to *that* extent: and divide this scale into 60 equal parts, for so many minutes of time. Then, as the time of New Moon, on the 14th of *July*, 1748, was 19 minutes, 58 seconds, after XI o'clock, take 19 minutes, 58 seconds, or, in this case, 20 minutes, (not regarding the 2 seconds) counted from A to *a* on the small scale A B in your compasses, and set them off, (in Fig. 1.) from the *middle point between* q and z, in the path of the penumbra's centre, to XI in that line; because the instant of tabular time of New Moon is exactly between the point q, where the axis C D of the Ecliptick, and the axis C M of the Moon's orbit, intersect the line, or path of the penumbra's centre on the Earth.

Take the whole length of the scale A B (Fig. 2.) in your compasses; and with that extent, make marks along the line IX X XI XII I, &c. both ways from XI; and set the hour-letters to these marks, as in the figure. Then, from the scale A B (Fig. 2.) divide each space, from mark to mark, into 60 equal parts, or horary minutes, which shall shew the points of the Earth's disk where the centre of the penumbra falls, at every hour and minute, during its transit over the Earth.

[*To the Binder.*—Let the Plate face this page, and unfold to the right.]

A Projection of a Lunar Eclipse observed at London April 10th. 1819. N.S.

Fig. 3

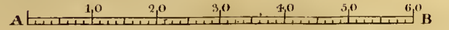

Fig 4.

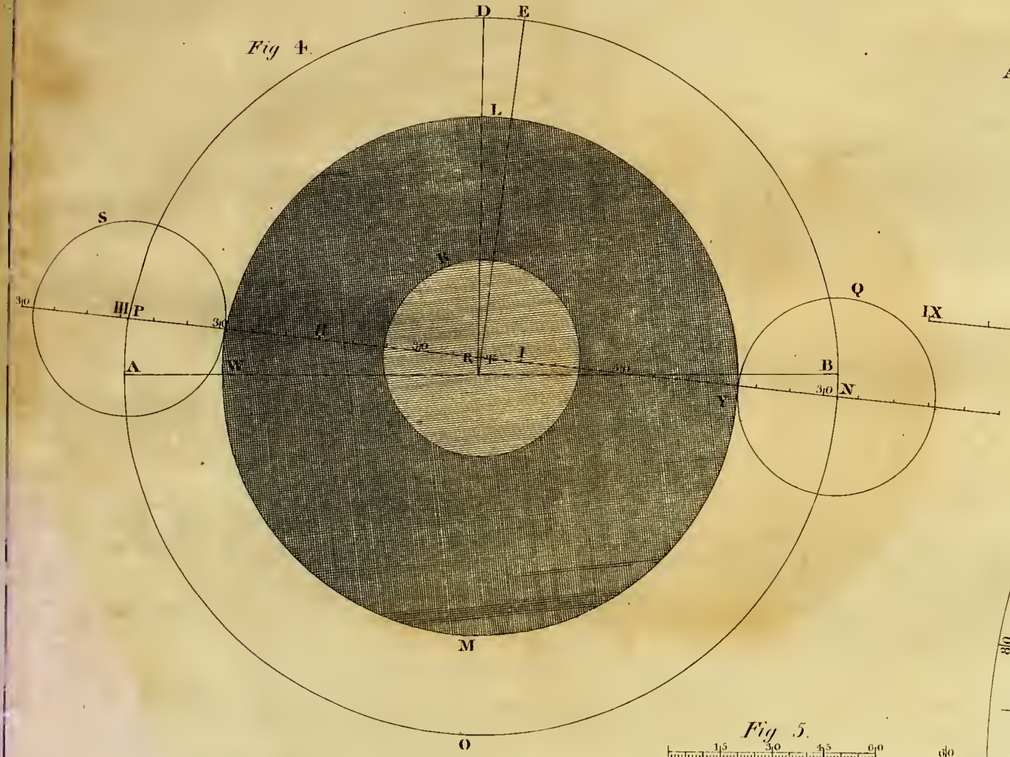

Fig. 2.

Fig. 5.

A Projection of the Sun's Eclipse observed at London, July 14th. 1748, Old Stile.

Fig 1

Apply one side of a square* to the line of the penumbra's path, IX X XI, &c. and move the square forward and backward till the other side cuts the same hour and minute, as at *r* and *s*, both in the path of the penumbra's centre, and the path of London: and the *minute* which the square cuts at the same instant in both these paths, is the *instant* of the visible conjunction of the Sun and Moon, or the greatest obscuration of the Sun, at the place for which the construction is made, namely, *London*, in the present example; and this instant, according to the projection, is at 34½ minutes past X o'clock in the morning.

Take the Sun's semidiameter 15' 51" in your compasses, from the scale A C, (Fig. 1.); and setting one foot at *r*, as a centre in the path of London, namely, at 34½ minutes past X, with the other foot describe the circle R S for the Sun, or which shall represent the Sun's disk as seen from London at the greatest obscuration.

Then take the Moon's semidiameter 14' 56" in your compasses from the scale; and setting one foot in the Moon's path at *s*, 34½ minutes past X, with the other foot describe the circle T U, for the Moon's disk, as seen from London, at the moment when the eclipse is at the greatest; and the portion of the Sun's disk which is hid or obscured by the Moon's, will shew the quantity of the eclipse at that time; which quantity may be measured on a line as, 1 2 3 4 5 6, &c. equal to the Sun's diameter, and divided into 12 equal parts for digits;† of which according to the present projection, there are 9$\frac{5}{6}$ digits eclipsed.

Lastly, take the semidiameter of the penumbra 30' 47" from the scale C A, (fig 1.) in your compasses; and setting one foot in the path of the penumbra's centre, direct the other foot to the path of London among the morning hours at the left hand; and carry that extent backward and forward, till both points of the compasses fall into the same instant in both the path's; and that instant will denote the time when the Eclipse *began* at London. Then, do the like on the right hand of the axis of the ecliptick; and where the points of the compasses fall into the same instant in both the paths, that instant will be the time when the Eclipse *ended* at London.

These trials give 7½ minutes after IX in the morning for the the beginning of the Eclipse: 34½ minutes after X, for the time of greatest obscuration; and 13¾ past XII, for the time when the Eclipse ended.

Note. *The learner will find it convenient to be provided with a small *wooden square*, the two sides of which are about 6 inches in length.

† A DIGIT is a 12th part of the apparent diameter of the Sun or Moon.

From these times we must subtract the equation of natural days, *viz.* 6 minutes, in *July* 14th. and we shall have the apparent times ; namely, 1 minute 30 seconds past IX, for the beginning of the Eclipse, 28 minutes 30 seconds past X, for the time of greatest obscuration, and 7½ minutes past XII for the time when the Eclipse ended. But the most convenient way is to apply this equation to the true equal time of New Moon, before the projection be begun, as is done in *Example* I.

For the motion or position of places on the Earth's disk, answer to apparent or solar time.—*(See Mr. Patterson's Edition of Fugeson's Astronomy, page* 340.—*also his introduction to Astronomy, p.* 163—171, *and End. Nat Phil. p.* 391—394.

TO PROJECT AN ECLIPSE OF THE MOON.

When the Moon is within 12 degrees of either of her Nodes, at the time when she is full, she will be eclipsed, otherwise she will not.

We find by Example III. page 178, that at the time of mean Full Moon in *April*, the Sun's distance from the ascending node was 11 *S* 26° 53' 2" ; that is only 3° 6' 58" short of her descending node, and the Moon being then opposite to the Sun, must have been just as near her ascending node, and was therefore eclipsed.

The Elements for constructing an Eclipse of the Moon are eight in number, as follows :

1. The true time of Full Moon : and at that time.
2. The Moon's horizontal parallax,
3. The Sun's semidiameter.
4. The Moon's semidiameter.
5. The semidiameter of the Earth's shadow at the Moon.
6. The Moon's Latitude.
7. The angle of the Moon's visible path with the Ecliptick.
8. The Moon's true horary motion from the Sun.

Therefore,

1. *To find the true time of Full Moon.* Work as already taught in the Precepts.—Thus we have the true time of Full Moon in *April*, 1819, *(see Example III. page* 178,*)* on the 10th day, at 11 minutes 48 seconds past I o'clock, P. M.

2. *To find the Moon's horizontal parallax.* Enter Table XVII. with the Moon's mean anomaly (at the above time of Full Moon) 2 *S* 27 ° 43 ' 47 ", and thereby take out her horizontal parallax ; which, by making the requisite proportion, will be found to be 57 ' 20 ".

3, 4. *To find the semidiameter of the Sun and Moon.* Enter Table XVII. with their respective anomalies, the Sun's being 9 *S* 7 ° 38 ' 53 ", (by the aforesaid Example) and the Moon's 2 *S* 27 ° 43 " 47 " ; and thereby take out their respective semidiameters : The Sun's 16 ' 4 ", and the Moon's 15 ' 38 ".

5. *To find the semidiameter of the Earth's shadow at the Moon.* Add the Sun's horizontal parallax, which is always 10 ", to the Moon's, which, in the present case is 57 ' 20 ", and the sum will be 57 ' 30 ", from which subtract the Sun's semidiameter 16 ' 4 ", and there will remain 41 ' 26 " for that part of the Earth's shadow which the Moon then passes through.

L. *To find the Moon's Latitude.* Find the Sun's true distance from the ascending node (as already taught in page 189) at the true time of Full Moon ; and this distance, increased by six signs, will be the Moon's true distance from the same node; and consequently the argument for finding her true latitude, as shewn in page 191.

Thus, in Example III. the Sun's mean distance from the ascending node, was 11*S* 26 ° 53 ' 2 ", at the time of mean Full Moon : but it appears by the Example, that the *true* time thereof, was 13 hours, 55 minutes, 33 seconds, later than the *mean* time, and therefore we must add the Sun's motion from the node (found in Table XII.) during this interval, to the above mean distance 11*S*. 26° 53' 2", in order to have his mean distance from it at the true time of Full Moon. Then to this apply the equation of his mean distance from the node (found in Table XV.) by his mean anomaly 9*S* 7° 38' 53" ; and lastly, add six signs : so shall the Moon's true distance from the ascending node be found, as follows :

	s	0	'	"
Sun's distance from node at mean Full Moon,	11	26	53	2
Add his motion from it in 13 hours			32	2
Add his motion from it in 55 minutes			2	15
Add his motion from it in 33 seconds,				1
Sun's mean distance at true Full Moon,	11	27	27	20
Equation of his mean distance, add		2	4	0
Sun's true distance from the node,	11	29	31	20
To which, add	6	0	0	0
And the sum will be	5	29	31	20

Which is the Moon's true distance from her ascending node at the true time of her being full ; and consequently the argument for finding her true latitude at that time. Therefore, with this argument, enter Table XVI. making proportion between the latitudes belonging to the 5th and 6th degree of

the argument at the right hand (the signs being at the bottom) for the 31' 20", and it will give 2' 41" for the Moon's true Latitude, which appears by the Table to be *North descending*.

7. *To find the angle of the Moon's visible path with the Ecliptick.* This may be reckoned 5° 35', without any perceivable errour in the projection of Eclipses.

8. *To find the Moon's true horary motion from the Sun.* With their respective anomalies take out their horary motions from Table XVII. and subtract the Sun's horary motion from the Moon's ; the difference will be the Moon's true horary motion from the Sun : in the present case 30' 49".

Now collect these elements together for use.

	D	H	M	S
1. True time of Full Moon in *April*, 1819	10	1	11	48

	°	'	"
2. Moon's horizontal parallax,	0	57	20
3. Sun's semidiameter	0	16	4
4. Moon's semidiameter,	0	15	38
5. Semidiameter of Earth's shadow at the Moon,	0	41	26
6. Moon's true Latitude, North descending,	0	2	41
7. Angle of the Moon's visible path with the Ecliptick,	5	35	0
8. Her true horary motion from the Sun.		30	49

These Elements being accurately prepared for the construction of the Moon's Eclipse in *April* 1819, proceed as follows :

Make a scale of any convenient length, A B, Fig. 3. and divide it into 60 equal parts, each part answering to a minute of a degree.

Draw the right line A B, (Fig. 4.) for part of the ecliptick, and R D perpendicular to A B for the northern part of its axis ; the Moon having North Latitude.

Add the semidiameters of the Moon and Earth's shadow together, which, in this Eclipse, will make 56' 4" ; and take this from the scale in your compasses, and setting one foot in the point where the axis R D of the Ecliptick meets the right line A B as a centre, describe the circle A D E N O ; in one point of which the Moon's centre will be at the beginning of the Eclipse, and in another point opposite to the former, at the end of the Eclipse.

N. B. If the Moon's NorthL atitude had been equal to twice her semidiameter, it would have been sufficient to describe only the semicircle A D E N.

But in case her Latitude had been South, and equal to twice her semidiameter ; we must have described the semicircle N O A. When her Latitude (whether North or South,) is less

than *twice* her semidiameter, it will be best to describe a complete circle, as in the Plate, fig. 4.

Take the semidiameter of the Earth's shadow, 41' 26", in your compasses from the scale, and setting one foot in the same point for a centre as before, with the other describe the circle W L Y M for the whole circumference of the Earth's shadow at the Moon, through which she passes at her full, *April* 1819.

Make R D the radius of a line of chords on the sector, and set off the angle of the Moon's visible path with the Ecliptick, 5° 35', from D to E; (or, by dividing the quadrant D E N into 90 equal parts, as in Fig. 1.) and draw the right line T E for the northern half of the axis of the Moon's orbit, lying to the right hand from the axis of the Ecliptick R D, because the Moon's lat. is North descending. It would have been the same way (on the *south* side of the ecliptick) if her Latitude had been South descending; but contrary in both cases, that is, to the right hand from the axis of the Ecliptick; if her Latitude had been either North *ascending* or South *ascending*.

Take the Moon's Latitude, 2' 41", from the scale in your compasses, and set off that extent from the point in which the perpendicular R D falls upon the right line A W B, to T in the axis of the Moon's orbit; and through the point T, at right angles to T E, draw the right line P R T N for the path of the Moon's centre.

Then, T shall be the point in the Earth's shadow, where the Moon's centre is at the middle of the Eclipse; the *midle point* between R and T (which was not designated for want of room,) will be the point where her centre is at the Tabular time of her being full; and R, the point where her centre is at the instant of ecliptical opposition.

Take the Moon's horary motion from the Sun, 30' 49", in your compasses from the scale A B (Fig 3.) and with that extent make the small scale (Fig. 5.) an divide it into 60 equal parts, or horary minutes.—Then as the true time of Full Moon in *April* 1819, was at 11 minutes 48 seconds, or $11\frac{4}{5}$ minutes past I o'clock; take $11\frac{4}{5}$ minutes from the (last mentioned) scale in your compasses, and set that extent from the point, signifying the instant of Full Moon (which is mid-way between R and T) to the *left* on the line (P R T N) of the Moon's centre, so shall *that* extent fix the point where the centre of the Moon is at the instant when it is I o'clock at *London*.

From this point I, with the whole length of the scale (Fig. 5.) in your compasses, make marks along the whole length of the line in the path of the Moon's centre, and set the hour letters to these marks, as in the figure: then divide each space from mark to mark, into 60 equal parts or horary minutes, as in (Fig. 5.)

Take the Moon's semidiameter, 15'38", in your compasses, from the scale A B, and with that extent, as a radius, upon the points N, T, and P, as centres, describe the circle Q for the Moon at the beginning of the Eclipse, when she touches the Earth's shadow at Y; the circle R for the Moon at the middle of the Eclipse, and the circle S for the Moon at the end of the Eclipse, just leaving the Earth's shadow at W.

The point N denotes the instant when the Eclipse begins, namely, at 25 minutes 30 seconds after XI in the morning: the point T the middle of the Eclipse, at 10 minutes 18 seconds past I o'clock in the afternoon; and the point P the end of the Eclipse, at 58 minutes after II.—Thus it appears, that the Moon was totally eclipsed for the space of 2 hours, 42 minutes, 30 seconds.

MORE EXAMPLES.

Exp. Let it be required to find the Elements for the Solar Eclipse which happened in *April* 1764, New Style.

	D. H. M. S.
1. True New Moon *April*, 1764.	1 10 30 25
	° ' "
2. Semidiameter of the Earth's disk,	0 54 43
3. Sun's distance from nearest solstice,	77 49 53
4. Sun's declination, North.	4 49 0
5. Moon's Latitude, North ascending.	0 40 18
6. Moon's horary motion from the Sun.	0 27 54
7. Angle of the Moon's visible path with the Ecliptick.	5 35 0
8. Sun's semidiameter,	16 6
9. Moon's semidiameter,	14 57
10. Semidiameter of the Penumbra.	31 3

This Eclipse was nearly central, and annular.*

* Note. In annular eclipses, the light of the Sun is left all around the Moon in a circular form. *Annular*, from the Latin *annulus*, a ring.

Exr. Let it be required to find the Elements for the Lunar Eclipse in *May*, 1762, N. S.

	D.	H.	M.	S.
1. True Full Moon in *May*, 1762.	8	5	50	50
		°	′	″
2. Moon's horizontal parallax.		0	57	20
3. Sun's semidiameter.		0	15	56
4. Moon's semidiameter.		0	15	39
5. Semidiameter of Earth's shadow at the Moon.		0	41	34
6. Moon's true Latitude, South descending.		0	32	21
7. Angle of the Moon's visible path with the Ecliptick.		5	35	0
8. Her horary motion from the Sun.		0	30	52

Ex. 3. Required the Elements for the Eclipse of the Sun, *April* 24th, 1819 ?

Ex. 4. Required the Elements for the Lunar Eclipse, *Oct.* 3d, 1819 ? (total.)

Ex. 4. Let it be required to calculate the Elements for the Lunar Eclipse, *March* 29th, 1820 ?

Ex. 6. In the year 1823, there will be four Eclipses ; namely, two of the Sun, one *February* 11th ; and the other *July* the 8th: and two of the Moon, one *January* 26th, and the other *July* 23d, (both total.) Let it be required to find the respective Elements for the construction of these Eclipses ?

Ex. 7. In the year 1826, there will be two Eclipses of the Moon ; viz. *May* 21st, and November 11th, (both total.) What are the Elements belonging to each ?

Ex. 8. What are the proper Elements for constructing an Eclipse of the Sun, which will happen *July* 29th, 1980, Old Style ?

To find the number of Eclipses there are in any given year, and in what Months they happen.

Precept. Enter Table XVIII. and take out the mean Longitude of the Moon's Nodes for the given year ; with which enter Table XIX. and find, in that table, when the Sun's Longitude will be nearly the *same* or six signs *different* ; and the day of the month in which these numbers are so found, will be the time required.

EXAMPLES.

1. It is required to find the number of Eclipses in the year 1796 ; and in what months they will happen.

The mean Longitude of the Moon's North Node, on the first of *January* 1796, is 3*S* 20° 35′, of the South Node (it being just

six signs distant) 9*S* 20° 35' ; wherefore the Node-months are *January*, *July*, and *December* ; consequently there were three Eclipses in that year.

2. Required the number of Eclipses in the year 1800, and in what months they happened.

3. Required the number of Eclipses in the year 1820, and the months in which they happen.

To find on what part of the Globe any given Eclipse of the Sun or Moon is visible.

This is most readily ascertained by means of an artificial globe ; as follows :

The day and hour being given when a Solar Eclipse will happen, to find where it will be visible.

PRECEPT. Find the Sun's declination, and elevate the pole agreeably to that declination ; bring the place, at which the hour is given, to that part of the brass meridian which is numbered from the equator towards the poles, and set the index of the hour circle to twelve ; then if the given time be before noon, turn the globe westward till the index has passed over as many hours as the given time wants of noon ; if the time be past noon, turn the globe eastward as many hours as it is past noon, and exactly under the degree of the Sun's declination on the brass meridian you will find the place on the globe where the Sun will be vertically eclipsed : at all places within 70 degrees of this place, the eclipse may* be visible, especially if it be a total eclipse.

Ex. On the 11th of February 1304, at 27 min. past 10 o'clock in the morning at London, there was an eclipse of the sun ; where was it visible, supposing the moon's penumbral shadow to extend northward 70 degrees from the place where the sun was vertically eclipsed ?

Ans. Britain, Ireland, France, Germany, &c.

The day and hour being given when a Lunar Eclipse will happen to find where it will be visible.

PRECEPT. Find the Sun's declination for the given day, and note whether it be north or south ; if it be north, elevate the south pole so many degrees above the horizon as are equal to

*Note. When the Moon is exactly in the node, and when the axis of the Moon's shadow and penumbra pass through the centre of the earth, the breadth of the earth's surface under the penumbral shadow is 70° 20' ; but the breadth of this shadow is variable ; and if it be not accurately determined by calculation, it is impossible to tell by the globe to what extent an eclipse of the sun will be visible.

the declination; if it be south, elevate the north pole in a similar manner; bring the place at which the hour is given to that part of the brass meridian which is numbered from the equator towards the poles, and set the index of the hour circle to twelve; then, if the given time be before noon, turn the globe westward as many hours as it wants of noon; if after noon, turn the globe eastward as many hours as it is past noon; the place exactly under the degree of the Sun's declination will be the antipodes of the place where the Moon is virtically eclipsed. Set the index of the hour circle again to twelve, and turn the globe on its axis till the index has passed over twelve hours; then to all places above the horizon the eclipse will be visible; to those places along the western edge of the horizon the moon will rise eclipsed; to those along the eastern edge she will set eclipsed; and to that place immediately under the Sun's declination the Moon will be virtically eclipsed.

Example. On the 26th of January 1804, at 58 min. past seven in the afternoon, at London, there was an eclipse of the Moon; where was it visible?

Answer. It was visible to the whole of Europe, Africa, and the continent of Asia.

A

NEW AND CORRECT

TABLE OF LOGARITHMS.

OF THE NATURAL NUMBERS FROM 1 TO 10.000,

EXTENDED TO SEVEN PLACES BESIDES THE INDEX; AND SO CONTRIVED, THAT THE LOGARITHM MAY BE EASILY FOUND TO ANY NUMBER BETWEEN 1 AND 10,000,000.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1	0.0000000	51	1.7075702	101	2.0043214	151	2.1789769	201	2.3031961
2	0.3010300	52	1.7160033	102	2.0086002	152	2.1818436	202	2.3053514
3	0.4771213	53	1.7242759	103	2.0128372	153	2.1846914	203	2.3074960
4	0.6020600	54	1.7323938	104	2.0170333	154	2.1875207	204	2.3096302
5	0.6989700	55	1.7403627	105	2.0211893	155	2.1903317	205	2.3117539
6	0.7781513	56	1.7481880	106	2.0253059	156	2.1931246	206	2.3138672
7	0.8450980	57	1.7558749	107	2.0293838	157	2.1958997	207	2.3159703
8	0.9030900	58	1.7634280	108	2.0334238	158	2.1986571	208	2.3180633
9	0.9542425	59	1.7708520	109	2.0374265	159	2.2013971	209	2.3201463
10	1.0000000	60	1.7781513	110	2.0413927	160	2.2041200	210	2.3222193
11	1.0413927	61	1.7853298	111	2.0453230	161	2.2068259	211	2.3242825
12	1.0791812	62	1.7923917	112	2.0492180	162	2.2095150	212	2.3263359
13	1.1139434	63	1.7993405	113	2.0530784	163	2.2121876	213	2.3283796
14	1.1461280	64	1.8061800	114	2.0569048	164	2.2148438	214	2.3304138
15	1.1760913	65	1.8129134	115	2.0606978	165	2.2174839	215	2.3324385
16	1.2041200	66	1.8195439	116	2.0644580	166	2.2201081	216	2.3344538
17	1.2304489	67	1.8260748	117	2.0681859	167	2.2227165	217	2.3364597
18	1.2552725	68	1.8325089	118	2.0718820	168	2.2253093	218	2.3384565
19	1.2787536	69	1.8388491	119	2.0755470	169	2.2278867	219	2.3404441
20	1.3010300	70	1.8450980	120	2.0791812	170	2.2304489	220	2.3424227
21	1.3222193	71	1.8512583	121	2.0827854	171	2.2329961	221	2.3443923
22	1.3424227	72	1.8573325	122	2.0863598	172	2.2355284	222	2.3463530
23	1.3617278	73	1.8633229	123	2.0899051	173	2.2380461	223	2.3483049
24	1.3802112	74	1.8692317	124	2.0934217	174	2.2405492	224	2.3502480
25	1.3979400	75	1.8750613	125	2.0969100	175	2.2430380	225	2.3521825
26	1.4149733	76	1.8808136	126	2.1003705	176	2.2455127	226	2.3541084
27	1.4313638	77	1.8864907	127	2.1038037	177	2.2479733	227	2.3560259
28	1.4471580	78	1.8920946	128	2.1072100	178	2.2504200	228	2.3579348
29	1.4623980	79	1.8976271	129	2.1105897	179	2.2528530	229	2.3598355
30	1.4771213	80	1.9030900	130	2.1139434	180	2.2552725	230	2.3617278
31	1.4913617	81	1.9084850	131	2.1172713	181	2.2576786	231	2.3636120
32	1.5051500	92	1.9138139	132	2.1205739	182	2.2600714	232	2.3654880
33	1.5185139	93	1.9190781	133	2.1238516	183	2.2624511	233	2.3673559
34	1.5314789	84	1.9242793	134	2.1271048	184	2.2648178	234	2.3692159
35	1.5440680	85	1.9294189	135	2.1303338	185	2.2671717	235	2.3710679
36	1.5563025	86	1.9344985	136	2.1335389	186	2.2695129	236	2.3729120
37	1.5682017	87	1.9395193	137	2.1367206	187	2.2718416	237	2.3747480
38	1.5797836	88	1.9444827	138	2.1398791	188	2.2741578	238	2.3765770
39	1.5910646	89	1.9493900	139	2.1430148	189	2.2764618	239	2.3783979
40	1.6020600	90	1.9542425	140	2.1461280	190	2.2787536	240	2.3802112
41	1.6127839	91	1.9590414	141	2.1492191	191	2.2810334	241	2.3820170
42	1.6232493	92	1.9637878	142	2.1522883	192	2.2833012	242	2.3838154
43	1.6334685	93	1.9684829	143	2.1553360	193	2.2855573	243	2.3856063
44	1.6434527	94	1.9731278	144	2.1583625	194	2.2878017	244	2.3873898
45	1.6532125	95	1.9777236	145	2.1613680	195	2.2900346	245	2.3891661
46	1.6627578	96	1.9822712	146	2.1643529	196	2.2922561	246	2.3909351
47	1.6720979	97	1.9867717	147	2.1673173	197	2.2944662	247	2.3926969
48	1.6812412	98	1.9912261	148	2.1702617	198	2.2966652	248	2.3944517
49	1.6901961	99	1.9956352	149	2.1731863	199	2.2988531	249	2.3961995
50	1.6989700	100	2.0000000	150	2.1760913	200	2.3010300	250	2.3979400

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
251	2.3996737	301	2.4785665	351	2.5453071	401	2.6031444	451	2.6541765
252	2.4014005	302	2.4800069	352	2.5465427	402	2.6042261	452	2.6551384
253	2.4031205	303	2.4814426	353	2.5477747	403	2.6053050	453	2.6560982
254	2.4048337	304	2.4828736	354	2.5490033	404	2.6063814	454	2.6570559
255	2.4065402	305	2.4842998	355	2.5502284	405	2.6074550	455	2.6580114
256	2.4082400	306	2.4857214	356	2.5514500	406	2.6085260	456	2.6589648
257	2.4099331	307	2.4871384	357	2.5526682	407	2.6095944	457	2.6599162
258	2.4116197	308	2.4885507	358	2.5538830	408	2.6106602	458	2.6608655
259	2.4132998	309	2.4899585	359	2.5550944	409	2.6117235	459	2.6618127
260	2.4149733	310	2.4913617	360	2.5563025	410	2.6127839	460	2.6627578
261	2.4166405	311	2.4927604	361	2.5575072	411	2.6138418	461	2.6637009
262	2.4183013	312	2.4941546	362	2.5587086	412	2.6148972	462	2.6646420
263	2.4199557	313	2.4955443	363	2.5599066	413	2.6159501	463	2.6655810
264	2.4216039	314	2.4969296	364	2.5611014	414	2.6170003	464	2.6665180
265	2.4232459	315	2.4983106	365	2.5622929	415	2.6180481	465	2.6674530
266	2.4248816	316	2.4996871	366	2.5634811	416	2.6190933	466	2.6683859
267	2.4265113	317	2.5010593	367	2.5646661	417	2.6201361	467	2.6693169
268	2.4281348	318	2.5024271	368	2.5658478	418	2.6211763	468	2.6702459
269	2.4297523	319	2.5037907	369	2.5670264	419	2.6222140	469	2.6711728
270	2.4313638	320	2.5051500	370	2.5682017	420	2.6232493	470	2.6720979
271	2.4329695	321	2.5065050	371	2.5693739	421	2.6242821	471	2.6730209
272	2.4345689	322	2.5078559	372	2.5705429	422	2.6253125	472	2.6739420
273	2.4361626	323	2.5092025	373	2.5717088	423	2.6263404	473	2.6748611
274	2.4377506	324	2.5105450	374	2.5728716	424	2.6273659	474	2.6757783
275	2.4393327	325	2.5118834	375	2.5740313	425	2.6283889	475	2.6766936
276	2.4400091	326	2.5132176	376	2.5751878	426	2.6294096	476	2.6776070
277	2.4424798	327	2.5145478	377	2.5763414	427	2.6304279	477	2.6785184
278	2.4440448	328	2.5158738	378	2.5774918	428	2.6314438	478	2.6794279
279	2.4456042	329	2.5171959	379	2.5786392	429	2.6324573	479	2.6803355
280	2.4471580	330	2.5185139	380	2.5797836	430	2.6334685	480	2.6812412
281	2.4487063	331	2.5198280	381	2.5809250	431	2.6344773	481	2.6821451
282	2.4502491	332	2.5211381	382	2.5820634	432	2.6354837	482	2.6830470
283	2.4517864	333	2.5224442	383	2.5831988	433	2.6364879	483	2.6839471
284	2.4533183	334	2.5237465	384	2.5843312	434	2.6374897	484	2.6848454
285	2.4548449	335	2.5250448	385	2.5854607	435	2.6384893	485	2.6857417
286	2.4563660	336	2.5263393	386	2.5865873	436	2.6394865	486	2.6866363
287	2.4578819	337	2.5276299	387	2.5877110	437	2.6404814	487	2.6875290
288	2.4593925	338	2.5289167	388	2.5888317	438	2.6414741	488	2.6884198
289	2.4608978	339	2.5301997	389	2.5899496	439	2.6424645	489	2.6893089
290	2.4623980	340	2.5314789	390	2.5910646	440	2.6434527	490	2.6901961
291	2.4638930	341	2.5327544	391	2.5921768	441	2.6444386	491	2.6910815
292	2.4653829	342	2.5340261	392	2.5932861	442	2.6454223	492	2.6919651
293	2.4668676	343	2.5352941	393	2.5943926	443	2.6464037	493	2.6928469
294	2.4683473	344	2.5365584	394	2.5954962	444	2.6473830	494	2.6937269
295	2.4698220	345	2.5378191	395	2.5965971	445	2.6483600	495	2.6946052
296	2.4712917	346	2.5390761	396	2.5976952	446	2.6493349	496	2.6954817
297	2.4727564	347	2.5403295	397	2.5987905	447	2.6503073	497	2.6963564
298	2.4742163	348	2.5415792	398	2.5998831	448	2.6512780	498	2.6972293
299	2.4756712	349	2.5428254	399	2.6009729	449	2.6522468	499	2.6981005
300	2.4771213	350	2.5440680	400	2.6020600	450	2.6532125	500	2.6989700

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
501	2.6998377	551	2.7411516	601	2.7788745	651	2.8135810	701	2.8457180
502	2.7007037	552	2.7419391	602	2.7795965	652	2.8142476	702	2.8463371
503	2.7015680	553	2.7427251	603	2.7803173	653	2.8149132	703	2.8469555
504	2.7024305	554	2.7435098	604	2.7810369	654	2.8155777	704	2.8475727
505	2.7032914	555	2.7442930	605	2.7817554	655	2.8162413	705	2.8481891
506	2.7041505	556	2.7450748	606	2.7824726	656	2.8169038	706	2.8488047
507	2.7050080	557	2.7458552	607	2.7831887	657	2.8175654	707	2.8494194
508	2.7058637	558	2.7466342	608	2.7839036	658	2.8182259	708	2.8500333
509	2.7067178	559	2.7474118	609	2.7846173	659	2.8188854	709	2.8506462
510	2.7075702	560	2.7481880	610	2.7853298	660	2.8195439	710	2.8512583
511	2.7084209	561	2.7489629	611	2.7860412	661	2.8202015	711	2.8518696
512	2.7092700	562	2.7497363	612	2.7867514	662	2.8208580	712	2.8524800
513	2.7101174	563	2.7505084	613	2.7874605	663	2.8215135	713	2.8530895
514	2.7109631	564	2.7512791	614	2.7881684	664	2.8221681	714	2.8536982
515	2.7118072	565	2.7520484	615	2.7888751	665	2.8228216	715	2.8543060
516	2.7126497	566	2.7528164	616	2.7895807	666	2.8234742	716	2.8549130
517	2.7134905	567	2.7535831	617	2.7902852	667	2.8241258	717	2.8555192
518	2.7143298	568	2.7543483	618	2.7909885	668	2.8247765	718	2.8561244
519	2.7151674	569	2.7551123	619	2.7916906	669	2.8254261	719	2.8567289
920	2.7160033	570	2.7558749	620	2.7923917	670	2.8260748	720	2.8573325
521	2.7168377	571	2.7566361	621	2.7930916	671	2.8267225	721	2.8579353
522	2.7176705	572	2.7573960	622	2.7937904	672	2.8273693	722	2.8585372
523	2.7185017	573	2.7581546	623	2.7944880	673	2.8280151	723	2.8591383
524	2.7193313	574	2.7589119	624	2.7951846	674	2.8286599	724	2.8597386
525	2.7201593	575	2.7596678	625	2.7958800	675	2.8293038	725	2.8603380
526	2.7209857	576	2.7604223	626	2.7965743	676	2.8299467	726	2.8609366
527	2.7218106	577	2.7611758	627	2.7972675	677	2.8305887	727	2.8615344
528	2.7226339	578	2.7619278	628	2.7979596	678	2.8312297	728	2.8621314
529	2.7234557	579	2.7626786	629	2.7986506	679	2.8318698	729	2.8627275
530	2.7242759	580	2.7634280	630	2.7993406	680	2.8325089	730	2.8633229
531	2.7250945	581	2.7641761	631	2.8000294	681	2.8331471	731	2.8639174
532	2.7259116	582	2.7649230	632	2.8007171	682	2.8337844	732	2.8645111
533	2.7267272	583	2.7656686	633	2.8014037	683	2.8344207	733	2.8651040
534	2.7275413	584	2.7664128	634	2.8020893	684	2.8350561	734	2.8656961
535	2.7283538	585	2.7671559	635	2.8027737	685	2.8356905	735	2.8662873
536	2.7291648	585	2.7678976	636	2.8034571	686	2.8363241	736	2.8668778
537	2.7299743	587	2.7686381	637	2.8041394	687	2.8369567	737	2.8674675
538	2.7307823	588	2.7693773	638	2.8048207	688	2.8375884	738	2.8680564
539	2.7315888	589	2.7701153	639	2.8055009	689	2.8382192	739	2.8686444
540	2.7323938	590	2.7708520	640	2.8061800	690	2.8388491	740	2.8692317
541	2.7331973	591	2.7715875	641	2.8068580	691	2.8394780	741	2.8698182
542	2.7339993	592	2.7723217	642	2.8075350	692	2.8401061	742	2.8704039
543	2.7347998	593	2.7730547	643	2.8082110	693	2.8407332	743	2.8709888
544	2.7355989	594	2.7737864	644	2.8088859	694	2.8413595	744	2.8715729
545	2.7363965	595	2.7745170	645	2.8095597	695	2.8419849	745	2.8721563
546	2.7371926	596	2.7752463	646	2.8102325	696	2.8426092	746	2.8727388
547	2.7379873	597	2.7759743	647	2.8109043	697	2.8432328	747	2.8743206
548	2.7387806	598	2.7767012	648	2.8115750	698	2.8438554	748	2.8739016
549	2.7395723	599	2.7774268	649	2.8122447	699	2.8444772	749	2.8744818
550	2.7403627	600	2.7781513	650	2.8129134	700	2.8450980	750	2.8750613

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
[illegible]	2.8755500	801	2.9036529	851	2.9299296	901	2.9547248	951	2.9781805
[illegible]	2.8762173	802	2.9041744	852	2.9304396	902	2.9552065	952	2.9786369
[illegible]	2.8768835	803	2.9047151	853	2.9309490	903	2.9556878	953	2.9790629
[illegible]	2.8775485	804	2.9052560	854	2.9314579	904	2.9561684	954	2.9795484
[illegible]	2.8779470	805	2.9057959	855	2.9319661	905	2.9566486	955	2.9800034
756	2.8785218	806	2.9063350	856	2.9324738	906	2.9571282	956	2.9804579
757	2.8790850	807	2.9068735	857	2.9329808	907	2.9572073	957	2.9809119
758	2.8796502	808	2.9074114	858	2.9334873	908	2.9580858	958	2.9813655
759	2.8802148	809	2.9079485	859	2.9339932	909	2.9585639	959	2.9818186
760	2.8808136	810	2.9084850	860	2.9344985	910	2.9590414	960	2.9822712
761	2.8813647	811	2.9090209	861	2.9350032	911	2.9595184	961	2.9827234
762	2.8819550	812	2.9095560	862	2.9355073	912	2.9599948	962	2.9831751
763	2.8825245	813	2.9100905	863	2.9360108	913	2.9604703	963	2.9836263
764	2.8830934	814	2.9106244	864	2.9365137	914	2.9609462	964	2.9840770
765	2.8836614	815	2.9111576	865	2.9370161	915	2.9614211	965	2.9845273
766	2.8842288	816	2.9116902	866	2.9375179	916	2.9688955	966	2.9849771
767	2.8847954	817	2.9122221	867	2.9380191	917	2.9623693	967	2.9854265
768	2.8853612	818	2.9127533	868	2.9385197	918	2.9618427	968	2.9858754
769	2.8859263	819	2.9132839	869	2.9390198	919	2.9633155	969	2.9863238
770	2.8864907	820	2.9138139	870	2.9395193	920	2.9637878	970	2.9867717
771	2.8870544	821	2.9143432	871	2.9400182	921	2.9642596	971	2.9972192
772	2.8876173	822	2.9148718	872	2.9405165	922	2.9647309	972	2.9876663
773	2.8881795	823	2.9153998	873	2.9410142	923	2.9652017	973	2.9881128
774	2.8887410	824	2.9159272	874	2.9415114	924	2.9656720	974	2.9885590
775	2.8893017	825	2.9164539	875	2.9420081	925	2.9661417	975	2.9890046
776	2.8898617	826	2.9169800	876	2.9425041	926	2.9666110	976	2.9894498
777	2.8904210	827	2.9175055	877	2.9429996	927	2.9670797	977	2.9898946
778	2.8909796	828	2.9180303	878	2.9434945	928	2.9675480	978	2.9903389
779	2.8915375	829	2.9185545	879	2.9439889	929	2.9680157	979	2.9907827
780	2.8920946	830	2.9190781	880	2.9444827	930	2.9684829	980	2.9912261
781	2.8926510	831	2.9196010	881	2.9489759	931	2.9689497	981	2.9916690
782	2.8932068	832	2.9201233	882	2.9454686	932	2.9694159	982	2.9921115
783	2.8937618	833	2.9206450	883	2.9459607	933	2.9698816	983	2.9925535
784	2.8943161	834	2.9211661	884	2.9464523	934	2.9703469	984	2.9929951
785	2.8948697	835	2.9216865	885	2.9469433	935	2.9708116	985	2.9934362
786	2.8954225	836	2.9222063	886	2.9474337	936	2.9712758	986	2.9938769
787	2.8959747	837	2.9227255	887	2.9479236	937	2.9717396	987	2.9943172
788	2.8965262	838	2.9232440	888	2.9484130	938	2.9722028	988	2.9947569
789	2.8970770	839	2.9237620	889	2.9489018	939	2.9726656	989	2.9951963
790	2.8976271	840	2.9242793	890	2.9493900	940	2.9731279	990	2.9956352
791	2.8981765	841	2.9247960	891	2.9498777	941	2.9735896	991	2.9960737
792	2.8987252	842	2.9253121	892	2.9503649	942	2.9740509	992	2.9965117
793	2.8992732	843	2.9258276	893	2.9508515	943	2.9745119	993	2.9969492
794	2.8998205	844	2.9263424	894	2.9513375	944	2.9749720	994	2.9973864
795	2.9003671	845	2.9268567	895	2.9518230	945	2.9754318	995	2.9978231
796	2.9009131	846	2.9273704	896	2.9523080	946	2.9758911	996	2.9982593
797	2.9014583	847	2.9278834	897	2.9527924	947	2.9763500	997	2.9986952
798	2.9020029	848	2.9283959	898	2.9532763	948	2.9768083	998	2.9991305
799	2.9025468	849	2.9289077	899	2.9537597	949	2.9772662	999	2.9995655
800	2.9030900	850	2.9294189	900	2.9542425	950	2.9777236	1000	3.0000000

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1001	3.0004341	1051	3.0216027	1101	3.0417873	1151	3.0610751	1201	3.0799430
1002	3.0008677	1052	3.0220157	1102	3.0421816	1152	3.0614525	1202	3.0799045
1003	3.0013009	1053	3.0224284	1103	3.0425755	1153	3.0618297	1203	3.0802656
1004	3.0017337	1054	3.0228406	1104	3.0429691	1154	3.0622058	1204	3.0806263
1005	3 0021661	1055	3.0232527	1105	3 0433623	1155	3.0625820	1205	3.0809870
1006	3.0025980	1056	3.0236639	1106	3.0437551	1156	3.0629578	1206	3.0813473
1007	3.0030295	1057	3.0240750	1107	3.0441476	1157	3.0633334	1207	3.0817073
1008	3.0034605	1058	3.0244857	1108	3.0445398	1158	3.0637086	1208	3.0820669
1009	3.0038912	1059	3 0248960	1109	3.0449315	1159	3 0640834	1209	3.0824263
1010	3.0043214	1060	3.0253059	1110	3.0453230	1160	3 0644580	1210	3.0827854
1011	3.0047512	1061	3.0257154	1111	3 0457141	1161	3.0648322	1211	3.0831441
1012	3.0051805	1062	3.0261245	1112	3.0461048	1162	3 0652061	1212	3.0835026
1013	3.0056094	1063	3 0265333	1113	3.0464952	1163	3.0655797	1213	3.0838608
1014	3.0060380	1064	3 0269416	1114	3.0468852	1164	3 0659530	1214	3 0842187
1015	3.0064660	1065	3 0273496	1115	3.0472749	1165	3.0663259	1215	3.0845763
1016	3.0068937	1066	3.0277572	1116	3.0476642	1166	3.0666986	1216	3.0849336
1017	3.0073210	1067	3.0281644	1117	3.0480532	1167	3.0670709	1217	3.0852906
1018	3.0077478	1068	3.0285713	1118	3.0484418	1168	3.0674428	1218	3.0856473
1019	3.0081742	1069	3.0289777	1119	3.0488301	1169	3.0678145	1219	3.0860037
1020	3.0086002	1070	3.0293838	1120	3.0492180	1170	3.0681859	1220	3.0863598
1021	3.0090257	1071	3.0297895	1121	3.0496056	1171	3.0685569	1221	3.0867157
1022	3.0094509	1072	3.0301948	1122	3.0499929	1172	3.0689276	1222	3.0870712
1023	3 0098756	1073	3 0305997	1123	3.0503798	1173	3.0692980	1223	3.0874265
1024	3 0103000	1074	3.0310043	1124	3.0507663	1174	3.0696681	1224	3.0877814
1025	3.0107239	1075	3.0314085	1125	3.0511525	1175	3.0700379	1225	3.0881361
1026	3.0111474	1076	3.0318123	1126	3.0515384	1176	3.0704073	1226	3.0884905
1027	3.0115704	1077	3.0322157	1127	3.0519239	1177	3.0707765	1227	3.0888446
1028	3 0119931	1078	3.0326188	1128	3.0523091	1178	3.0711453	1228	3.0891984
1029	3.0124154	1079	3.0330214	1129	3.0526939	1179	3 0715138	1229	3.0895519
1030	3.0128372	1080	3.0334238	1130	3.0530784	1180	3 0718820	1230	3.0899051
1031	3.0132587	1081	3.0338257	1131	3 0534626	1181	3.0722499	1231	3.0902581
1032	3.0136797	1082	3.0342273	1132	3.0538464	1182	3.0726175	1232	3.0906107
1033	3.0141003	1083	3.0346285	1133	3.0542299	1183	3.0729847	1233	3.0909631
1034	3.0145205	1084	3.0350293	1134	3.0546131	1184	3.0733517	1234	3.0913152
1055	3.0149403	1085	3.0354297	1135	3 0549959	1185	3.0737184	1235	3.0916670
1036	3.0153598	1086	3.0358298	1136	3.0553783	1186	3.0740847	1236	3.0920185
1037	3.0157788	1087	3.0362295	1137	3.0557605	1187	3.0744507	1237	3.0923697
1038	3.0161974	1088	3.0366289	1138	3.0561423	1188	3.0748164	1238	3.0927206
1039	3.0166155	1089	3.0370279	1139	3.0565237	1189	3.0751819	1239	3.0930713
1040	3.0170333	1090	3.0374265	1140	3.0569049	1190	3.0755470	1240	3.0934217
1041	3.1074507	1091	3.0378248	1141	3.0572856	1191	3.0759118	1241	3 0937718
1042	3.0178677	1092	3.0382226	1142	3.0576661	1192	3.0762763	1242	3.0941216
1043	3.0182845	1093	3.0386202	1143	3.0580462	1193	3.0766404	1243	3.0944711
1044	3 0187005	1094	3.0390173	1144	3.0584260	1194	3.0770043	1244	3.0948208
1045	3.0191163	1095	3.0394141	1145	3.0588053	1195	3 0773679	1245	3.0951694
1046	3.0195317	1096	3.0398106	1146	3.0591846	1196	3.0777312	1246	3 0955180
1047	3.0199467	1097	3 0402066	1147	3.0595634	1197	3.0780942	1247	3.0958665
1048	3.0203613	1098	3.0406023	1148	3.0599419	1198	3.0784568	1248	3 0962146
1049	3.0207755	1099	3.0409977	1149	3.0603200	1199	3.0788192	1249	3.0965624
1050	3.0211893	1100	3 0413027	1150	3.0606978	1200	3 0791812	1250	3 0969100

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1251	3.0972575	1301	3.1142773	1351	3.1306553	1401	3.1464381	1451	3.1616674
1252	3.0976043	1302	3.1146110	1352	3.1309769	1402	3.1467480	1452	3.1619660
1253	3.0979511	1303	3.114 444	1353	3.1312978	1403	3.1470577	1453	3.1622656
1254	3.0982975	1304	3.1152776	1354	3.1316187	1404	3.1473671	1454	3.1625644
1255	3.0986437	1305	3.1156105	1355	3.1319393	1405	3.1476763	1455	3.1628630
1256	3.0989896	1306	3.1159432	1356	3.1322597	1406	3.1479853	1456	3.1631614
1257	3.0993353	1307	3.1162756	1357	3.1325798	1407	3.1482941	1457	3.1634595
1258	3.0996806	1308	3.1166077	1358	3.1328998	1408	3.1486027	1458	3.1637575
1259	3.1000257	1309	3.1169396	1259	3.1332195	1409	3.1489110	1459	3.1640553
1260	3.1003705	1310	3.1172713	1360	3.1335389	1410	3.1492191	1460	3.1643529
1261	3.1007151	1311	3.1176027	1361	3.1338581	1411	3.1495270	1461	3.1646502
1262	3.1010594	1312	3.1179338	1362	3.1341771	1412	3.1498347	1462	3.1649474
1263	3.1014034	1313	3.1182647	1363	3.1344959	1413	3.1501422	1463	3.1652443
1264	3.1017471	1314	3.1185954	1364	3.1348144	1414	3.1504494	1464	3.1655411
1265	3.1020905	1315	3.1189258	1365	3.1351327	1415	3.1507564	1465	3.1658376
1266	3.1024337	1316	3.1192559	1366	3.1354507	1416	3.1510633	1466	3.1661340
1267	3.1027766	1317	3.1195858	1367	3.1357685	1417	3.1513699	1467	3.1664301
1268	3.1031193	1318	3.1199154	1368	3.1360861	1418	3.1516762	1468	3.1667260
1269	3.1034616	1319	3.1202448	1369	3.1364034	1419	3.1519824	1469	3.1670218
1270	3.1038037	1320	3.1205739	1370	3.1367206	1420	3.1522881	1470	3.1673173
1271	3.1041456	1321	3.1209028	1371	3.1370375	1421	3.1525941	1471	3.1676127
1272	3.1044871	1322	3.1212315	1372	3.1373541	1422	3.1528996	1472	3.1679078
1273	3.1048284	1323	3.1215598	1373	3.1376705	1423	3.1532049	1473	3.1682027
1274	3.1051694	1324	3.1218880	1374	3.1379867	1424	3.1535100	1474	3.1684975
1275	3.1055102	1325	3.1222155	1375	3.1383027	1225	3.1538149	1475	3.1687920
1276	3.1058507	1326	3.1225439	1376	3.1386184	1426	3.1541195	1476	3.1690864
1277	3.1061909	1327	3.1228709	1377	3.1389339	1427	3.1544240	1477	3.1693803
1278	3.1065309	1328	3.1231981	1378	3.1392492	1428	3.1547282	1478	3.1696744
1279	3.1068705	1329	3.1235250	1379	3.1395643	1429	3.1550322	1479	3.1699682
1280	3.1072100	1330	3.1238516	1380	3.1398791	1430	3.1553360	1480	3.1702617
1281	3.1075491	1331	3.1241781	1381	3.1401937	1431	3.1556396	1481	3.1705551
1282	3.1078880	1332	3.1245042	1382	3.1405080	1432	3.1559430	1482	3.1708482
1283	3.1082267	1333	3.1248301	1383	3.1408222	1433	3.1562462	1383	3.1711412
1284	3.1085650	1334	3.1251558	1384	3.1413612	1434	3.1565492	1484	3.1714339
1285	3.1089031	1335	3.1254813	1385	3.1414498	1435	3.1568519	1485	3.1717265
1286	3.1092410	1336	3.1258065	1386	3.1417632	1436	3.1571544	1486	3.1720188
1287	3.1095785	1337	3.1261314	1387	3.1420765	1437	3.1574568	1487	3.1723110
1288	3.1099159	1338	3.1264561	1388	3.1423895	1438	3.1577589	1488	3.1726029
1289	3.1102529	1339	3.1267806	1389	3.1427022	1439	3.1580608	1489	3.1728947
1290	3.1105897	1340	3.1271048	1390	3.1430148	1440	3.1583625	1490	3.1731863
1291	3.1109262	1341	3.1274288	1391	3.1433271	1441	3.1586640	1491	3.1734776
1292	3.1112625	1342	3.1277525	1392	3.1436392	1442	3.1589653	1492	3.1737685
1293	3.1115985	1343	3.1280760	1393	3.1439511	1443	3.1592663	1493	3.1740598
1294	3.1119343	1344	3.1283993	1394	3.1442628	1444	3.1595672	1494	3.1743506
1295	3.1122698	1345	3.1287223	1395	3.1445742	1445	3.1598678	1495	3.1746412
1296	3.1126050	1346	3.1290451	1396	3.1448854	1446	3.1601683	1496	3.1749316
1297	3.1129400	1347	3.1293676	1397	3.1451964	1447	3.1604685	1497	3.1752218
1298	3.1132747	1348	3.1296899	1398	3.1455072	1448	3.1607686	1498	3.1755118
1299	3.1136092	1349	3.1300119	1399	3.1458177	1449	3.1610684	1499	3.1758016
1300	3.1139434	1350	3.1303338	1400	3.1461280	1450	3.1613680	1500	3.1760913

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1501	3.1763807	1551	3.1906118	1601	3.2045913	1651	3.2177471	1701	3.2307043
1502	3.1766699	1552	3.1908917	1602	3.2046625	1652	3.2180100	1702	3.2309596
1503	3.1769590	1553	3.1911715	1603	3.2049335	1653	3.2182729	1703	3.2312146
1504	3.1772478	1554	3.1914510	1604	3.2052044	1654	3.2185355	1704	3.2314696
1505	3.1775365	1555	3.1917304	1605	3.2054750	1655	3.2187980	1705	3.2317244
1506	3.1778250	1556	3.1920096	1606	3.2057455	1656	3.2190605	1706	3.2319790
1507	3.1781133	1557	3.1922886	1607	3.2060159	1657	3.2193225	1707	3.2322335
1508	3.1784013	1558	3.1925675	1608	3.2062860	1658	3.2195845	1708	3.2324879
1509	3.1786892	1559	3.1928461	1609	3.2065560	1659	3.2198464	1709	3.2327421
1510	3.1789769	1560	3.1931246	1610	3.2068259	1660	3.2201081	1710	3.2329961
1511	3.1792645	1561	3.1934029	1611	3.2070955	1661	3.2203696	1711	3.2332500
1512	3.1795518	1562	3.1936810	1612	3.2073650	1662	3.2206310	1712	3.2335038
1513	3.1798389	1563	3.1939590	1613	3.2076344	1663	3.2208922	1713	3.2337574
1514	3.1801259	1564	3.1942367	1614	3.2079035	1664	3.2211533	1714	3.2340108
1515	3.1804126	1565	3.1945143	1615	3.2081725	1665	3.2214142	1715	3.2342641
1516	3.1806992	1566	3.1947918	1616	3.2084414	1666	3.2216750	1716	3.2345173
1517	3.1809856	1567	3.1950690	1617	3.2087100	1667	3.2219356	1717	3.2347703
1518	3.1812718	1568	3.1953461	1618	3.2089785	1668	3.2221960	1718	3.2350232
1519	3.1815578	1569	3.1956229	1619	3.2092468	1669	3.2224563	1719	3.2352759
1520	3.1818436	1570	3.1958997	1620	3.2095150	1670	3.2227165	1720	3.2355284
1521	3.1821292	1571	3.1961762	1621	3.2097830	1671	3.2229764	1721	3.2357809
1522	3.1824147	1572	3.1964525	1622	3.2100508	1672	3.2232363	1722	3.2360331
1523	3.1826999	1573	3.1967287	1623	3.2103185	1673	3.2234959	1723	3.2362853
1524	3.1829850	1574	3.1970047	1624	3.2105860	1674	3.2237555	1724	3.2365373
1525	3.1832698	1575	3.1972805	1625	3.2108534	1675	3.2240148	1725	3.2367891
1526	3.1835545	1576	3.1975562	1626	3.2111205	1676	3.2242740	1726	3.2370408
1527	3.1838390	1577	3.1978317	1627	3.2113876	1677	3.2245331	1727	3.2372923
1528	3.1841234	1578	3.1981070	1628	3.2116544	1678	3.2247920	1728	3.2375437
1529	3.1844075	1579	3.1983821	1629	3.2119211	1679	3.2250507	1729	3.2377950
1530	3.1846914	1580	3.1986571	1630	3.2121876	1680	3.2253093	1730	3.2380461
1531	3.1849752	1581	3.1989319	1631	3.2124540	1681	3.2255677	1731	3.2382971
1532	3.1852588	1582	3.1992065	1632	3.2127202	1682	3.2258260	1732	3.2385547
1533	3.1855422	1583	3.1994809	1633	3.2129862	1683	3.2260841	1733	3.2387986
1534	3.1858254	1584	3.1997552	1634	3.2132521	1684	3.2263421	1734	3.2390491
1535	3.1861084	1585	3.2000293	1635	3.2135178	1685	3.2266000	1735	3.2392995
1536	3.1863912	1586	3.2003032	1636	3.2137835	1686	3.2268576	1736	3.2395497
1537	3.1866739	1587	3.2005769	1637	3.2052044	1687	3.2271151	1737	3.2397998
1538	3.1869563	1588	3.2008505	1638	3.2062860	1688	3.2273724	1738	3.2400498
1539	3.1872386	1589	3.2011239	1639	3.2065560	1689	3.2276296	1739	3.2402996
1540	3.1875207	1590	3.2013971	1640	3.2068259	1690	3.2278867	1740	3.2405493
1541	3.1878026	1591	3.2016702	1641	3.2070955	1691	3.2281436	1741	3.2407988
1542	3.1880844	1592	3.2019431	1642	3.2073732	1692	3.2284004	1742	3.2410482
1543	3.1883659	1593	3.2022158	1643	3.2076376	1693	3.2286570	1743	3.2412974
1544	3.1886473	1594	3.2024883	1644	3.2159018	1694	3.2289134	1744	3.2415465
1545	3.1889285	1595	3.2027607	1645	3.2161659	1695	3.2291697	1745	3.2417954
1546	3.1892095	1596	3.2030329	1646	3.2164298	1696	3.2294258	1746	3.2420442
1547	3.1894903	1597	3.2033049	1647	3.2166936	1697	3.2296818	1747	3.2422929
1548	3.1897710	1598	3.2035768	1648	3.2169572	1698	3.2299377	1748	3.242541
1549	3.1900514	1599	3.2038485	1649	3.2172207	1699	3.2301934	1749	3.242789
1550	3.1903317	1600	3.2041200	1650	3.2174839	1700	3.2304489	1750	3.243038

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1751	3.2432361	1801	3.2555137	1851	3.2674064	1901	3.2789821	1951	3.2902573
1752	3.2435341	1802	3.2557549	1852	3.2676410	1902	3.2792105	1952	3.2904798
1753	3.2437819	1803	3.2559957	1853	3.2678754	1903	3.2794388	1953	3.2907022
1754	3.2440296	1804	3.2562365	1854	3.2681097	1904	3.2796669	1954	3.2909240
1755	3.2442771	1805	3.2564772	1855	3.2683439	1905	3.2798950	1955	3.2911468
1756	3.2445245	1806	3.2567177	1856	3.2685780	1906	3.2801229	1956	3.2913689
1757	3.2447718	1807	3.2569582	1857	3.2688119	1907	3.2803507	1957	3.2915908
1758	3.2450189	1808	3.2571984	1858	3.2690457	1908	3.2805784	1958	3.2918127
1759	3.2452658	1809	3.2574386	1859	3.2692794	1909	3.2808059	1959	3.2920344
1760	3.2455127	1810	3.2576786	1860	3.2695129	1910	3.2810334	1960	3.2922561
1761	3.2457594	1811	3.2579185	1861	3.2697464	1911	3.2812607	1961	3.2924776
1762	3.2460059	1812	3.2581582	1862	3.2699797	1912	3.2814879	1962	3.2916990
1763	3.2462523	1813	3.2583978	1863	3.2702129	1913	3.2817150	1963	3.2929205
1764	3.2464986	1814	3.2586373	1864	3.2704459	1914	3.2819419	1964	3.2931415
1765	3.2467447	1815	3.2588766	1865	3.2706788	1915	3.2821688	1965	3.2933626
1766	3.2469907	1816	3.2591158	1866	3.2709116	1916	3.2823955	1966	3.2935835
1767	3.2472365	1817	3.2593549	1867	3.2711443	1917	3.2826221	1967	3.2938044
1768	3.2474823	1818	3.2595939	1868	3.2713769	1918	3.2828486	1968	3.2940251
1769	3.2477278	1819	3.2598327	1869	3.2716093	1919	3.2830750	1969	3.2942457
1770	3.2479733	1820	3.2600714	1870	3.2718416	1920	3.2833012	1970	3.2944662
1771	3.2482186	1821	3.2603099	1871	3.2720738	1921	3.2835274	1971	3.2946866
1772	3.2484637	1822	3.2605484	1872	3.2723058	1922	3.2837534	1972	3.2949069
1773	3.2487087	1823	3.2607867	1873	3.2725378	1923	3.2839793	1973	3.2951271
1774	3.2489536	1824	3.2610248	1874	3.2727696	1924	3.2842051	1974	3.2953471
1775	3.2491984	1825	3.2612629	1875	3.2730013	1925	3.2844307	1975	3.2955671
1776	3.2494430	1826	3.2615008	1876	3.2732328	1926	3.2846563	1976	3.2957869
1777	3.2496874	1827	3.2617385	1877	3.2734643	1927	3.2848817	1977	3.2960067
1778	3.2499318	1828	3.2619762	1878	3.2736956	1928	3.2851070	1978	3.2962263
1779	3.2501759	1829	3.2622137	1879	3.2739268	1929	3.2853322	1979	3.2964458
1780	3.2504200	1830	3.2624511	1880	3.2741578	1930	3.2855573	1980	3.2966652
1781	3.2506639	1831	3.2626883	1881	3.2743888	1931	3.2857823	1981	3.2968845
1782	3.2509077	1832	3.2629255	1882	3.2746196	1932	3.2860071	1982	3.2971037
1783	3.2511513	1833	3.2631625	1883	3.2748502	1933	3.2862319	1983	3.2973227
1784	3.2513949	1834	3.2633993	1884	3.2750809	1934	3.2864565	1984	3.2975417
1785	3.2516382	1835	3.2636361	1885	3.2753114	1935	3.2866810	1985	3.2977605
1786	3.2518815	1836	3.2638727	1886	3.2755417	1936	3.2869054	1986	3.2979792
1787	3.2521246	1837	3.2641092	1887	3.2757719	1937	3.2871296	1987	3.2981979
1788	3.2523675	1838	3.2643455	1888	3.2760020	1938	3.2873538	1988	3.2984164
1789	3.2526103	1839	3.2645817	1889	3.2762320	1939	3.2875778	1989	3.2986348
1790	3.2528530	1840	3.2648178	1890	3.2764618	1940	3.2878017	1990	3.2988531
1791	3.2530956	1841	3.2650538	1891	3.2766915	1941	3.2880255	1991	3.2990713
1792	3.2533380	1842	3.2652896	1892	3.2769211	1942	3.2882492	1992	3.2992893
1793	3.2535803	1843	3.2655253	1893	3.2771506	1943	3.2884728	1993	3.2995073
1794	3.2538224	1844	3.2657609	1894	3.2773800	1944	3.2886963	1994	3.2997252
1795	3.2540645	1845	3.2659964	1895	3.2776092	1945	3.2889196	1995	3.2999429
1796	3.2543063	1846	3.2662317	1896	3.2778383	1946	3.2891429	1996	3.3001605
1797	3.2545481	1847	3.2664669	1897	3.2780673	1947	3.2893660	1997	3.3003781
1798	3.2547897	1848	3.2667020	1898	3.2782962	1948	3.2895890	1998	3.3005955
1799	3.2550312	1849	3.2669369	1899	3.2785250	1949	3.2898118	1999	3.3008128
1800	3.2552725	1850	3.2671717	1900	3.2787536	1950	3.2900346	2000	3.3010300

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2001	3.3012471	2051	3.2902573	2101	3.3224261	2151	3.3326404	2201	3.3426200
2002	3.3014641	2052	3.2904798	2102	3.3226337	2152	3.3328423	2202	3.3428173
2003	3.3016809	2053	3.2907022	2103	3.3228393	2153	3.3330440	2203	3.3430145
2004	3.3018977	2054	3.2909240	2104	3.3230457	2154	3.3332457	2204	3.3432116
2005	3.3021144	2055	3.2911468	2105	3.3232521	2155	3.3334473	2205	3.3434086
2006	3.3023309	2056	3.2913689	2106	3.3234584	2156	3.3336484	2206	3.3436055
2007	3.3025474	2057	3.2915908	2107	3.3236645	2157	3.3338501	2207	3.3438023
2008	3.3027637	2058	3.2918127	2108	3.3238706	2158	3.3340514	2208	3.3439991
2009	3.3029799	2059	3.2920344	2109	3.3240766	2159	3.3342526	2209	3.3441957
2010	3.3031961	2060	3.2922561	3110	3.3242825	2160	3.3344538	2210	3.3443923
2011	3.3034121	2061	3.2924776	2111	3.3244882	2161	3.3346548	2211	3.3445887
2012	3.3036280	2062	3.2926990	2112	3.3246939	2162	3.3348557	2212	3.3447851
2013	3.3038438	2063	3.2929203	2113	3.3248995	2163	3.3350565	2213	3.3449814
2014	3.3040595	2064	3.2931415	2114	3.3251050	2164	3.3352573	2214	3.3451776
2015	3.3042751	2065	3.2933626	2115	3.3253104	2165	3.3354579	2215	3.3453737
2016	3.3044905	2066	3.2935838	2116	3.3255157	2166	3.3356585	2216	3.3455698
2017	3.3047059	2067	3.2938044	3117	3.3257269	2167	3.3358589	2217	3.3457657
2018	3.3049212	2068	3.2940251	2118	3.3259260	2168	3.3360593	2218	3.3459615
2019	3.3051363	2069	3.2942457	3119	3.3261310	2169	3.3362596	2219	3.3461573
2020	3.3053514	2070	3.2944662	2120	3.3263359	2170	3.3364597	2220	3.3463530
2021	3.3055663	2071	3.2946866	2121	3.3265407	2171	3.3366598	2221	3.3465486
2022	3.3057812	2072	3.2949069	2122	3.3267454	2172	3.3368598	2222	3.3467441
2023	3.3059959	2073	3.2951271	2123	3.3269500	2173	3.3370597	2223	3.3469395
2024	3.3062105	2074	3.2953471	2124	3.3271545	2174	3.3372595	2224	3.3471348
2025	3.3064250	2075	3.2955671	2125	3.3273589	2175	3.3374593	2225	3.3473300
2026	3.3066394	2076	3.2957869	2126	3.3275633	2176	3.3376589	2226	3.3475252
2027	3.3068537	2077	3.2960067	2127	3.3277675	2177	3.3378584	2227	3.3477202
2028	3.3070680	2078	3.2962263	2128	3.3279716	2178	3.3380579	2228	3.3479152
2029	3.3072820	2079	3.2964458	2129	3.3281757	2179	3.3382572	2229	3.3481101
2030	3.3074960	2080	3.2966652	2130	3.3283796	2180	3.3384565	2230	3.3483049
2031	3.3077099	2081	3.3182721	2131	3.3285834	2181	3.3386557	2231	3.3484996
2032	3.3079237	2082	3.3184807	2132	3.3287872	2182	3.3388547	2232	3.3486942
2033	3.3081374	2083	3.3186893	2133	3.3289909	2183	3.3390537	2233	3.3488887
2034	3.3083509	2084	3.3188977	2134	3.3291944	2184	3.3392526	2234	3.3490832
2035	3.3085644	2085	3.3191061	2135	3.3293979	2185	3.3394514	2235	3.3492775
2036	3.3087778	2086	3.3193143	2136	3.3296012	2186	3.3396502	2236	3.3494718
2037	3.3089910	2087	3.3195224	2137	3.3298045	2187	3.3398488	2237	3.3496660
2038	3.3092042	2088	3.3197305	2138	3.3300077	2188	3.3400473	2238	3.3498601
2039	3.3094172	2089	3.3199384	2139	3.3302108	2189	3.3402458	2239	3.3500541
2040	3.3096302	2090	3.3201463	2140	3.3304138	2190	3.3404441	2240	3.3502480
2041	3.2880255	2091	3.3203540	2141	3.3306167	2191	3.3406424	2241	3.3504419
2042	3.2882492	2092	3.3205617	3142	3.3308195	2192	3.3408405	2242	3.3506356
2043	3.2884728	2093	3.3207692	2143	3.3310222	2193	3.3410386	2243	3.3508293
2044	3.2886963	2094	3.3209767	2144	3.3312248	2194	3.3412366	2244	3.3510229
2045	3.2889196	2095	3.3211840	2145	3.3314273	2195	3.3414345	2245	3.3512163
2046	3.2891429	2096	3.3213913	2146	3.3316297	2196	3.3416323	2246	3.3514098
2047	3.2893660	2097	3.3215984	2147	3.3318320	2197	3.3418301	2247	3.3516031
2048	3.2895890	2098	3.3218055	2148	3.3320343	2198	3.3420277	2248	3.3517963
2049	3.2898118	2099	3.3220124	2149	3.3322364	2199	3.3422252	2249	3.3519895
2050	3.2900346	2100	3.3222193	2150	3.3324385	2200	3.3424227	2250	3.3521825

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2252	3 3525684	2302	3 3621053	2352	3 3714373	2402	3 3808730	2452	3 3899205
2253	3 3527612	2303	3 3622939	2353	3 3716219	2403	3 3809538	2453	3 3899975
2254	3 3529539	2304	3 3624825	2354	3 3718055	2404	3 3809345	2454	3 3898746
2255	3 3531465	2305	3 3626709	2355	3 3719909	2405	3 3811151	2455	3 3900515
2256	3 3533391	2306	3 3628593	2356	3 3721753	2406	3 3812956	2456	3 3902284
2257	3 3535316	2307	3 3630476	2357	3 3723590	2407	3 3814761	2457	3 3904052
2258	3 3537239	2308	3 3632358	2358	3 3725438	2408	3 3816565	2458	3 3905819
2259	3 3539162	2309	3 3634239	2359	3 3727279	2409	3 3818368	2459	3 3907585
2260	3 3541084	2310	3 3636120	2360	3 3729120	2410	3 3820170	2460	3 3909351
2261	3 3543005	2311	3 3637999	2361	3 3730950	2411	3 3821972	2461	3 3911116
2262	3 3544926	2312	3 3639878	2362	3 3732799	2412	3 3823773	2462	3 3912880
2263	3 3546846	2313	3 3641755	2363	3 3734637	2413	3 3825573	2463	3 3914644
2264	3 3548764	2314	3 3643633	2364	3 3736475	2414	3 3827373	2464	3 3916407
2265	3 3550682	2315	3 3645510	2365	3 3738311	2415	3 3829171	2465	3 3918169
2266	3 3552599	2316	3 3647386	2366	3 3740147	2416	3 3830969	2466	3 3919931
2267	3 3554515	2317	3 3649260	2367	3 3741983	2417	3 3832766	2467	3 3921691
2268	3 3556431	2318	3 3651134	2368	3 3743817	2418	3 3834563	2468	3 3923452
2269	3 3558345	2319	3 3653007	2369	3 3745651	2419	3 3836359	2469	3 3925211
2270	3 3560259	2320	3 3654880	2370	3 3747483	2420	3 3838154	2470	3 3926970
2271	3 3562171	2321	3 3656751	2371	3 3749316	2421	3 3839948	2471	3 3928727
2272	3 3564083	2322	3 3658622	2372	3 3751147	2422	3 3841741	2472	3 3930485
2273	3 3565994	2323	3 3660492	2373	3 3752977	2423	3 3843534	2473	3 3932241
2274	3 3567905	2324	3 3662361	2374	3 3754807	2424	3 3845326	2474	3 3933997
2275	3 3569814	2325	3 3664230	2375	3 3756636	2425	3 3847117	2475	3 3935752
2276	3 3571723	2326	3 3666097	2376	3 3758464	2426	3 3848908	2476	3 3937506
2277	3 3573630	2327	3 3667964	2377	3 3760292	2427	3 3850698	2477	3 3939260
2278	3 3575537	2328	3 3669830	2378	3 3762118	2428	3 3852487	2478	3 3941013
2279	3 3577443	2329	3 3671695	2379	3 3763944	2429	3 3854275	2479	3 3942765
2280	3 3579348	2330	3 3673559	2380	3 3765770	2430	3 3856063	2480	3 3944517
2281	3 3581253	2331	3 3675423	2381	3 3767594	2431	3 3857850	2481	3 3946268
2282	3 3583158	2332	3 3677285	2382	3 3769418	2432	3 3859636	2482	3 3948018
2283	3 3585059	2333	3 3679147	2383	3 3771240	2433	3 3861421	2483	3 3949767
2284	3 3586961	2334	3 3681008	2384	3 3773062	2434	3 3863206	2484	3 3951516
2285	3 3588862	2335	3 3682869	2385	3 3774884	2435	3 3864990	2485	3 3953264
2286	3 3590762	2336	3 3684728	2386	3 3776704	2436	3 3866773	2486	3 3955011
2287	3 3592662	2337	3 3686587	2387	3 3778524	2437	3 3868555	2487	3 3956758
2288	3 3594560	2338	3 3688445	2388	3 3780343	2438	3 3870337	2488	3 3958504
2289	3 3596458	2339	3 3690302	2389	3 3782161	2439	3 3872118	2489	3 3960249
2290	3 3598355	2340	3 3692159	2390	3 3783979	2440	3 3873898	2490	3 3961993
2291	3 3600251	2341	3 3694014	2391	3 3785796	2441	3 3875678	2491	3 3963737
2292	3 3602146	2342	3 3695869	2392	3 3787612	2442	3 3877457	2492	3 3965480
2293	3 3604041	2343	3 3697723	2393	3 3789427	2443	3 3879235	2493	3 3967223
2294	3 3605934	2344	3 3699576	2394	3 3791241	2444	3 3881012	2494	3 3968964
2295	3 3607827	2345	3 3701428	2395	3 3793055	2445	3 3882789	2495	3 3970705
2296	3 3609719	2346	3 3703280	2396	3 3794868	2446	3 3884565	2496	3 3972446
2297	3 3611611	2347	3 3705131	2397	3 3796680	2447	3 3886340	2497	3 3974185
2298	3 3613500	2348	3 3706981	2398	3 3798492	2448	3 3888114	2498	3 3975924
2299	3 3615390	2349	3 3708830	2399	3 3800302	2449	3 3889888	2499	3 3977663
2300	3 3617278	2350	3 3710679	2400	3 3802112	2450	3 3891661	2500	3 3979400

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2502	3.3982873	2552	3.4068807	2602	3.4153073	2652	3.4235735	2702	3.4316855
2503	3.3984604	2553	3.4070508	2603	3.4154742	2653	3.4237372	2703	3.4318460
2504	3.3986343	2554	3.4072209	2604	3.4156410	2654	3.4239009	2704	3.4320067
2505	3.3988077	2555	3.4073909	2605	3.4158077	2655	3.4240645	2705	3.4321673
2506	3.3989811	2556	3.4075608	2606	3.4159744	2656	3.4242281	2706	3.4323278
2507	3.3991543	2557	3.4077307	2607	3.4161410	2657	3.4243916	2707	3.4324883
2508	3.3993257	2558	3.4079005	2608	3.4163076	2658	3.4245550	2708	3.4326487
2509	3.3995007	2559	3.4080703	2609	3.4164741	2659	3.4247185	2709	3.4328090
2510	3.3996737	2560	3.4082400	2610	3.4166405	2660	3.4248818	2710	3.4329693
2511	3.3998467	2561	3.4084096	2611	3.4168069	2661	3.4250452	2711	3.4331295
2512	3.4000196	2562	3.4085791	2612	3.4169732	2662	3.4252081	2712	3.4332897
2513	3.4001960	2563	3.4087486	2613	3.4171394	2663	3.4253712	2713	3.4334498
2514	3.4003652	2564	3.4089180	2614	3.4173056	2664	3.4255342	2714	3.4336098
2515	3.4005380	2565	3.4090874	2615	3.4174717	2665	3.4256972	2715	3.4337698
2516	3.4007106	2566	3.4092567	2616	3.4176377	2666	3.4258601	2716	3.4339298
2517	3.4008832	2567	3.4094259	2617	3.4178037	2667	3.4260230	2717	3.4340896
2518	3.4010557	2568	3.4095950	2618	3.4179696	2668	3.4261858	2718	3.4342495
2519	3.4012282	2569	3.4097641	2619	3.4181355	2669	3.4263486	2719	3.4344092
2520	3.4014005	2570	3.4099331	2620	3.4183013	2670	3.4265113	2720	3.4345689
2521	3.4015728	2571	3.4101021	2621	3.4184670	2671	3.4266739	2721	3.4347285
2522	3.4017451	2572	3.4102710	2622	3.4186327	2672	3.4268365	2722	3.4348881
2523	3.4019173	2573	3.4104398	2623	3.4187983	2673	3.4269990	2723	3.4350476
2524	3.4020893	2574	3.4106085	2624	3.4189638	2674	3.4271614	2724	3.4352071
2525	3.4022614	2575	3.4107772	2625	3.4191293	2675	3.4273238	2725	3.4353665
2526	3.4024333	2576	3.4109459	2626	3.4192947	2676	3.4274861	2726	3.4355259
2527	3.4026052	2577	3.4111144	2627	3.4194601	2677	3.4276484	2727	3.4356851
2528	3.4027771	2578	3.4112829	2628	3.4196254	2678	3.4278106	2728	3.4358444
2529	3.4029488	2579	3.4114513	2629	3.4197906	2679	3.4279727	2729	3.4360035
2530	3.4031205	2580	3.4116197	2630	3.4199557	2680	3.4281348	2730	3.4361626
2531	3.4032921	2581	3.4117880	2631	3.4201208	2681	3.4282968	2731	3.4363217
2532	3.4034637	2582	3.4119562	2632	3.4202859	2682	3.4284588	2732	3.4364807
2533	3.4036352	2583	3.4121244	2633	3.4204509	2683	3.4286207	2733	3.4366396
2534	3.4038066	2584	3.4122925	2634	3.4206158	2684	3.4287825	2734	3.4367985
2535	3.4039780	2585	3.4124605	2635	3.4207806	2685	3.4289443	2735	3.4369573
2536	3.4041493	2586	3.4126285	2636	3.4209454	2686	3.4291060	2736	3.4371161
2537	3.4043205	2587	3.4127964	2637	3.4211101	2687	3.4292677	2737	3.4372748
2538	3.4044915	2588	3.4129643	2638	3.4212748	2688	3.4294293	2738	3.4374334
2539	3.4046627	2589	3.4131320	2639	3.4214394	2689	3.4295908	2739	3.4375920
2540	3.4048337	2590	3.4132998	2640	3.4216039	2690	3.4297523	2740	3.4377506
2541	3.4050047	2591	3.4134674	2641	3.4217684	2691	3.4299137	2741	3.4379090
2542	3.4051755	2592	3.4136350	2642	3.4219328	2692	3.4300751	2742	3.4380674
2543	3.4053464	2593	3.4138025	2643	3.4220972	2693	3.4302363	2743	3.4382258
2544	3.4055171	2594	3.4139700	2644	3.4222614	2694	3.4303976	2744	3.4383841
2545	3.4056878	2595	3.4141374	2645	3.4224257	2695	3.4305588	2745	3.4385423
2546	3.4058584	2596	3.4143047	2646	3.4225898	2696	3.4307199	2746	3.4387005
2547	3.4060289	2597	3.4144719	2647	3.4227539	2697	3.4308809	2747	3.4388587
2548	3.4061994	2598	3.4146391	2648	3.4229180	2698	3.4310419	2748	3.4390167
2549	3.4063698	2599	3.4148063	2649	3.4230820	2699	3.4312029	2749	3.4391747
2550	3.4065402	2600	3.4149733	2650	3.4232459	2700	3.4313638	2750	3.4393327

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2752	3.4396184	2802	3.4474681	2852	3.4551495	2902	3.4626974	2952	3.4701164
2753	3.4398962	2803	3.4476251	2853	3.4553018	2903	3.4628470	2453	3.4702634
2754	3.4399639	2804	3.4477780	2854	3.4554540	2904	3.4629965	2954	3.4704105
2755	3.4401216	2805	3.4479329	2855	3.4556061	2905	3.4631461	2955	3.4705575
2756	3.4402792	2806	3.4480877	2856	3.4557582	2906	3.4632956	2956	3.4707044
2757	3.4404368	2807	3.4482424	2857	3.4559102	2907	3.4634450	2957	3.4708513
2758	3.4405943	2808	3.4483971	2858	3.4560622	2908	3.4635944	2958	3.4709982
2759	3.4407517	2809	3.4485517	2859	3.4562142	2909	3.4637437	2959	3.4711450
2760	3.4409091	2810	3.4487063	2860	3.4563660	2910	3.4638930	2960	3.4712917
2761	3.4410664	2811	3.4488608	2861	3.4565179	2911	3.4640422	2961	3.4714384
2762	3.4412237	2812	3.4490153	2862	3.4566696	2912	3.4641914	2962	3.4715851
2763	3.4413809	2813	3.4491697	2863	3.4568213	2913	3.4643405	2963	3.4717317
2764	3.4415380	2814	3.4493241	2864	3.4569730	2914	3.4644895	2964	3.4718782
2765	3.4416951	2815	3.4494784	2865	3.4571246	2915	3.4646386	2965	3.4720247
2766	3.4418522	2816	3.4496327	2866	3.4572762	2916	3.4647875	2966	3.4721711
2767	3.4420092	2817	3.4497868	2867	3.4574277	2917	3.4649364	2967	3.4723175
2768	3.4421661	2818	3.4499410	2868	3.4575791	2918	3.4650853	2968	3.4724639
2769	3.4423230	2819	3.4500951	2869	3.4577305	2919	3.4652341	2969	3.4726102
2770	3.4424798	2820	3.4502491	2870	3.4578819	2920	3.4653829	2970	3.4727564
2771	3.4426365	2821	3.4504031	2871	3.4580332	2921	3.4655316	2971	3.4729027
2772	3.4427932	2822	3.4505570	2872	3.8581844	2922	3.4656802	2972	3.4730488
2773	3.4429499	2823	3.4507109	2873	3.4583356	2923	3.4658288	2973	3.4731949
2774	3.4431065	2824	3.4508647	2874	3.4584868	2924	3.4659774	2974	3.4733410
2775	3.4432630	2825	3.4510185	2875	3.4586378	2925	3.4661259	2975	3.4734870
2776	3.4434195	2826	3.4511722	2876	3.4587889	2926	3.4662743	2976	3.4736329
2777	3.4435759	2827	3.4513258	2877	3.4589399	2927	3.4664227	2977	3.4737788
2778	3.4437322	2828	3.4514794	2878	3.4590908	2928	3.4665711	2978	3.4739247
2779	3.4438885	2829	3.4516329	2879	3.4592417	2929	3.4667194	2979	3.4740705
2780	3.4440448	2830	3.4517864	2880	3.4593925	2930	3.4668676	2980	3.4742163
2781	3.4442010	2831	3.4519399	2881	3.4595433	2931	3.4670158	2981	3.4743620
2782	3.4443571	2832	3.4520932	2882	3.4596940	2932	3.4671640	2982	3.4745076
2783	3.4445132	2833	3.4522466	2883	3.4598446	2933	3.4673121	2983	3.4746533
2784	3.4446692	2834	3.4523998	2884	3.4599953	2934	3.4674601	2984	3.4747988
2785	3.4448252	2835	3.4525531	2885	3.4601458	2935	3.4676081	2985	3.4749443
2786	3.4449821	2836	3.4527062	2886	3.4602963	2936	3.4677561	2986	3.4750898
2787	3.4451370	2837	3.4528593	2887	3.4604468	2937	3.4679039	2987	3.4752352
2788	3.4452928	2838	3.4530124	2888	3.4605972	2938	3.4680518	2988	3.4753806
2789	3.4454485	2839	3.4531654	2889	3.4607475	2939	3.4681996	2989	3.4755259
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2792	3.4459154	2842	3.4536241	2892	3.4611983	2942	3.4686427	2992	3.4759610
2793	3.4460709	2843	3.4537769	2893	3.4613484	2943	3.4687903	2993	3.4761067
2794	3.4462264	2844	3.4539296	2894	3.4614985	2944	3.4689378	2994	3.4762518
2795	3.4463818	2845	3.4540823	2895	3.4616486	2945	3.4690853	2995	3.4763968
2796	3.4465372	2846	3.4542349	8896	3.4617986	2946	3.4692327	2996	3.4765418
2797	3.4466925	2847	3.4543875	2897	3.4619485	2947	3.4693801	2997	3.4766867
2798	3.4468477	2848	3.4545400	2898	3.4620984	2948	3.4695275	2998	3.4768316
2799	3.4470029	2849	3.4546924	2899	3.4622482	2949	3.4696748	2999	3.4769765
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3002	3.4774107	3052	3.4845845	3102	3.4916418	3152	3.4985862	3202	3.5054213
3003	3.4775553	3053	3.4847268	3103	3.4917818	3153	3.4987240	3203	3.5055569
3004	3.4776999	3054	3.4848690	3104	3.4919217	3154	3.4988617	3204	3.5056925
3005	3.4778445	3055	3.4850112	3105	3.4920616	3155	3.4989994	3205	3.5058280
3006	3.4779890	3056	3.4851533	3106	3.4922015	3156	3.4991370	3206	3.5059635
3007	3.4781334	3057	3.4852954	3107	3.4923413	3157	3.4992746	3207	3.5060990
3008	3.4782778	3058	3.4854375	3108	3.4924810	3158	3.4994121	3208	3.5062344
3009	3.4784222	3059	3.4855795	3109	3.4926207	3159	3.4995496	3209	3.5063697
3010	3.4785665	3060	3.4857214	3110	3.4927604	3160	3.4996871	3210	3.5065050
3011	3.4787108	3061	3.4858633	3111	3.4929000	3161	3.4998245	3211	3.5066403
3012	3.4788550	3062	3.4860052	3112	3.4930396	3162	3.4999696	3212	3.5067755
3013	3.4789991	3063	3.4861470	3113	3.4931791	3163	3.5000992	3213	3.5069107
3014	3.4791432	3064	3.4862888	3114	3.4933186	3164	3.5002365	3214	3.5070459
3015	3.4792873	3065	3.4864305	3115	3.4934581	3165	3.5003737	3215	3.5071810
3016	3.4794313	3066	3.4865722	3116	3.4935974	3166	3.5005109	3216	3.5073160
3017	3.4795753	3067	3.4867138	3117	3.4937368	3167	3.5006481	3217	3.5074511
3018	3.4797192	3068	3.4868554	3118	3.4938761	3168	3.5007852	3218	3.5075860
3019	3.4798631	3069	3.4869969	3119	3.4940154	3169	3.5009222	3219	3.5077210
3020	3.4800069	3070	3.4871384	3120	3.4941546	3170	3.5010593	3220	3.5078559
3021	3.4801507	3071	3.4872798	3121	3.4942938	3171	3.5011962	3221	3.5079907
3022	3.4802945	3072	3.4874212	3122	3.4944329	3172	3.5013332	3222	3.5081255
3023	3.4804381	3073	3.4875626	3123	3.4945720	3173	3.5014701	3223	3.5082603
3024	3.4805818	3074	3.4877039	3124	3.4947110	3174	3.5016069	3224	3.5083950
3025	3.4807254	3075	3.4878451	3125	3.4948500	3175	3.5017437	3225	3.5085297
3026	3.4808689	3076	3.4879863	3126	3.4949890	3176	3.5018805	3226	3.5086644
3027	3.4810124	3077	3.4881275	3127	3.4951279	3177	3.5020172	3227	3.5087990
3028	3.4811559	3078	3.4882686	3128	3.4952667	3178	3.5021539	3228	3.5089335
3029	3.4812993	3079	3.4884097	3129	3.4954056	3179	3.5022905	3229	3.5090680
3030	3.4814426	3080	3.4885507	3130	3.4955443	3180	3.5024271	3230	3.5092025
3031	3.4815859	3081	3.4886917	3131	3.4956831	3181	3.5025637	3231	3.5093370
3032	3.4817292	3082	3.4888326	3132	3.4958218	3182	3.5027002	3232	3.5094714
3033	3.4818724	3083	3.4889735	3133	3.4959604	3183	3.5028366	3233	3.5096057
3034	3.4820156	3084	3.4891144	3134	3.4960990	3184	3.5029731	3234	3.5097400
3035	3.4821587	3085	3.4892552	3135	3.4962375	3185	3.5031094	3235	3.5098743
3036	3.4823018	3086	3.4893959	3136	3.4963767	3186	3.5032458	3236	3.5100085
3037	3.4824448	3087	3.4895366	3137	3.4965145	3187	3.5033821	3237	3.5101427
3038	3.4825878	3088	3.4896773	3138	3.4966529	3188	3.5035183	3238	3.5102768
3039	3.4827307	3089	3.4898179	3139	3.4967913	3189	3.5036545	3239	3.5104109
3040	3.4828736	3090	3.4899585	3140	3.4969296	3190	3.5037907	3240	3.5105450
3041	3.4830164	3091	3.4900990	3141	3.4970679	3191	3.5039268	3241	3.5106790
3042	3.4831592	3092	3.4902395	3142	3.4972062	3192	3.5040629	3242	3.5108130
3043	3.4833020	3093	3.4903799	3143	3.4973444	3193	3.5041989	3243	3.5109469
3044	3.4834446	3094	3.4905203	3144	3.4974825	3194	3.5043349	3244	3.5110808
3045	3.4835873	3095	3.4906607	3145	3.4976206	3195	3.5044709	3245	3.5112147
3046	3.4837299	3096	3.4908010	3146	3.4977587	3196	3.5046068	3246	3.5113485
3047	3.4838725	3097	3.4909412	3147	3.4978967	3197	3.5047426	3247	3.5114823
3048	3.4840150	3098	3.4910814	3148	3.4980347	3198	3.5048785	3248	3.5116160
3049	3.4841574	3099	3.4912216	3149	3.4981727	3199	3.5050142	3249	3.5117497
3050	3.4842998	3100	3.4913617	3150	3.4983106	3200	3.5051500	3250	3.5118834

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
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3252	3.5121595	3302	3.5187771	3352	3.5253040	3402	3.5317343	3452	3.5380708
3253	3.5122841	3303	3.5189089	3353	3.5254336	3403	3.5318619	3453	3.5381966
3254	3.5124175	3304	3.5190400	3354	3.5255631	3404	3.5319896	3454	3.5383223
3255	3.5125510	3305	3.5191715	3355	3.5256925	3405	3.5311171	3455	3.5384481
3256	3.5126844	3306	3.5193028	3356	3.5258220	3406	3.5322446	3456	3.5385737
3257	3.5128178	3307	3.5194342	3357	3.5259513	3407	3.5323721	3457	3.5386994
3258	3.5129512	3308	3.5195655	3358	3.5260807	3408	3.5324996	3458	3.5388250
3259	3.5130844	3309	3.5196968	3359	3.5262100	3409	3.5326270	3459	3.5389500
3260	3.5132176	3310	3.5198280	3360	3.5265393	3410	3.5327544	3460	3.5390761
3261	3.5133508	3311	3.5199592	3361	3.5264685	3411	3.5328817	3461	3.5392016
3262	3.5134840	3312	3.5200903	3362	3.5265977	3412	3.5330090	3462	3.5393271
3263	3.5136171	3313	3.5202214	3363	3.5267269	3413	3.5331363	3463	3.5394525
3264	3.5137502	3314	3.5203525	3364	3.5268560	3414	3.5332635	3464	3.5395779
3265	3.5138832	3315	3.5204835	3365	3.5269851	3415	3.5333907	3465	3.5397032
3266	3.5140162	3316	3.5206145	3366	3.5271141	3416	3.5335179	3466	3.5398286
3267	3.5141491	3317	3.5207455	3367	3.5272431	3417	3.5336450	3467	3.5399538
3268	3.5142820	3318	3.5208764	3368	3.5273721	3418	3.5337721	3468	3.5400791
3269	3.5144149	3319	3.5210073	3369	3.5275010	3419	3.5338991	3469	3.5402043
3270	3.5145478	3320	3.5211381	3370	3.5276299	3420	3.5340261	3470	3.5403295
3271	3.5146805	3321	3.5212689	3371	3.5277588	3421	3.5341531	3471	3.5404546
3272	3.5148133	3322	3.5213996	3372	3.5278876	3422	3.5342800	3472	3.5405797
3273	3.5149460	3323	3.5215303	3373	3.5280163	3423	3.5344069	3473	3.5407048
3274	3.5150787	3324	3.5216610	3374	3.5281451	3424	3.5345338	3474	3.5408298
3275	3.5152113	3325	3.5217916	3375	3.5282738	3425	3.5346606	3475	3.5409548
3276	3.5153439	3326	3.5219222	3376	3.5284024	3426	3.5347874	3476	3.5410797
3277	3.5154764	3327	3.5220528	3377	3.5285311	3427	3.5349141	3477	3.5412047
3278	3.5156089	3328	3.5221833	3378	3.5286596	3428	3.5350408	3478	3.5413296
3279	3.5157414	3329	3.5223138	3379	3.5287882	3429	3.5351675	3479	3.5414544
3280	3.5158738	3330	3.5224442	3380	3.5289167	3430	3.5352941	3480	3.5415792
3281	3.5160062	3331	3.5225786	3381	3.5290452	3431	3.5354207	3481	3.5417040
3282	3.5161386	3332	3.5227050	3382	3.5291736	3432	3.5355473	3482	3.5418288
3283	3.5162709	3333	3.5228353	3383	3.5293020	3433	3.5356738	3483	3.5419535
3284	3.5164031	3334	3.5229656	3384	3.5293304	3434	3.5358003	3484	3.5420781
3285	3.5165354	3335	3.5230958	3385	3.5295587	3435	3.5359267	3485	3.5422028
3285	3.5166676	3336	3.5232260	3386	3.5296870	3436	3.5360532	3486	3.5423274
3287	3.5167997	3337	3.5233562	3387	3.5298152	3437	3.5361795	3487	3.5424519
3288	3.5169318	3338	3.5234863	3388	3.5299434	3438	3.5363059	3488	3.5425765
3289	3.5170639	3339	3.5236164	3389	3.5300716	3439	3.5364322	3489	3.5427010
3290	3.5171959	3340	3.5237465	3390	3.5301997	3440	3.5365584	3490	3.5428254
3291	3.5173279	3341	3.5238765	3391	3.5303278	3441	3.5366847	3491	3.5429498
3292	3.5174598	3342	3.5240064	3392	3.5304558	3442	3.5368109	3492	3.5430742
3293	3.5175917	3343	3.5241361	3393	3.5305839	3443	3.5369370	3493	3.5431986
3294	3.5177236	3344	3.5242663	3394	3.5307118	3444	3.5370631	3494	3.5433229
3295	3.5178554	3345	3.5243961	3395	3.5308398	3445	3.5371892	3495	3.5434472
3296	3.5179872	3346	3.5245259	3396	3.5309675	3446	3.5373152	3496	3.5435714
3297	3.5181189	3347	3.5246557	3397	3.5310955	3447	3.5374413	3497	3.5436956
3298	3.5182507	3348	3.5247854	3398	3.5312234	3448	3.5375673	3498	3.5438198
3299	3.5183823	3349	3.5249151	3399	3.5313513	3449	3.5376932	3499	3.5439439
3300	3.5185139	3350	3.5250448	3400	3.5314789	3450	3.5378191	3500	3.5440680

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
3501	3.5441921	3551	3.5503507	3601	3.5564231	3651	3.5624118	3701	3.5683191
3502	3.5443161	3552	3.5504730	3602	3.5565437	3652	3.5625308	3702	3.5684364
3503	3.5444401	3553	3.5505952	3603	3.5566645	3653	3.5626497	3703	3.5685537
3504	3.5445641	3554	3.5507174	3604	3.5567848	3654	3.5627685	3704	3.5686710
3505	3.5446880	3555	3.5508396	3605	3.5569053	3655	3.5628874	3705	3.5687882
3506	3.5448119	3556	3.5509618	3606	3.5570257	3656	3.5630062	3706	3.5689054
3507	3.5449358	3557	3.5510839	3607	3.5571461	3657	3.5631250	3707	3.5690226
3508	3.5450596	3558	3.5512059	3608	3.5572665	3658	3.5632437	3708	3.5691397
3509	3.5451834	3559	3.5513280	3609	3.5573869	3659	3.5633624	3709	3.5692568
3510	3.5453071	3560	3.5514500	3610	3.5575072	3660	3.5634811	3710	3.5693739
3511	3.5454308	3561	3.5515720	3611	3.5576275	3661	3.5635997	3711	3.5694910
3512	3.5455545	3562	3.5516939	3612	3.5577477	3662	3.5637183	3712	3.5696080
3513	3.5456781	3563	3.5518158	3613	3.5578686	3663	3.5638369	3713	3.5697249
3514	3.5458018	3564	3.5519377	3614	3.5579881	3664	3.5639555	3714	3.5698419
3515	3.5459253	3565	3.5520595	3615	3.5581033	3665	3.5640740	3715	3.5699588
3516	3.5460489	3566	3.5521813	3616	3.5582284	3666	3.5641925	3716	3.5700757
3517	3.5461724	3567	3.5523031	3617	3.5583485	3667	3.5643109	3717	3.5701926
3518	3.5462958	3568	3.5524248	3618	3.5584686	3668	3.5644293	3718	3.5703094
3519	3.5464193	3569	3.5525465	3619	3.5585886	3669	3.5645477	3719	3.5704262
3520	3.5465427	3570	3.5526682	3620	3.5587086	3670	3.5646661	3720	3.5705429
3521	3.5466660	3571	3.5527899	3621	3.5588285	3671	3.5647844	3721	3.5706597
3522	3.5467894	3572	3.5529115	3622	3.5589484	3672	3.5649027	3722	3.5707764
3523	3.5469126	3573	3.5531030	3623	3.5590683	3673	3.5650209	3723	3.5708930
3524	3.5470359	3574	3.5531545	3624	3.5591882	3674	3.5651392	3724	3.5710097
3525	3.5471591	3575	3.5532760	3625	3.5593080	3675	3.5652573	3725	3.5711263
3526	3.5472823	3576	3.5533975	3626	3.5594278	3676	3.5653755	3726	3.5712429
3527	3.5474055	3577	3.5535189	3627	3.5595476	3677	3.5654936	3727	3.5713598
3528	3.5475286	3578	3.5536403	3628	3.5596673	3678	3.5656117	3728	3.5714759
3529	3.5476517	3579	3.5537617	3629	3.5597870	3679	3.5657297	3729	3.5715924
3530	3.5477747	3580	3.5538830	3630	3.5599066	3680	3.5658478	3730	3.5717088
3531	3.5478977	3581	3.5540043	3631	3.5600262	3681	3.5659658	3731	3.5718252
3532	3.5480207	3582	3.5541256	3632	3.5601458	3682	3.5660838	3732	3.5719416
3533	3.5481436	3583	3.5542468	3633	3.5602654	3683	3.5662017	3733	3.5720580
3534	3.5482665	3584	3.5543680	3634	3.5603849	3684	3.5663196	3734	3.5721743
3535	3.5483894	3585	3.5544892	3635	3.5605044	3685	3.5664375	3735	3.5722906
3536	3.5485123	3586	3.5546103	3636	3.5606239	3686	3.5665553	3736	3.5724169
3537	3.5486351	3587	3.5547314	3637	3.5607433	3687	3.5666731	3737	3.5725231
3538	3.5487578	3588	3.5548524	3638	3.5608627	3688	3.5667909	3738	3.5726393
3539	3.5488806	3589	3.5549735	3639	3.5609821	3689	3.5669087	3739	3.5727554
3540	3.5490033	3590	3.5550944	3640	3.5611014	3690	3.5670264	3740	3.5728716
3541	3.5491259	3591	3.5552154	3641	3.5612207	3691	3.5671440	3741	3.5729877
3542	3.5492486	3592	3.5553363	3642	3.5613399	3692	3.5672617	3742	3.5731098
3543	3.5493712	3593	3.5554572	3643	3.5614592	3693	3.5673793	3743	3.5732198
3544	3.5494937	3594	3.5555781	3644	3.5615784	3694	3.5674969	3744	3.5733358
3545	3.5496162	3595	3.5556989	3645	3.5616975	3695	3.5676144	3745	3.5734518
3546	3.5497387	3596	3.5558197	3646	3.5618167	3696	3.5677320	3746	3.5735678
3547	3.5498612	3597	3.5559404	3647	3.5619358	3697	3.5678495	3747	3.5736837
3548	3.5499836	3598	3.5560612	3648	3.5620548	3698	3.5679559	3748	3.5737996
3549	3.5501060	3599	3.5561818	3649	3.5621739	3699	3.5680843	3749	3.5739154
3550	3.5502284	3600	3.5563025	3650	3.5622929	3700	3.5682017	3750	3.5740313

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
3751	3.5741471	3801	3.5798979	3851	3.5855735	3901	3.5911760	3951	3.5967070
3752	3.5742628	3802	3.5800121	3852	3.5856863	3902	3.5912873	3952	3.5968169
3753	3.5743786	3803	3.5801263	3853	3.5857990	3903	3.5913986	3953	3.5969268
3754	3.5744943	3804	3.5802405	3854	3.5859117	3904	3.5915098	3954	3.5970367
3755	3.5746099	3805	3.5803547	3855	3.5860244	3905	3.5916210	3955	3.5971465
3756	3.5747256	3806	3.5804688	3856	3.5861370	3906	3.5917322	3956	3.5972563
3757	3.5748413	3807	3.5805829	3857	3.5862496	3907	3.5918434	3957	3.5973661
3758	3.5749568	3808	3.5806969	3858	3.5863622	3908	3.5919546	3958	3.5974758
3759	3.5750723	3809	3.5808110	3859	3.5864748	3909	3.5920657	3959	3.5975855
3760	3.5751878	3810	3.5809250	3860	3.5865873	3910	3.5921768	3960	3.5976952
3761	3.5753033	3811	3.5810389	3861	3.5866998	3911	3.5922878	3961	3.5978048
3762	3.5754188	3812	3.5811529	3862	3.5868123	3912	3.5923988	3962	3.5979145
3763	3.5755342	3813	3.5812668	3863	3.5869247	3913	3.5925098	3963	3.5980241
3764	3.5756496	3814	3.5813807	3864	3.5870371	3914	3.5926208	3964	3.5981336
3765	3.5757650	3815	3.5814945	3865	3.5871495	3915	3.5927318	3965	3.5982432
3766	3.5758803	3816	3.5816084	3866	3.5872618	3916	3.5928427	3966	3.5983527
3767	3.5759956	3817	3.5817222	3867	3.5873743	3917	3.5929536	3967	3.5984692
3768	3.5761109	3818	3.5818359	3868	3.5874865	3918	3.5930644	3968	3.5985717
3770	3.5762261	3819	3.5819497	3869	3.5875987	3919	3.5931753	3969	3.5986811
3770	3.5763414	3820	3.5820634	3870	3.5877110	3920	3.5932861	3970	3.5987905
3771	3.5764565	3821	3.5821770	3871	3.5878232	3921	3.5933968	3971	3.5988999
3772	3.5765717	3822	3.5822907	3872	3.5879353	3922	3.5935076	3972	3.5990092
3773	3.5766868	3823	3.5824043	3873	3.5880475	3923	3.5936083	3973	3.5991186
3774	3.5768019	3824	3.5825179	3874	3.5881596	3924	3.5937290	3974	3.5992279
3775	3.5769170	3825	3.5826314	3875	3.5882717	3925	3.5938397	3975	3.5993371
3776	3.5770320	3826	3.5827450	3876	3.5883838	3926	3.5939503	3976	3.5994464
3777	3.5771470	3827	3.5828585	3877	3.5884958	3927	3.5940609	3977	3.5995556
3778	3.5772620	3828	3.5829719	3878	3.5886078	3928	3.5941715	3978	3.5996648
3779	3.5773769	3829	3.5830854	3879	3.5887198	3929	3.5942820	3979	3.5997739
3780	3.5774918	3830	3.5831988	3880	3.5888317	3930	3.5943926	3980	3.5998831
3781	3.5776067	3831	3.5833122	3881	3.5889436	3931	3.5945030	3981	3.5999922
3782	3.5777215	3832	3.5834255	3882	3.5890555	3932	3.5946135	3982	3.6001013
3783	3.5778363	3833	3.5835388	3883	3.5891674	3933	3.5947239	3983	3.6002103
3784	3.5779511	3834	3.5836521	3884	3.5892792	3934	3.5948344	3984	3.6003193
3785	3.5780659	3835	3.5837654	3885	3.5893910	3935	3.5949447	3985	3.6004283
3786	3.5781806	3836	3.5838786	3886	3.5895028	3936	3.5950551	3986	3.6005373
3787	3.5782953	3837	3.5839918	3887	3.5896145	3937	3.5951654	3987	3.6006462
3788	3.5784100	3838	3.5841050	3888	3.5897263	3938	3.5952757	3988	3.6007551
3789	3.5785246	3839	3.5842181	3889	3.5898379	3939	3.5953860	3989	3.6008640
3790	3.5786392	3840	3.5843312	3890	3.5899496	3940	3.5954962	3990	3.6009729
3791	3.5787538	3841	3.5844443	3891	3.5900612	3941	3.5956064	3991	3.6010817
3792	3.5788683	3842	3.5845574	3892	3.5901728	3942	3.5957166	3992	3.6011905
3793	3.5789828	3843	3.5846704	3893	3.5902844	3943	3.5958268	3993	3.6012993
3794	3.5790973	3844	3.5847834	3894	3.5903959	3944	3.5959369	3994	3.6014080
3795	3.5792118	3845	3.5848963	3895	3.5905075	3945	3.5960470	3995	3.6015168
3796	3.5793262	3846	3.5850092	3896	3.5906189	3946	3.5961571	3996	3.6016255
3797	3.5794406	3847	3.5851222	3897	3.5907304	3947	3.5962671	3997	3.6017341
3798	3.5795550	3848	3.5852351	3898	3.5908418	3948	3.5963771	3998	3.6018428
3799	3.5796693	3849	3.5853479	3899	3.5909532	3949	3.5964871	3999	3.6019514
3800	3.5797836	3850	3.5854607	3900	3.5910646	3950	3.5965971	4000	3.6020600

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4002	3.6022771	4052	3.6076694	4102	3.6129957	4152	3.6182573	4202	3.6234560
4003	3.6023857	4053	3.6077766	4103	3.6131015	4153	3.6183619	4203	3.6235594
4004	3.6024941	4054	3.6078837	4104	3.6132074	4154	3.6184665	4204	3.6236627
4005	3.6026025	4055	3.6079909	4105	3.6133132	4155	3.6185710	4205	3.6237660
4006	3.6027109	4056	3.6080979	4106	3.6134189	4156	3.6186755	4206	3.6238693
4007	3.6028193	4057	3.6082050	4107	3.6135247	4157	3.6187800	4207	3.6239725
4008	3.6029277	4058	3.6083120	4108	3.6136304	4158	3.6188845	4208	3.6240757
5009	3.6030361	4059	3.6084190	4109	3.6137361	4159	3.6189889	4209	3.6241789
4010	3.6031444	4060	3.6085260	4110	3.6138418	4160	3.6190933	4210	3.6242821
4011	3.6032527	4061	3.6086330	4111	3.6139475	4161	3.6191977	4211	3.6243852
4012	3.6033609	4062	3.6087399	4112	3.6140531	4162	3.6193021	4212	3.6244884
4013	3.6034692	4063	3.6088468	4113	3.6141587	4163	3.6194064	4213	3.6245915
4014	3.6035774	4064	3.6089537	4114	3.6142643	4164	3.6195107	4214	3.6246945
4015	3.6036855	4065	3.6090605	4115	3.6143698	4165	3.6196150	4215	3.6247976
4016	3.6037937	4066	3.6091674	4116	3.6144754	4166	3.6197193	4216	3.6249006
4017	3.6039018	4067	3.6092742	4117	3.6145809	4167	3.6198235	4217	3.6250036
4018	3.6040099	4068	3.6093809	4118	3.6146863	4168	3.6199277	4218	3.6251066
4019	3.6041180	4069	3.6094877	4119	3.6147918	4169	3.6200319	4219	3.6252095
4020	3.6042261	4070	3.6095944	4120	3.6148972	4170	3.6201361	4220	3.6253125
4021	3.6043341	4071	3.6097011	4121	3.6150026	4171	3.6202402	4221	3.6254154
4022	3.6044421	4072	3.6098078	4122	3.6151080	4172	3.6203443	4222	3.6255182
4023	3.6045500	4073	3.6099144	4123	3.6152133	4173	3.6204484	4223	3.6256211
4024	3.6046580	4074	3.6100210	4124	3.6153187	4174	3.6205524	4224	3.6257239
4025	3.6047659	4075	3.6101276	4125	3.6154240	4175	3.6206565	4225	3.6258267
4026	3.6048738	4076	3.6102342	4126	3.6155292	4176	3.6207605	4226	3.6259295
4027	3.6049816	4077	3.6103407	4127	3.6156345	4177	3.6208645	4227	3.6260322
4028	3.6050895	4078	3.6104472	4128	3.6157397	4178	3.6209684	4228	3.6261350
4029	3.6051973	4079	3.6105537	4129	3.6158449	4179	3.6210724	4229	3.6262377
4030	3.6053050	4080	3.6106602	4130	3.6159501	4180	3.6211763	4230	3.6263404
4031	3.6054128	4081	3.6107666	4131	3.6160552	4181	3.6212802	4231	3.6264430
4032	3.6055205	4082	3.6108730	4132	3.6161603	4182	3.6213840	4232	3.6265457
4033	3.6056282	4083	3.6109794	4133	3.6162654	4183	3.6214879	4233	3.6266483
4034	3.6057359	4084	3.6110857	4134	3.6163705	4184	3.6215917	4234	3.6267509
4035	3.6058435	4085	3.6111921	4135	3.6164755	4185	3.6216955	4235	3.6268534
4036	3.6059512	4086	3.6112984	4136	3.6165805	4186	3.6217992	4236	3.6269560
4037	3.6060587	4087	3.6114046	4137	3.6166855	4187	3.6219030	4237	3.6270585
4038	3.6061663	4088	3.6115109	4138	3.6167905	4188	3.6220067	4238	3.6271610
4039	3.6062739	4089	3.6116171	4139	3.6168954	4189	3.6221104	4239	3.6272634
4040	3.6063814	4090	3.6117233	4140	3.6170003	4190	3.6222140	4240	3.6273659
4041	3.6064889	4091	3.6118295	4141	3.6171050	4191	3.6223177	4241	3.6274683
4042	3.6065963	4092	3.6119356	4142	3.6172101	4192	3.6224213	4242	3.6275707
4043	3.6067037	4093	3.6120417	4143	3.6173149	4193	3.6225249	4243	3.6276730
4044	3.6068111	4094	3.6121478	4144	3.6174197	4194	3.6226284	4244	3.6277754
4045	3.6069185	4095	3.6122539	4145	3.6175245	4195	3.6227320	4245	3.6278777
4046	3.6070259	4096	3.6123599	4146	3.6176293	4196	3.6228355	4246	3.6279800
4047	3.6071332	4097	3.6124660	4147	3.6177340	4197	3.6229390	4247	3.6280820
4048	3.6072405	4098	3.6125720	4148	3.6178387	4198	3.6230424	4248	3.6281845
4049	3.6073478	4099	3.6126779	4149	3.6179434	4199	3.6231458	4249	3.6282865
4050	3.6074550	4100	3.6127839	4150	3.6180481	4200	3.6232493	4250	3.6283887

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
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4252	3 6285933	4302	3 6336704	4352	3 6386889	4402	3 6436500	4452	3 6485552
4253	3 6286954	4303	3 6337713	4353	3 6387887	4403	3 6437487	4453	3 6486427
4254	3 6287975	4304	3 6338723	4354	3 6388884	4404	3 6438473	4454	3 6487502
4255	3 6288996	4305	3 6339732	4355	3 6389882	4405	3 6439459	4455	3 6488477
4256	3 6290015	4306	3 6340740	4356	3 6390879	4406	3 6440445	4456	3 6489452
4257	3 6291037	4307	3 6341749	4357	3 6391876	4407	3 6441431	4457	3 6490426
4258	3 6292057	4308	3 6342757	4358	3 6392872	4408	3 6442416	4458	3 6491401
4259	3 6293076	4309	3 6343765	4359	3 6393869	4409	3 6443401	4459	3 6492375
4260	3 6294096	4310	3 6344773	4360	3 6394865	4410	3 6444386	4460	3 6493349
4261	3 6295115	4311	3 6345780	4361	3 6395861	4411	3 6445371	4461	3 6494322
4262	3 6296134	4312	3 6346788	4362	3 6396857	4412	3 6446355	4462	3 6495296
4263	3 6297153	4313	3 6347795	4363	3 6397852	4413	3 6447339	4463	3 6496269
4264	3 6298172	4314	3 6348801	4364	3 6398847	4414	3 6448323	4464	3 6497242
4265	3 6299190	4315	3 6349808	4365	3 6399842	4415	3 6449307	4465	3 6498215
4266	3 6300207	4316	3 6350814	4366	3 6400837	4416	3 6450291	4466	3 6499187
4267	3 6301226	4317	3 6351820	4367	3 6401832	4417	3 6451274	4467	3 6500160
4268	3 6302244	4318	3 6352826	4368	3 6402826	4418	3 6452257	4468	3 6501132
4269	3 6303222	4319	3 6353832	4369	3 6403820	4419	3 6453240	4469	3 6502104
4270	3 6304279	4320	3 6354837	4370	3 6404814	4420	3 6454223	4470	3 6503075
4271	3 6305296	4321	3 6355843	4371	3 6405808	4421	3 6455205	4471	3 6504047
4272	3 6306312	4322	3 6356848	4372	3 6406802	4422	3 6456187	4472	3 6505018
4273	3 6307329	4323	3 6357852	4373	3 6407795	4423	3 6457169	4473	3 6505989
4274	3 6308345	4324	3 6358857	4374	3 6408788	4424	3 6458151	4474	3 6506960
4275	3 6309361	4325	3 6359861	4375	3 6409781	4425	3 6459133	4475	3 6507930
4276	3 6310377	4326	3 6360865	4376	3 6410773	4426	3 6460114	4476	3 6508901
4277	3 6311393	4327	3 6361869	4377	3 6411765	4427	3 6461095	4477	3 6509871
4278	3 6312408	4328	3 6362873	4378	3 6412758	4428	3 6462076	4478	3 6510841
4279	3 6313423	4329	3 6363876	4379	3 6413749	4429	3 6463057	4479	3 6511811
4280	3 6314438	4330	3 6364879	4380	3 6414741	4430	3 6464037	4480	3 6512780
4281	3 6315452	4331	3 6365882	4381	3 6415733	4431	3 6465018	4481	3 6513749
4282	3 6316467	4332	3 6366884	4382	3 6416724	4432	3 6465998	4482	3 6514719
4283	3 6317481	4333	3 6367887	4383	3 6417715	4433	3 6466917	4483	3 6515687
4284	3 6318495	4334	3 6368889	4384	3 6418705	4434	3 6467957	4484	3 6516656
4285	3 6319508	4335	3 6369891	4385	3 6419696	4435	3 6468936	4485	3 6517624
4286	3 6320522	4336	3 6370893	4386	3 6420686	4436	3 6469915	4486	3 6518593
4287	3 6321535	4337	3 6371894	4387	3 6421676	4437	3 6470894	4487	3 6519561
4288	3 6322548	4338	3 6372895	4388	3 6422666	4438	3 6471873	4488	3 6520528
4289	3 6323560	4339	3 6373897	4389	3 6423656	4439	3 6472851	4489	3 6521496
4290	3 6324573	4340	3 6374897	4390	3 6424645	4440	3 6473830	4490	3 6522463
4291	3 6325585	4341	3 6375898	4391	3 6425634	4441	3 6474808	4491	3 6523431
4292	3 6326597	4342	3 6376898	4392	3 6426623	4442	3 6475785	4492	3 6524397
4293	3 6327609	4343	3 6377898	4393	3 6427612	4443	3 6476763	4493	3 6525364
4294	3 6328620	4344	3 6378898	4394	3 6428601	4444	3 6477741	4494	3 6526331
4295	3 6329632	4345	3 6379898	4395	3 6429589	4445	3 6478718	4495	3 6527297
4296	3 6330643	4346	3 6380897	4396	3 6430577	4446	3 6479695	4496	3 6529263
4297	3 6331654	4347	3 6381896	4397	3 6431565	4447	3 6480671	4497	3 6529229
4298	3 6332664	4348	3 6382895	4398	3 6432552	4448	3 6481648	4498	3 6530195
4299	3 6333674	4349	3 6383894	4399	3 6433540	4449	3 6482624	4499	3 6531160
4300	3 6334685	4350	3 6384893	4400	3 6434527	4450	3 6483600	4500	3 6532125

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4502	3.6534055	4552	3.6582023	4602	3.6629466	4652	3.6676397	4702	3.6722826
4503	3.6535019	4553	3.6582977	4603	3.6630410	4653	3.6677331	4703	3.6723750
4504	3.6535984	4554	3.6583930	4604	3.6631353	4654	3.6678264	4704	3.6724673
4505	3.6536948	4555	3.6584884	4605	3.6632296	4655	3.6679197	4705	3.6725596
4506	3.6537912	4556	3.6585837	4606	3.6633239	4656	3.6680130	4706	3.6726519
4507	3.6538876	4557	3.6586790	4607	3.6634182	4657	3.6681062	4707	3.6727442
4508	3.6539839	4558	3.6587743	4608	3.6635125	4658	3.6681995	4708	3.6728365
4509	3.6540802	4559	3.6588696	4609	3.6636 67	4659	3.6682927	4709	3.6729287
4510	3.6541765	4560	3.6589648	4610	3.6637009	4660	3.6683859	4710	3.6730209
4511	3.6542728	4561	3.6590601	4611	3.6637951	4661	3.6684791	4711	3.6731131
4512	3.6543691	4562	3.6591553	4612	3.6638893	4662	3.6685723	4712	3.6732053
4513	3.6544653	4563	3.6592505	4613	3.6639835	4663	3.6686654	4713	3.6732974
4514	3.6545616	4564	3.6593456	4614	3.6640776	4664	3.6687585	4714	3.6733896
4515	3.6546578	4565	3.6594408	4615	3.6641717	4665	3.6688516	4715	3.6734817
4516	3.6547539	4566	3.6595359	4616	3.6642658	4666	3.6689447	4716	3.6735738
4517	3.6548501	4567	3.6596310	4617	3.6643599	4667	3.6690378	4717	3.6736659
4518	3.6549462	4568	3.6597261	4618	3.6644539	4668	3.6691308	4718	3.6737579
4519	3.6550423	4569	3.6598212	4619	3.6645480	4669	3.6692239	4719	3.6738500
4520	3.6551384	4570	3.6599162	4620	3.6646420	4670	3.6693169	4720	3.6739420
4521	3.6552345	4571	3.6600112	4621	3.6647360	4671	3.6694099	4721	3.6740340
4522	3.6553306	4572	3.6601062	4622	3.6648299	4672	3.6695028	4722	3.6741260
4523	3.6554266	4573	3.6602012	4623	3.6649239	4674	3.6695958	4723	3.6742179
4524	3.6555226	4574	3.6602962	4624	3.6650178	4674	3.6696887	4724	3.6743099
4525	3.6556186	4575	3.6603911	4625	3.6651117	4675	3.6697816	4725	3.6744018
4526	3.6557145	4576	3.6604860	4626	3.6652056	4676	3.6698745	4726	3.6744937
4527	3.6558105	4577	3.6605809	4627	3.6652995	4677	3.6699674	4727	3.6745856
4528	3.6559064	4578	3.6606758	4628	3.6653934	4678	3.6700602	4728	3.6746775
4529	3.6560023	4579	3.6607706	4629	3.6654872	4679	3.6701530	4729	3.6747693
4530	3.6560982	4580	3.6608655	4630	3.6655810	4680	3.6702459	4730	3.6748611
4531	3.6561941	4581	3.6609603	4631	3.6656748	4681	3.6703386	4731	3.6749529
4532	3.6562899	4582	3.6610551	4632	3.6657686	4682	3.6704314	4732	3.6750447
4533	3.6563857	4583	3.6611499	4633	3.6658623	4683	3.6705242	4733	3.6751365
4534	3.6564815	4584	3.6612446	4634	3.6659560	4684	3.6706169	4734	3.6752283
4535	3.6565773	4585	3.6613393	4635	3.6660497	4685	3.6707096	4735	3.6753200
4536	3.6566730	4586	3.6614341	4636	3.6661434	4686	3.6708023	4736	3.6754117
4537	3.6567688	4587	3.6615287	4637	3.6662371	4687	3.6708950	4737	3.6755034
4538	3.6568645	4588	3.6616234	4638	3.6663307	4688	3.6709876	4738	3.6755951
4539	3.6569002	4589	3.6617181	4639	3.6664244	4689	3.6710802	4739	3.6756867
4540	3.6570559	4590	3.6618127	4640	3.6665180	4690	3.6711728	4740	3.6757783
4541	3.6571515	4591	3.6619073	4641	3.6666116	4691	3.6712654	4741	3.6758700
4542	3.6572471	4592	3.6620019	4642	3.6667957	4692	3.6713589	4742	3.6759615
4543	3.6573427	4193	3.6620964	4643	3.6667987	4693	3.6714506	4743	3.6760531
4544	3.6574383	4594	3.6621910	4644	3.6668922	4694	3.6715431	4744	3.6761447
4545	3.6575339	4595	3.6622855	4645	3.6669857	4695	3.6716356	4745	3.6762362
4546	3.6576294	4596	3.6623800	4646	3.6670792	4696	3.6717281	4746	3.6763277
4547	3.6577250	4597	3.6624745	4647	3.6671727	4697	3.6718206	4747	3.6764192
4548	3.6578205	4598	3.6625690	4648	3.6672661	4698	3.6719130	4748	3.6765107
4549	3.6579159	4599	3.6626634	4649	3.6673595	4699	3.6720054	4749	3.6766022
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4752	3.6768564	4802	3.6814222	4852	3.6859208	1902	3.6903733	4952	3.6947806
4753	3.6769678	4803	3.6815126	4853	3.6860103	4903	3.6904019	4953	3.6948683
4754	3.6770502	4804	3.6816010	4854	3.6860998	4904	3.6905505	4954	3.6949500
4755	3.6771505	4805	3.6816934	4855	3.6861892	4905	3.6906390	4955	3.6950437
4756	3.6772418	4806	3.6817838	4856	3.6862787	4906	3.6907275	4956	3.6951313
4757	3.6773332	4807	3.6818741	4857	3.6863681	4907	3.6908161	4957	3.6952189
4758	3.6774244	4808	3.6819645	4858	3.6864575	4908	3.6909046	4958	3.6953065
4759	3.6775157	4809	3.6820548	4859	3.6865469	4909	3.6909930	4959	3.6953941
4760	3.6776070	4810	3.6821451	4860	3.6866363	4910	3.6910815	4960	3.6954817
4761	3.6776982	4811	3.6822354	4861	3.6867256	4911	3.6911699	4961	3.6955692
4762	3.6777894	4812	3.6823256	4862	3.6868150	4912	3.6912584	4962	3.6956568
4763	3.6778806	4813	3.6824159	4863	3.6869043	4913	3.6913468	4963	3.6957443
4764	3.6779718	4814	3.6825061	4864	3.6869936	4914	3.6914352	4964	3.6958318
4765	3.6780629	4815	3.6825963	4865	3.6870828	4915	3.6915235	4965	3.6959193
4766	3.6781540	4816	3.6826865	4866	3.6871721	4916	3.6916119	4966	3.6960067
4767	3.6782452	4817	3.6827766	4867	3.6872613	4917	3.6917002	4967	3.6960942
4768	3.6783362	4818	3.6828668	4868	3.6873506	4918	3.6917885	4968	3.6961816
4769	3.6784273	4819	3.6829569	4869	3.6874398	4919	3.6918768	4969	3.6962690
4770	3.6785184	4820	3.6830470	4870	3.68 5290	4920	3.6919651	4970	3.6963564
4771	3.6785094	4821	3.6831371	4871	3.6876181	4921	3.6920534	4971	3.6964438
4772	3.6787004	4822	3.6832272	4872	3.6877073	4922	3.6921416	4972	3.6965311
4773	3.6787914	4823	3.6833173	4873	3.6877964	4923	3.6922298	4973	3.6966185
4774	3.6788824	4824	3.6834073	4874	3.6878855	4924	3.6923180	4974	3.6967058
4775	3.6789734	4825	3.6834973	4875	3.6879746	4925	3.6924062	4975	3.6967931
4776	3.6790643	4826	3.6835873	4876	3.6880637	4926	3.6924944	4976	3.6968804
4777	3.6791552	4827	3.6836773	4877	3.6881528	4927	3.6925826	4977	3.6969676
4778	3.6792461	4828	3.6837673	4878	3.6882418	4928	3.6926707	4978	3.6970549
4779	3.6793370	4829	3.6838572	4879	3.6883308	4929	3.6927588	4979	3.6971421
4780	3.6794279	4830	3.6839471	4880	3.6884198	4930	3.6928469	4980	3.6972293
4781	3.6795187	4831	3.6840370	4881	3.6885088	4931	3.6929350	4981	3.6973165
4782	3.6796096	4832	3.6841269	4882	3.6885978	4932	3.6930231	4982	3.6974037
4783	3.6797004	4833	3.6842168	4883	3.6886867	4933	3.6931111	4983	3.6974909
4784	3.6797912	4834	3.6843066	4884	3.6887757	4934	3.6931991	4984	3.6975780
4785	3.6799819	4835	3.6843965	4885	3.6888646	4935	3.6932872	4985	3.6976652
4786	3.6799727	4836	3.6844863	4886	3.6889535	4936	3.6933752	4986	3.6977523
4787	3.6800634	4837	3.6845761	4887	3.6890423	4937	3.6934631	4987	3.6978394
4788	3.6801541	4838	3.6846659	4888	3.6891312	4938	3.6935511	4988	3.6979264
4789	3.6802448	4839	3.6847556	4889	3.6892200	4939	3.6936390	4989	3.6980135
4790	3.6803355	4840	3.6848454	4890	3.6893089	4940	3.6937269	4990	3.6981005
4791	3.6804262	4841	3.6849351	4891	3.6893977	4941	3.6938149	4991	3.6981876
4792	3.6805168	4842	3.6850248	4892	3.6894864	4942	3.6939027	4992	3.6982746
4793	3.6806074	4843	3.6851145	4893	3.6895752	4943	3.6 39906	4993	3.6983616
4794	3.6309680	4844	3.6852041	4894	3.6896640	4944	3.6940785	4994	3.6984485
4795	3.6307886	4845	3.6852938	4895	3.6897527	4945	3.6941663	4995	3.6985355
4796	3.6808792	4846	3.6853834	4896	3.6898414	4946	3.6942541	4996	3.6986224
4797	3.6809692	4847	3.6854730	4897	3.6899301	4947	3.6943419	4997	3.6987093
4798	3.6810502	4848	3.6855626	4898	3.6900188	4948	3.6944297	4998	3.6987963
4799	3.6811507	4849	3.6856522	4899	3.6901074	4949	3.6945174	4999	3.6988831
4800	3.6812412	4850	3.6857417	4900	3.6901961	4950	3.6946052	5000	3.6989700

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
5001	3.6990569	5051	3.7033774	5101	3.7076553	5151	3.7118915	5201	3.7160869
5002	3.6991437	5052	3.7034633	5102	3.7077405	5152	3.7119759	5202	3.7161703
5003	3.6992305	5053	3.7035498	5103	3.7078256	5153	3.7120601	5203	3.7162538
5004	3.6993173	5054	3.7036352	5104	3.7079107	5154	3.7121444	5204	3.7163373
5005	3.6994041	5055	3.7037212	5105	3.7079957	5155	3.7122287	5205	3.7164207
5006	3.6994908	5056	3.7038071	5106	3.7080808	5156	3.7123129	5206	3.7165042
5007	3.6995776	5057	3.7038930	5107	3.7081659	5157	3.7123971	5207	3.7165876
5008	3.6996643	5058	3.7039788	5108	3.7082509	5158	3.7124813	5208	3.7166710
5009	3.6997510	5059	3.7040647	5109	3.7083359	5159	3.7125655	5209	3.7167544
5010	3.6998377	5060	3.7041505	5110	3.7084209	5160	3.7126497	5210	3.7168377
5011	3.6999244	5061	3.7042363	5111	3.7085059	5161	3.7127339	5211	3.7169211
5012	3.7000111	5062	3.7043221	5112	3.7085908	5162	3.7128180	5212	3.7170044
5013	3.7000977	5063	3.7044079	5113	3.7086758	5163	3.7129021	5213	3.7170877
5014	3.7001843	5064	3.7044937	5114	3.7087607	5164	3.7129862	5214	3.7171710
5015	3.7002709	5065	3.7045794	5115	3.7088456	5165	3.7130703	5215	3.7172543
5016	3.7003575	5066	3.7046653	5116	3.7089305	5166	3.7131544	5216	3.7173376
5017	3.7004441	5067	3.7047509	5117	3.7090154	5167	3.7132385	5217	3.7174208
5018	3.7005307	5068	3.7048366	5118	3.7091003	5168	3.7133225	5218	3.7175041
5019	3.7006172	5069	3.7049223	5119	3.7091851	5169	3.7134065	5219	3.7175873
5020	3.7007037	5070	3.7050080	5120	3.7092700	5170	3.7134905	5220	3.7176705
5021	3.7007902	5071	3.7050936	5121	3.7093548	5171	3.7135745	5221	3.7177537
5022	3.7008762	5072	3.7051792	5122	3.7094396	5172	3.7136585	5222	3.7178369
5023	3.7009632	5073	3.7052649	5123	3.7095244	5173	3.7137425	5223	3.7179200
5024	3.7010496	5074	3.7053505	5124	3.7096091	5174	3.7138264	5224	3.7180032
5025	3.7011361	5075	3.7054360	5125	3.7096939	5175	3.7139104	5225	3.7180863
5026	3.7012225	5076	3.7055216	5126	3.7097786	5176	3.7139943	5226	3.7181694
5027	3.7013089	5077	3.7056072	5127	3.7098633	5177	3.7140782	5227	3.7182525
5028	3.7013953	5078	3.7056927	5128	3.7099480	5178	3.7141620	5228	3.7183356
5029	3.7014816	5079	3.7057782	5129	3.7100327	5179	3.7142459	5229	3.7184186
5030	3.7015680	5080	3.7058637	5130	3.7101174	5180	3.7143298	5230	3.7185017
5031	3.7016543	5081	3.7059492	5131	3.7102020	5181	3.7144136	5231	3.7185847
5032	3.7017406	5082	3.7060347	5132	3.7102866	5182	3.7144974	5232	3.7186677
5033	3.7018269	5083	3.7061201	5133	3.7103713	5183	3.7145812	5233	3.7187507
5034	3.7019132	5084	3.7062055	5134	3.7104559	5184	3.7146650	5234	3.7188337
5035	3.7019995	5085	3.7062910	5135	3.7105404	5185	3.7147488	5235	3.7189167
5036	3.7020857	5086	3.7063764	5136	3.7106250	5186	3.7148325	5236	3.7189996
5037	3.7021720	5087	3.7064617	5137	3.7107096	5187	3.7149162	5237	3.7190826
5038	3.7022582	5088	3.7065471	5138	3.7107941	5188	3.7150000	5238	3.7191655
5039	3.7023444	5089	3.7066325	5139	3.7108786	5189	3.7150837	5239	3.7192444
5040	3.7024305	5090	3.7067178	5140	3.7109631	5190	3.7151674	5240	3.7193313
5041	3.7025167	5091	3.70.8031	5141	3.7110476	5191	3.7152510	5241	3.7194142
5042	3.7026028	5092	3.7068884	5142	3.7111321	5192	3.7153347	5242	3.7194970
5043	3.7026890	5093	3.7069737	5143	3.7112165	5193	3.7154183	5243	3.7195799
5044	3.7027751	5094	3.7070589	5144	3.7113010	5194	3.7155019	5244	3.7196627
5045	3.7028612	5095	3.7071442	5145	3.7113854	5195	3.7155856	5245	3.7197455
5046	3.7029472	5096	3.7072294	5146	3.7114698	5196	3.7156691	5246	3.7198283
5047	3.7030333	5097	3.7073146	5147	3.7115542	5197	3.7157527	5247	3.7199111
5048	3.7031193	5098	3.7073998	5148	3.7116385	5198	3.7158363	5248	3.7199938
5049	3.7032054	5099	3.7074850	5149	3.7117229	5199	3.7159198	5249	3.7200766
5050	3.7032914	5100	3.7075702	5150	3.7118072	5200	3.7160033	5250	3.7201593

No.	Log.	No.	Log.	No.	Log	No.	Log.	No.	Log.
5251	3.7202429	5301	3.7243578	5351	3.7284350	5401	3.7324743	5451	3.7364762
5252	3.7203247	5302	3.7244397	5352	3.7285161	5402	3.7325546	5452	3.7365558
5253	3.7204074	5303	3.7245216	5353	3.7285972	5403	3.7326350	5453	3.7366355
5254	3.7204901	5304	3.7246035	5354	3.7286784	5404	3.7327153	5454	3.7367151
5255	3.7205727	5305	3.7246854	5355	3.7287595	5405	3.7327957	5455	3.7367948
5256	3.7206554	5306	3.7247672	5356	3.7288406	5406	3.7328760	5456	3.7368744
5257	3.7207380	5307	3.7248491	5357	3.7289216	5407	3.7329564	5457	3.7369540
5258	3.7208206	5308	3.7249309	5358	3.7290027	5408	3.7330367	5458	3.7370355
5259	3.7209032	5309	3.7250127	5359	3.7290838	5409	3.7331170	5459	3.7371152
5260	3.7209857	5310	3.7250945	5360	3.7291648	5410	3.7331975	5460	3.7371926
5261	3.7210683	5311	3.7251763	5361	3.7292458	5411	3.7332775	5461	3.7372722
5262	3.7211508	5312	3.7252581	5362	3.7293268	5412	3.7333578	5462	3.7373517
5263	3.7212334	5313	3.7253398	5363	3.7294078	5413	3.7334380	5463	3.7374312
5264	3.7213159	5314	3.7254216	5364	3.7294888	5414	3.7335183	5464	3.7375107
5265	3.7213984	5315	3.7255033	5365	3.7295697	5415	3.7335985	5465	3.7375902
5266	3.7214809	5316	3.7255850	5366	3.7296507	5416	3.7336787	5466	3.7376696
5267	3.7215633	5317	3.7256667	5367	3.7297316	5417	3.7337588	5467	3.7377491
5268	3.7216458	5318	3.7257483	5368	3.7298125	5418	3.7338390	5468	3.7378285
5269	3.7217282	5319	3.7258300	5369	3.7298934	5419	3.7339192	5469	3.7379079
5270	3.7218106	5320	3.7259116	5370	3.7299743	5420	3.7339993	5470	3.7379873
5271	3.7218930	5321	3.7259933	5371	3.7300552	5421	3.7340794	5471	3.7380667
5272	3.7219754	5322	3.7260749	5372	3.7301360	5422	3.7341595	5472	3.7381462
5273	3.7220578	5323	3.7261565	5373	3.7302168	5423	3.7342396	5473	3.7382254
5274	3.7221401	5324	3.7262380	5374	3.7302977	5424	3.7343197	5474	3.7383048
5275	3.7222225	5325	3.7263196	5375	3.7303785	5425	3.7343997	5475	3.7383841
5276	3.7223048	5326	3.7264012	5376	3.7304593	5426	3.7344798	5476	3.7384634
5277	3.7223871	5327	3.7264827	5377	3.7305400	5427	3.7345598	5477	3.7385427
5278	3.7224694	5328	3.7265642	5378	3.7306208	5428	3.7346398	5478	3.7386220
5279	3.7225517	5329	3.7266457	5379	3.7307015	5429	3.7347198	5479	3.7387013
5280	3.7226339	5330	3.7267272	5380	3.7307823	5430	3.7347998	5480	3.7387806
5281	3.7221162	5331	3.7268087	5381	3.7308630	5431	3.7348798	5481	3.7388598
5282	3.7227984	5332	3.7268901	5382	3.7309437	5432	3.7349598	5482	3.7389390
5283	3.7228806	5333	3.7269916	5383	3.7310244	5433	3.7350397	5483	3.7390182
5284	3.7229628	5334	3.7270530	5384	3.7311051	5434	3.7351196	5484	3.7390974
5285	3.7230450	5335	3.7271344	5385	3.7311857	5435	3.7351995	5485	3.7391766
5286	3.7231272	5336	3.7272158	5386	3.7312663	5436	3.7352794	5486	3.7392558
5287	3.7232093	5337	3.7272972	5387	3.7313470	5437	3.7353593	5487	3.7393350
5288	3.7232914	5338	3.7273786	5388	3.7314276	5438	3.7354392	5488	3.7394141
5289	3.7233736	5339	3.7274599	5389	3.7315082	5439	3.7355191	5489	3.7394932
5290	3.7234557	5340	3.7275413	5390	3.7315888	5440	3.7355989	5490	3.7395725
5291	3.7235378	5341	3.7276226	5391	3.7316693	5441	3.7356787	5491	3.7396514
5292	3.7236198	5342	3.7277039	5392	3.7317499	5442	3.7357585	5492	3.7397305
5293	3.7237019	5343	3.7277852	5393	3.7318304	5443	3.7358383	5493	3.7398096
5294	3.7237839	5344	3.7278664	5394	3.7319109	5444	3.7359181	5494	3.7398887
5295	3.7238660	5345	3.7279477	5395	3.7319914	5445	3.7359979	5495	3.7399677
5296	3.7239480	5346	3.7280290	5396	3.7320719	5446	3.7360776	5496	3.7400467
5297	3.7240300	5347	3.7281102	5397	3.7321524	5447	3.7361574	5497	3.7401257
5298	3.7241120	5348	3.7281914	5398	3.7322329	5448	3.7362371	5498	3.7402047
5299	3.7241939	5349	3.7282726	5399	3.7323133	5449	3.7363168	5499	3.7402837
5300	3.7242759	5350	3.7283533	5400	3.7323938	5450	3.7363965	5500	3.7403627

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
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5502	3.7405206	5552	3.7444495	5602	3.7483431	5652	3.7522022	5702	3.7560272
5503	3.7405995	5553	3.7445277	5603	3.7484206	5653	3.7522791	5703	3.7561034
5504	3.7406784	5554	3.7446059	5604	3.7484885	5654	3.7523558	5704	3.7561795
5505	3.7407573	5555	3.7446841	5605	3.7485756	5655	3.7524326	5705	3.7562556
5506	3.7408302	5556	3.7447622	5606	3.7486531	5656	3.7525094	5706	3.7563318
5507	3.7409151	5557	3.7448404	5607	3.7487306	5657	3.7525862	5707	3.7564079
5508	3.7409939	5558	3.7449185	5608	3.7488080	5658	3.7526629	5708	3.7564840
5509	3.7410728	5559	3.7449967	5609	3.7488854	5659	3.7527397	5709	3.7565600
5510	3.7411516	5560	3.7450748	5610	3.7489609	5660	3.7528164	5710	3.7566361
5511	3.7412304	5561	3.7451529	5611	3.7490403	5661	3.7528932	5711	3.7567122
5512	3.7413092	5562	3.7452310	5612	3.7491177	5662	3.7529699	5712	3.7567882
5513	3.7413880	5563	3.7453091	5613	3.7491950	5663	3.7530466	5713	3.7568642
5514	3.7414668	5564	3.7453871	5614	3.7492724	5664	3.7531232	5714	3.7569402
5515	3.7415455	5565	3.7454652	5615	3.7493498	5665	3.7531999	5715	3.7570162
5516	3.7416243	5566	3.7455432	5616	3.7494271	5666	3.7532766	5716	3.7570922
5517	3.7417030	5567	3.7456212	5617	3.7495044	5667	3.7533532	5717	3.7571682
5518	3.7417817	5568	3.7456992	5618	3.7495817	5668	3.7534298	5718	3.7572442
5519	3.7418604	5569	3.7457772	5619	3.7496590	5669	3.7535065	5719	3.7573201
5520	3.7419391	5570	3.7458552	5620	3.7497363	5670	3.7535831	5720	3.7573960
5521	3.7420177	5571	3.7459332	5621	3.7498136	5671	3.7536596	5721	3.7574719
5522	3.7420964	5572	3.7460111	5622	3.7498908	5672	3.7537362	5722	3.7575479
5523	3.7421750	5573	3.7460890	5623	3.7499681	5673	3.7538128	5723	3.7576237
5524	3.7422537	5574	3.7461670	5624	3.7500453	5674	3.7538893	5724	3.7576996
5525	3.7423323	5575	3.7462449	5625	3.7501225	5675	3.7539659	5725	3.7577755
5526	3.7424109	5576	3.7463228	5626	3.7501997	5676	3.7540424	5726	3.7578513
5527	3.7424895	5577	3.7464006	5627	3.7502769	5677	3.7541189	5727	3.7579272
5528	3.7425680	5578	3.7464785	5628	3.7503541	5678	3.7541954	5728	3.7580030
5529	3.7426462	5579	3.7465564	5629	3.7504312	5679	3.7542719	5729	3.7580788
5530	3.7427251	5580	3.7466342	5630	3.7505084	5680	3.7543483	5730	3.7581546
5531	3.7428037	5581	3.7467120	5631	3.7505855	5681	3.7544248	5731	3.7582304
5532	3.7428822	5582	3.7467898	5632	3.7506626	5682	3.7545012	5732	3.7583062
5533	3.7429607	5583	3.7468676	5633	3.7507398	5683	3.7545777	5733	3.7583819
5534	3.7430392	5584	3.7469454	5634	3.7508168	5684	3.7546541	5734	3.7584577
5535	3.7431176	5585	3.7470232	5635	3.7508939	5685	3.7547305	5735	3.7585334
5536	3.7431961	5586	3.7471009	5636	3.7509710	5686	3.7548069	5736	3.7586091
5537	3.7432745	5587	3.7471787	5637	3.7510480	5687	3.7548832	5737	3.7586848
5538	3.7433530	5588	3.7472564	5638	3.7511251	5688	3.7549596	5738	3.7587605
5539	3.7434314	5589	3.7473341	5639	3.7512021	5689	3.7550359	5739	3.7588362
5540	3.7435098	5590	3.7474118	5640	3.7512791	5690	3.7551123	5740	3.7589119
5541	3.7435881	5591	3.7474895	5641	3.7513561	5691	3.7551886	5741	3.7589875
5542	3.7436665	5592	3.7475672	5642	3.7514331	5692	3.7552649	5742	3.7590632
5543	3.7437449	5593	3.7476448	5643	3.7515101	5693	3.7553412	5743	3.7591388
5544	3.7438232	5594	3.7477225	5644	3.7515870	5694	3.7554175	5744	3.7592144
5545	3.7439016	5595	3.7478001	5645	3.7516639	5695	3.7554937	5745	3.7592900
5546	3.7439799	5596	3.7478777	5646	3.7517409	5696	3.7555700	5746	3.7593656
5547	3.7440582	5597	3.7479553	5647	3.7518178	5697	3.7556462	5747	3.7594412
5548	3.7441365	5598	3.7480329	5648	3.7518947	5698	3.7557224	5748	3.7594168
5549	3.7442147	5599	3.7481105	5649	3.7519716	5699	3.7557987	5749	3.7595923
5550	3.7442930	5600	3.7481880	5650	3.7520484	5700	3.7558749	5750	3.7596677

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5752	3.7598189	5802	3.7635777	5852	3.7673045	5902	3.7709992	5952	3.7746629
5753	3.7598944	5803	3.7636526	5853	3.7673785	5903	3.7710728	5953	3.7747359
5754	3.7599699	5804	3.7637274	5854	3.7674527	5904	3.7711463	5954	3.7748088
5755	3.7600453	5805	3.7638022	5855	3.7675269	5905	3.7712199	5955	3.7748818
5756	3.7601208	5806	3.7638777	5856	3.7676011	5906	3.7712934	5956	3.7749547
5757	3.7601962	5807	3.7639518	5857	3.7676752	5907	3.7713670	5957	3.7750276
5758	3.7602714	5808	3.7640266	5858	3.7677494	5908	3.7714405	5958	3.7751005
5759	3.7603471	5809	3.7641014	5859	3.7678235	5909	3.7715140	5959	3.7751734
5760	3.7604225	5810	3.7641761	5860	3.7678976	5910	3.7715875	5960	3.7752463
5761	3.7604979	5811	3.7642509	5861	3.7679717	5911	3.7716610	5961	3.7753191
5762	3.7605733	5812	3.7643256	5862	3.7680458	5912	3.7717344	5962	3.7753920
5763	3.7606485	5813	3.7644003	5863	3.7681199	5913	3.7718079	5963	3.7754684
5764	3.7607240	5814	3.7644750	5864	3.7681940	5914	3.7718813	5964	3.7755379
5765	3.7607993	5815	3.7645497	5865	3.7682680	5915	3.7719547	5965	3.7756104
5766	3.7608746	5816	3.7646244	5866	3.7683421	5916	3.7720282	5966	3.7756832
5767	3.7609500	5817	3.7646991	5867	3.7684161	5917	3.7721016	5967	3.7757560
5768	3.7610253	5818	3.7647737	5868	3.7684901	5918	3.7721750	5968	3.7758288
5769	3.7611005	5819	3.7648484	5869	3.7685641	5919	3.7722483	5969	3.7759016
5770	3.7611758	5820	3.7649230	5870	3.7686381	5920	3.7723218	5970	3.7759743
5771	3.7612511	5821	3.7649976	5871	3.7687121	5921	3.7723951	5971	3.7760471
5772	3.7613265	5822	3.7650722	5872	3.7687860	5922	3.7724684	5972	3.7761198
5773	3.7614016	5823	3.7651468	5873	3.7688600	5923	3.7725417	5973	3.7761925
5774	3.7614768	5824	3.7652214	5874	3.7689339	5924	3.7726150	5974	3.7762652
5775	3.7615520	5825	3.7652959	5875	3.7690079	5925	3.7726883	5975	3.7763379
5776	3.7616272	5826	3.7653705	5876	3.7690818	5926	3.7727610	5976	3.7764106
5777	3.7617024	5827	3.7654450	5877	3.7691557	5927	3.7728349	5977	3.7764833
5778	3.7617775	5828	3.7655195	5878	3.7692296	5928	3.7729082	5978	3.7765559
5779	3.7618527	5829	3.7655941	5879	3.7693035	5929	3.7729814	5979	3.7766285
5780	3.7619288	5830	3.7656686	5880	3.7693773	5930	3.7730547	5980	3.7767012
5781	3.7620030	5831	3.7657430	5881	3.7694512	5931	3.7731279	5981	3.7767738
5782	3.7620781	5832	3.7658175	5882	3.7695250	5932	3.7732011	5982	3.7768464
5783	3.7621532	5833	3.7658920	5883	3.7695988	5933	3.7732743	5983	3.7769190
5784	3.7622283	5834	3.7659664	5884	3.7696727	5934	3.7733475	5984	3.7769916
5785	3.7623034	5835	3.7660409	5885	3.7697465	5935	3.7734207	5985	3.7770641
5786	3.7623784	5836	3.7661153	5886	3.7698203	5936	3.7734939	5986	3.7771367
5787	3.7624535	5837	3.7661897	5887	3.7698940	5937	3.7735670	5987	3.7772093
5788	3.7625285	5838	3.7662641	5888	3.7699678	5938	3.7736402	5988	3.7772818
5789	3.7626035	5839	3.7663385	5889	3.7700415	5939	3.7737133	5989	3.7773543
5790	3.7626786	5840	3.7664128	5890	3.7701153	5940	3.7737864	5990	3.7774268
5791	3.7627536	5841	3.7664872	5891	3.7701890	5941	3.7738595	5991	3.7774993
5792	3.7628286	5842	3.7665615	5892	3.7702627	5942	3.7739326	5992	3.7775718
5793	3.7629035	5843	3.7666358	5893	3.7703364	5943	3.7740057	5993	3.7776443
5794	3.7629785	5844	3.7667101	5894	3.7704101	5944	3.7740788	5994	3.7777167
5795	3.7630534	5845	3.7667845	5895	3.7704838	5945	3.7741519	5995	3.7777892
5796	3.7631284	5846	3.7668588	5896	3.7705575	5946	3.7742249	5996	3.7778616
5797	3.7632033	5847	3.7669331	5897	3.7706311	5947	3.7742979	5997	3.7779340
5798	3.7632782	5848	3.7670074	5898	3.7707048	5948	3.7743710	5998	3.7780065
5799	3.7633531	5849	3.7670816	5899	3.7707784	5949	3.7744440	5999	3.7780789
5800	3.7634280	5850	3.7671559	5900	3.7708520	5950	3.7745170	6000	3.7781512

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6002	3.7782960	6052	3.7818989	6102	3.7854732	6152	3.7890163	6202	3.7925318
6003	3.7783683	6053	3.7819707	6103	3.7855434	6153	3.7890869	6203	3.7926018
6004	3.7784407	6054	3.7820424	6104	3.7856143	6154	3.7891575	6204	3.7926718
6005	3.7785130	6055	3.7821141	6105	3.7856855	6155	3.7892281	6205	3.7927418
6006	3.7785853	6056	3.7821859	6106	3.7857566	6156	3.7892986	6206	3.7928118
6007	3.7786576	6057	3.7822576	6107	3.7858278	6157	3.7893691	6207	3.7928817
6008	3.7787299	6058	3.7823293	6108	3.7858990	6158	3.7894397	6208	3.7929517
6009	3.7788022	6059	3.7824009	6109	3.7859701	6159	3.7895102	6209	3.7930217
6010	3.7788745	6060	3.7824726	6110	3.7860412	6160	3.7895807	6210	3.7930966
6011	3.7789467	6061	3.7825443	6111	3.7861123	6161	3.7896512	6211	3.7931615
6012	3.7790190	6062	3.7826159	6112	3.7861833	6162	3.7897217	6212	3.7932314
6013	3.7790912	6063	3.7826876	6113	3.7862544	6163	3.7897922	6213	3.7933013
6014	3.7791634	6064	3.7827592	6114	3.7863254	6164	3.7898626	6214	3.7933712
6015	3.7792356	6065	3.7828308	6115	3.7863965	6165	3.7899331	6215	3.7934411
6016	3.7793078	6066	3.7829024	6116	3.7864675	6166	3.7900035	6216	3.7935110
6017	3.7793800	6067	3.7829740	6117	3.7865384	6167	3.7900739	6217	3.7935809
6018	3.7794522	6068	3.7830459	6118	3.7866095	6168	3.7901444	6218	3.7936507
6019	3.7795243	6069	3.7831171	6119	3.7866804	6169	3.7902148	6219	3.7937206
6020	3.7795965	6070	3.7831887	6120	3.7867514	6170	3.7902852	6220	3.7937904
6021	3.7796686	6071	3.7832602	6121	3.7868224	6171	3.7903555	6221	3.7938602
6022	3.7797407	6072	3.7833318	6122	3.7868933	6172	3.7904259	6222	3.7939300
6023	3.7798129	6073	3.7834033	6123	3.7869643	6173	3.7904963	6223	3.7939998
6024	3.7798850	6074	3.7834748	6124	3.7870352	6174	3.7905666	6224	3.7940696
6025	3.7799570	6075	3.7835463	6125	3.7871061	6175	3.7906370	6225	3.7941394
6026	3.7800291	6076	3.7836178	6126	3.7871770	6176	3.7907073	6226	3.7942091
6027	3.7801012	6077	3.7836892	6127	3.7872479	6177	3.7907776	6227	3.7942789
6028	3.7801732	6078	3.7837607	6128	3.7873188	6178	3.7908479	6228	3.7943486
6029	3.7802453	6079	3.7838322	6129	3.7873896	6179	3.7909182	6229	3.7944183
6030	3.7803173	6080	3.7839036	6130	3.7874605	6180	3.7909885	6230	3.7944880
6031	3.7803893	6081	3.7839750	6131	3.7875313	6181	3.7910587	6231	3.7945577
6032	3.7804613	6082	3.7840464	6132	3.7876021	6182	3.7911290	6232	3.7946274
6033	3.7805333	6083	3.7841178	6133	3.7876730	6183	3.7911993	6233	3.7946971
6034	3.7806053	6084	3.7841892	6134	3.7877438	6184	3.7912695	6234	3.7947668
6035	3.7806773	6085	3.7842607	6135	3.7878146	6185	3.7913397	6235	3.7948365
6036	3.7807492	6086	3.7843319	6136	3.7878853	6186	3.7914099	6236	3.7949061
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6038	3.7808931	6088	3.7844746	6138	3.7880269	6188	3.7915503	6238	3.7950454
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6041	3.7811088	6091	3.7846886	6141	3.7882391	6191	3.7917608	6241	3.7952542
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6043	3.7812526	6093	3.7848312	6143	3.7883805	6193	3.7919011	6243	3.7953933
6044	3.7813245	6094	3.7849024	6144	3.7884512	6194	3.7919712	6244	3.7954629
6045	3.7813963	6095	3.7849737	6145	3.7885219	6195	3.7920413	6245	3.7955324
6046	3.7814681	6096	3.7850450	6146	3.7885926	6196	3.7921114	6246	3.7956020
6047	3.7815400	6097	3.7851162	6147	3.7886632	6197	3.7921815	6247	3.7956715
6048	3.7816118	6098	3.7851874	6148	3.7887339	6198	3.7922516	6248	3.7957410
6049	3.7816836	6099	3.7852586	6149	3.7888045	6199	3.7923216	6249	3.7958105
6050	3.7817554	6100	3.7853298	6150	3.7888751	6200	3.7923917	6250	3.7958800

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6252	3 7960190	6302	3 7994784	6352	3 8029105	6402	3 8063157	6452	3 8096944
6253	3 7960884	6303	3 7995473	6353	3 8029789	6403	3 8063839	6453	3 8097617
6254	3 7961579	6304	3 7996162	6354	3 8030472	6404	3 8064515	6454	3 8098290
6255	3 7962273	6305	3 7996851	6355	3 8031155	6405	3 8065191	6455	3 8098962
6256	3 7962967	6306	3 7997540	6356	3 8031839	6406	3 8065869	6456	3 8099635
6257	3 7963662	6307	3 7998228	6357	3 8032522	6407	3 8066547	6457	3 8100318
6258	3 7964356	6308	3 7998917	6358	3 8033205	6408	3 8067225	6458	3 8100980
6259	3 7965040	6309	3 7999605	6359	3 8033888	6409	3 8067905	6459	3 8101655
6260	3 7965743	6310	3 8000294	6360	3 8034571	6410	3 8068580	6460	3 8102325
6261	3 7966437	6311	3 8000982	6361	3 8035254	6411	3 8069258	6461	3 8102997
6262	3 7967131	6312	3 8001670	6362	3 8035937	6412	3 8069935	6462	3 8103669
6263	3 7967824	6313	3 8002358	6363	3 8036619	6413	3 8070613	6463	3 8104342
6264	3 7968517	6314	3 8003046	6364	3 8037302	6414	3 8071290	6464	3 8105013
6265	3 7 69211	6315	3 8003734	6365	3 8037984	6415	3 8071967	6465	3 8105685
6266	3 7969904	6315	3 8004421	6366	3 8038666	6416	3 8072644	6466	3 8106357
6267	3 7970597	6317	3 8005100	6367	3 8039343	6417	3 8073320	6467	3 8107029
6268	3 7971290	6318	3 8005796	6368	3 8040030	6418	3 8073997	6468	3 8107700
6269	3 7971983	6319	3 8006484	6369	3 8040712	6419	3 8074674	6469	3 8103371
6270	3 7972675	6320	3 8007171	6370	3 8041394	6420	3 8075350	6470	3 8109043
6271	3 7973369	6321	3 8007858	6371	3 8042076	6421	3 8076027	6471	3 8109714
6272	3 7974060	6322	3 8008545	6372	3 8042758	6422	3 8076703	6472	3 8110385
6273	3 7974753	6323	3 8009232	6373	3 8043439	6423	3 8077379	6473	3 8111056
6274	3 7975445	6324	3 8009919	6374	3 8044121	6424	3 8078055	6474	3 8111727
6275	3 7976137	6325	3 8010605	6375	3 8044802	6425	3 8078731	6475	3 8112398
6276	3 7976829	6326	3 8011292	6376	3 8045483	6426	3 8079407	6476	3 8113068
6277	3 7977521	6327	3 8011978	6377	3 8046164	6427	3 8080083	6477	3 8113739
6278	3 7978213	6328	3 8012665	6378	3 8046845	6428	3 8080759	6478	3 8114409
6279	3 7978905	6329	3 8013351	6379	3 8047526	6429	3 8081434	6479	3 8115080
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6281	3 7980288	6331	3 8014723	6381	3 8048887	6431	3 8082785	6481	3 8116420
6282	3 7980979	6332	3 8015409	6382	3 8049568	6432	3 8083460	6482	3 8117090
6283	3 7681671	6333	3 8016095	6383	3 8050248	6433	3 8084135	6483	3 8117760
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6289	3 7985816	6339	3 8020208	6389	3 8054329	6439	3 8088184	6489	3 8121778
6290	3 7986506	6340	3 8020893	6390	3 8055009	6440	3 8088859	6490	3 8122447
6291	3 7987197	6341	3 8021578	6391	3 8055688	6441	3 8089533	6491	3 8123116
6292	3 7987887	6342	3 8022262	6392	3 8056368	6442	3 8090207	6492	3 8123785
6293	3 7988577	6343	3 8022947	6393	3 8057047	6443	3 8090881	6493	3 8124454
6294	3 7989267	6344	3 8023032	6394	3 8057726	6444	3 8091555	6494	3 8125123
6295	3 7989957	6345	3 8024316	6395	3 8058405	6445	3 8092229	6495	3 8125792
6296	3 7990647	6346	3 8025001	6396	3 8059085	6446	3 8092903	6496	3 8126460
6297	3 7991337	6347	3 8025685	6397	3 8059763	6447	3 8093350	6497	3 8127129
6298	3 7992026	6348	3 8026369	6398	3 8060442	6448	3 8094250	6498	3 8127797
6299	3 7992716	6349	3 8027053	6399	3 8061121	6449	3 8094924	6499	3 8128465
6300	3 7993405	6350	3 8027737	6400	3 8061800	6450	3 8095597	6500	3 8129134

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6501	3.8129602	6551	3.8163076	6601	3.8196097	6651	3.8228869	6701	3.8261396
6502	3.8130470	6552	3.8163739	6602	3.8196775	6652	3.8229522	6702	3.8262044
6503	3.8131137	6553	3.8164402	6603	3.8197413	6653	3.8230175	6703	3.8262692
6504	3.8131805	6554	3.8165064	6604	3.8198071	6654	3.8230828	6704	3.8263340
6505	3.8132473	6555	3.8165727	6605	3.8198728	6655	3.8231481	6705	3.8263988
6506	3.8133141	6556	3.8166389	6606	3.8199386	6656	3.8232133	6706	3.8264635
6507	3.8133808	6557	3.8167052	6607	3.8200043	6657	3.8232786	6707	3.8265283
6508	3.8134475	6558	3.8167714	6608	3.8200700	6658	3.8233438	6708	3.8265931
6509	3.8135143	6559	3.8168376	6609	3.8201357	6659	3.8234090	6709	3.8266578
6510	3.8135810	6560	3.8169038	6610	3.8202015	6660	3.8234742	6710	3.8267225
6511	3.8136477	6561	3.8169700	6611	3.8202672	6661	3.8235394	6711	3.8267872
6512	3.8137144	6562	3.8170362	6612	3.8203328	6662	3.8236046	6712	3.8268519
6513	3.8137811	6563	3.8171024	6613	3.8203985	6663	3.8236698	6713	3.8269166
6514	3.8138477	6564	3.8171686	6614	3.8204642	6664	3.8237350	6714	3.8269813
6515	3.8139144	6565	3.8172347	6615	3.8205298	6665	3.8238002	6715	3.8270460
6516	3.8139811	6566	3.8173009	6616	3.8205955	6666	3.8238653	6716	3.8271107
6517	3.8140478	6567	3.8173670	6617	3.8206611	6667	3.8239305	6717	3.8271753
6518	3.8141144	6568	3.8174331	6618	3.8207268	6668	3.8239956	6718	3.8272400
6519	3.8141810	6569	3.8174993	6619	3.8207924	6669	3.8240607	6719	3.8273046
6520	3.8142476	6570	3.8175654	6620	3.8208580	6670	3.8241253	6720	3.8273693
6521	3.8143142	6571	3.8176315	6621	3.8209236	6671	3.8241909	6721	3.8274339
6522	3.8143808	6572	3.8176975	6622	3.8209892	6672	3.8242560	6722	3.8274985
6523	3.8144474	6573	3.8177636	6623	3.8210548	6674	3.8243211	6723	3.8275631
6524	3.8145139	6574	3.8178297	6624	3.8211203	6674	3.8243862	6724	3.8276277
6525	3.8145805	6575	3.8178958	6625	3.8211859	6675	3.8244513	6725	3.8276923
6526	3.8146471	6576	3.8179618	6626	3.8212514	6676	3.8245163	6726	3.8277569
6527	3.8147136	6577	3.8180278	6627	3.8213170	6677	3.8245814	6727	3.8278214
6528	3.8147801	6578	3.8180939	6628	3.8213825	6678	3.8246464	6728	3.8278860
6529	3.8148467	6579	3.8181599	6629	3.8214480	6679	3.8247114	6729	3.8279505
6530	3.8149132	6580	3.8182259	6630	3.8215135	6680	3.8247765	6730	3.8280951
6531	3.8149797	6581	3.8182919	6631	3.8215790	6681	3.8248415	6731	3.8280769
6532	3.8150462	6582	3.8183579	6632	3.8216445	6682	3.8249065	6732	3.8281441
6533	3.8151127	6583	3.8184239	6633	3.8217100	6683	3.8249715	6733	3.8282086
6534	3.8151791	6584	3.8184898	6634	3.8217755	6684	3.8250364	6734	3.8282731
6535	3.8152456	6585	3.8185558	6635	3.8218409	6685	3.8251014	6735	3.8283376
6536	3.8153120	6586	3.8186217	6636	3.8219064	6686	3.8251664	6736	3.8284021
6537	3.8153785	6587	3.8186877	6637	3.8219718	6687	3.8252313	6737	3.8284665
6538	3.8154449	6588	3.8187536	6638	3.8220372	6688	3.8252963	6738	3.8285310
6539	3.8155113	6589	3.8188195	6639	3.8221027	6689	3.8253612	6739	3.8285955
6540	3.8155777	6590	3.8188854	6640	3.8221681	6690	3.8254261	6740	3.8286599
6541	3.8156441	6591	3.8189513	6641	3.8222335	6691	3.8254910	6741	3.8287243
6542	3.8157105	6592	3.8190172	6642	3.8222989	6692	3.8255559	6742	3.8287887
6543	3.8157769	6593	3.8190831	6643	3.8223642	6693	3.8256208	6743	3.8288532
6544	3.8158433	6594	3.8191489	6644	3.8224296	6694	3.8256857	6744	3.8289176
6545	3.8159096	6595	3.8192148	6645	3.8224950	6695	3.8257506	6745	3.8289820
6546	3.8159760	6596	3.8192806	6646	3.8225603	6696	3.8258154	6746	3.8290463
6547	3.8160423	6597	3.8193465	6647	3.8226257	6697	3.8258803	6747	3.8291107
6548	3.8161087	6598	3.8194123	6648	3.8226910	6698	3.8259451	6748	3.8291751
6549	3.8161750	6599	3.8194781	6649	3.8227563	6699	3.8260100	6749	3.8292394
6550	3.8162413	6600	3.8195439	6650	3.8228216	6700	3.8260748	6750	3.8293036

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
6751	3.8293581	6801	3.8325728	6851	3.8357540	6901	3.838 120	6951	3.8420473
6752	3.8294324	6802	3.8326356	6852	3.8358174	6902	3.8389750	6952	3.8421098
6753	3.4294967	6803	3.8327005	6853	3.8358807	6903	3.8390379	6953	3.8421722
6754	3.8295611	6804	3.8327643	6854	3.8359441	6904	3.8391008	6954	3.8422347
6755	3.2296254	6805	3.8328281	6855	3.8360075	6905	3.8391637	6955	3.8422971
6756	3.8296896	6806	3.8328919	6856	3.8360708	6906	3.8392266	6956	3.8423596
6757	3.8297539	6807	3.8329558	6857	3.8361341	6907	3.8392895	6957	3.8424220
6758	3.8298182	6808	3.8330195	6858	3.8361975	6908	3.8393523	6958	3.8424844
6759	3.8298824	6809	3.8330833	6859	3.8362608	6909	3.8394152	6959	3.8425468
6760	3.8299467	6810	3.8331471	6860	3.8363241	6910	3.8394780	6960	3.8426092
6761	3.8300109	6811	3.8332209	6861	3.8363874	6911	3.8395409	6961	3.8426716
6762	3.8300752	6812	3.8332746	6862	3.8364507	6912	3.8396037	6962	3.8427340
6763	3.8301394	6813	3.8333384	6863	3.8365140	6913	3.8396666	6963	3.8427964
6764	3.8302036	6814	3.8334021	6864	3.8365773	6914	3.8397294	6964	3.8428588
6765	3.8302678	6815	3.8334659	6865	3.8366405	6915	3.8397922	6965	3.8429211
6766	3.8303320	6816	3.8335296	6866	3.8367038	6916	3.8398550	6966	3.8429835
6767	3.8303962	6817	3.8335933	6867	3.8367670	6917	3.8399178	6967	3.8430458
6768	3.8304604	6818	3.8336570	6868	3.8368303	6918	3.8399806	6968	3.8431081
6769	3.3305245	6819	3.8337207	6869	3.8368935	6919	3.8400433	6969	3.8431705
6770	3.83 5887	6820	3.8337844	6870	3.8369567	6920	3.8401061	6970	3.8432328
6771	3.8306528	6821	3.8338480	6871	3.8370199	6921	3.8401688	6971	3.8432951
6772	3.8307169	6822	3.8339117	6872	3.8370832	6922	3.8402316	6972	3.8433574
6773	3.8307811	6823	3.8339754	6873	3.8371463	6923	3.8402943	6973	3.8434197
6774	3.8308452	6824	3.8340390	6874	3.8372095	6924	3.8403541	6974	3.8434819
6775	3.8309093	6825	3.8341027	6875	3.8372727	6925	3.8404198	6975	3.8435442
6776	3.8309734	6826	3.8341663	6876	3.8373359	6926	3.8404825	6976	3.8436065
6777	3.8310375	6827	3.8342299	6877	3.8373990	6927	3.8405452	6977	3.8436687
6778	3.8311016	6828	3.8342935	6878	3.8378622	6928	3.8406079	6978	3.8437310
6779	3.8311656	6829	3.8343571	6879	3.8375253	2929	3.8406706	6979	3.8437932
6780	3.8312297	6830	3.8344207	6880	3.8375884	6930	3.8407332	6980	3.8438554
6781	3.8312937	6831	3.8344443	6881	3.8376516	6931	3.8407959	6981	3.8439176
6782	3.8313578	6832	3.8345479	6882	3.8377147	6932	3.8408586	6982	3.8439798
6783	3.8314218	6833	3.8346114	6883	3.8377778	6933	3.8409212	6983	3.8440420
6784	3.8314858	6834	3.8346750	6884	3.8378409	6934	3.8410988	6984	3.8441042
6785	3.8315499	6835	3.8347385	6885	3.8379039	6935	3.8410465	6985	3.8441664
6786	3.8316139	6836	3.8348021	6886	3.8379670	6936	3.8411091	6986	3.8442286
6787	3.8316778	6837	3.8348655	6887	3.8380301	6937	3.8411717	6987	3.8442907
6788	3.8317418	6838	3.8349291	6888	3.8380931	6938	3.8412343	6988	3.8443529
6789	3.8318058	6839	3.8349926	6889	3.8381562	6939	3.8412969	6989	3.8444150
6790	3.8318698	6840	3.8350561	6890	3.8382192	6940	3.8413595	6990	3.8444772
6791	3.8319337	6841	3.8351196	6891	3.8382822	6941	3.8414220	6991	3.8445393
6792	3.8319977	6842	3.8351831	6892	3.8383453	6942	3.8414846	6992	3.8446014
6793	3.8320616	6843	3.8352465	6893	3.8384083	6943	3.8415472	6993	3.8446635
6794	3.8321255	6844	3.8353100	6894	3.8384713	6944	3.8416097	6994	3.8447256
6795	3.8321895	6845	3.8353735	6895	3.8385343	6945	3.8416723	6995	3.8447877
6796	3.8322534	6846	3.8354369	6896	3.8385973	6946	3.8417343	6996	3.8448498
6797	3.8323173	6847	3.8355003	6897	3.8386602	6947	3.8417973	6997	3.8449119
6798	3.8323812	6848	3.8355638	6898	3.838 232	6948	3.8418598	6998	3.8449739
6799	3.8324450	6849	3.8356272	6899	3.8387861	6949	3.8419224	6999	3.8450360
6800	3.8325089	6850	3.8356905	6900	3.8388491	6950	3.8419848	7000	3.8450980

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
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7002	3.8452221	7052	3.8483123	7102	3.8513807	7152	3.8544275	7202	3.8574531
7003	3.8452841	7053	3.8483739	7103	3.8514418	7153	3.8544882	7203	3.8575134
7004	3.8453461	7054	3.8484355	7104	3.8515030	7154	3.8545489	7204	3.8575737
7005	3.8454081	7055	3.8484970	7105	3.8515641	7155	3.8546096	7205	3.8576340
7006	3.8454701	7056	3.8485586	7106	3.8516252	7156	3.8546703	7206	3.8576943
7007	3.8455321	7057	3.8486201	7107	3.8516863	7157	3.8547310	7207	3.8577545
7008	3.8455941	7058	3.8486817	7108	3.8517474	7158	3.8547917	7208	3.8578148
7009	3.8456561	7059	3.8487432	7109	3.8518085	7159	3.8548524	7209	3.8578750
7010	3.4457180	7060	3.8488047	7110	3.8518696	7160	3.8549130	7210	3.8579353
7011	3.8457800	7061	3.8488662	7111	3.8519307	7161	3.8549737	7211	3.8579955
7012	3.8458419	7062	3.8489277	7112	3.8519917	7162	3.8550343	7212	3.8580557
7013	3.8459038	7063	3.8489892	7113	3.8520528	7163	3.8550950	7213	3.8581156
7014	3.8459658	7064	3.8490507	7114	3.8521139	7164	3.8551556	7214	3.8581761
7015	3.8460277	7065	3.8491122	7115	3.8521749	7165	3.8552162	7215	3.8582363
7016	3.8460896	7066	3.8491736	7116	3.8522359	7166	3.8552768	7216	3.8582965
7017	3.8461515	7067	3.8492351	7117	3.8522970	7167	3.8553374	7217	3.8583567
7018	3.8462134	7068	3.8492965	7118	3.8523580	7168	3.8553980	7218	3.8584169
7019	3.8462752	7069	3.8493580	7119	3.8524190	3169	3.8554586	7219	3.8584770
7020	3.8463371	7070	3.8494194	7120	3.8524800	7170	3.8555192	7220	3.8585372
7021	3.8463990	7071	3.8494808	7121	3.8525410	7171	3.8555797	7221	3.8585973
7022	3.8464608	7072	3.8495423	7122	3.8526020	7172	3.8556403	7222	3.8586575
7023	3.8465227	7073	3.8496037	7123	3.8526629	7173	3.8557008	7223	3.8587176
7024	3.8465845	7074	3.8496651	7124	3.8527239	7174	3.8557614	7224	3.8587777
7025	3.8466463	7075	3.8497264	7125	3.8527849	7175	3.8558219	7225	3.8588379
7026	3.8467081	7076	3.8497878	7126	3.8528458	7176	3.8558824	7226	3.8588980
7027	3.8467700	6077	3.8498492	7127	3.8529068	7177	3.8559429	7227	3.8589581
7028	3.8468318	7078	3.8499106	7128	3.8529677	7178	3.8560035	7228	3.8590181
7029	3.8468935	7079	3.8499719	7129	3.8530286	7179	3.8560640	7229	3.8590782
7030	3.8469553	7080	3.8500333	7130	3.8530895	7180	3.8561244	7230	3.8591383
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7032	3.8470789	7082	3.8501559	7132	3.8532113	7182	3.8562454	7232	3.8592588
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7037	3.8473876	7087	3.8504625	7137	3.8535157	7187	3.8565476	7237	3.8595986
7038	3.8474493	7088	3.8505237	7138	3.8535765	7188	3.8566081	7238	3.8596186
7039	3.8475110	7089	3.8505850	7139	3.8536374	7189	3.8566685	7239	3.8596786
7040	3.8475727	7090	3.8506462	7140	3.8536982	7190	3.8567289	7240	3.8597386
7041	3.8476343	7091	3.8507075	7141	3.8537590	7191	3.8567893	7241	3.8597985
7042	3.8476900	7092	3.8507687	7142	3.8538198	7192	3.8567849	7242	3.8598583
7043	3.8477577	7093	3.8508300	7143	3.8538807	7193	3.8569101	7243	3.8599181
7044	3.8478193	7094	3.8508912	7144	3.8539414	7194	3.8569704	7244	3.8599784
7045	3.8478810	7095	3.8509524	7145	3.8540022	7195	3.8570308	7245	3.8600384
7046	3.8479426	7096	3.8510136	7146	3.8540630	7196	3.8570912	7246	3.8600983
7047	3.8480043	7097	3.8510748	7147	3.8541238	7197	3.8571515	7247	3.8601583
7048	3.8480659	7098	3.8511560	7148	3.8541845	7198	3.8572118	7248	3.8602182
7049	3.8481275	7099	3.8511972	7149	3.8542453	7199	3.8572721	7249	3.8602781
7050	3.8481891	7100	3.8512583	7150	3.8543060	7200	3.8573325	7250	3.8603380

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7252	3.8604578	7302	3 8634418	7352	3 8664063	7402	3.8693491	7502	3.8751771
7253	3.8605177	7303	3.8635013	7353	3.8664646	7403	3.8694077	7503	3.8752349
7254	3 8605776	7304	3 8635608	7354	3.8665230	7404	3.8694664	7504	3,8752928
7255	3 8606374	7305	3.8636202	7355	3.8665827	7405	3.8695251	7505	3.8753507
7256	3 8606973	7306	3.8636797	7356	3 8666417	7406	3.8695837	7506	3.8754086
7257	3.8607571	7307	3.8637391	7357	3 8667008	7407	3.8696423	7507	3.8754664
7258	3.8608170	7308	3.8637985	7358	3.8667598	7408	3.8697010	7508	3 8755243
7259	3.8608768	7309	3.8638580	7359	3.8668188	7409	3.8697596	7509	3.8755821
7260	3.8609366	7310	3.8639174	7360	3 8668778	7410	3.8698182	7510	3.8756399
7261	3.8609964	7311	3.8639768	7361	3.8669368	7411	3.8698768	7511	3.8756978
7262	3.8610562	7312	3.8640363	7362	3.8669958	7412	3.8699354	7512	3.8757556
7263	3.8611160	7313	3.8640956	7363	3.86705 8	7413	3.8699940	7513	3.8758134
7264	3.8611758	7314	3.8641550	7364	3.8671138	7414	3.8700526	7514	3.8758712
7265	3 8612356	7315	3 8642143	7365	3 8671723	7415	3 8701112	7515	3.8759290
7266	3.8612954	7316	3.8642737	7366	3.8672317	7416	3.8701697	7516	3.8759868
7267	3.8613552	7317	3.8643331	7367	3 8672907	7417	3.8702283	7517	3 8760445
7268	3.8614149	7318	3.8643924	7368	3 8673496	7418	3.8702868	7518	3.8761023
7269	3.8614747	7319	3.8644517	7369	3 8674086	7419	3 8703454	7519	3.8761601
7270	3.8615344	7320	3.8645111	7370	3 8674675	7420	3.8704039	7520	3 8762178
7271	3.8615941	7321	3.8645704	7371	3.8675264	7421	3 8704624	7521	3.8762656
7272	3 8616539	7322	3.8646297	7372	3.8675853	7422	3.8705210	7522	3 8763333
7273	3.8617136	7323	3.8646890	7373	3.8676442	7423	3.8705794	7523	3.8763911
7274	3.8617733	7324	3.8647483	7374	3 8677031	7424	3.8706380	7524	3.8764488
7275	3 8618330	7325	3.8648076	7375	3.8677620	7425	3 8706965	7525	3.8765065
7276	3.8618927	7326	3.8648669	7376	3.8678209	7426	3.8707549	7526	3.8765642
7277	3.8619524	7327	3.8649262	7377	3.8678798	7427	3.8708134	7527	3.8766219
7278	3.8620121	7328	3.8649855	7378	3.8679387	7428	3.8708719	7528	3 8766796
7279	3.8620717	7329	3.8650447	7379	3.8679975	7429	3.8709304	7529	3.8767373
7280	3 8621314	7330	3.8651040	7380	3.8680564	7430	3.8709888	7530	3.8767950
7281	3.8621910	7331	3.8651632	7381	3.8081152	7431	3 8710423	7531	3.8768526
7282	3.8622507	7332	3 8652225	7382	3.8681740	7432	3 8711057	7532	3.8769103
7283	3.8623103	7333	3.8652817	7383	3.8682329	7433	3.8711621	7533	3.8769680
7284	3 8623699	7334	3.8653409	7384	3.8682917	7434	3.8712226	7534	3.8770256
7285	3.8624296	7335	3.8654001	7385	3.8683505	7435	3.8712810	7535	3.8770833
7286	3.8624892	7336	3.8654593	7386	3.8684093	7436	3 8713394	7536	3.8771409
7287	3 8665488	7337	3.8655185	7387	3.8684681	7437	3.8713978	7537	3.8771985
7283	3.8626084	7338	3.8655777	7388	3.8685269	7438	3 8714562	7538	3 8772561
7289	3.8626680	7339	3 8656369	7389	3.8685857	7439	3 8715146	7539	3.8773137
7290	3 8627275	7340	3 8656961	7390	3.8686444	7440	3.8715729	7540	3.8773713
7291	3 8627871	7341	3.8657552	7391	3.8687032	7491	3.8745398	7541	3.8774289
7292	3 8628467	7342	3 8658144	7392	3.8687620	7492	3.8745978	7542	3.8774865
7293	3.8629063	7343	3 8658735	7393	3 8688207	7493	3.8746557	7543	3.8775441
7294	3 8629658	7344	3.8659327	7394	3 8688794	7494	3.8747137	7544	3 8776077
7295	3.8630253	7345	3 8659948	7395	3.8689382	7495	3.8747716	7545	3.8776592
7296	3.8630848	7346	3.8660509	7396	3.8689969	7496	3.8748296	7546	3.8777168
7297	3 8631443	7347	3.8661100	7397	3 8690556	7497	3 8748875	7547	3.8777743
7298	3 8632039	7348	3 8661691	7398	3.8691143	7498	3.8749454	7548	3 8778319
7299	3.8632634	7349	3 8662283	7399	3.8691730	7499	3.8750034	7549	3 8778894
7300	3.8633229	7350	3 8662873	7400	3 8692717	7500	3 8750613	7550	3 8779470

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
7551	3.8780045	7601	3.8808707	7651	3.8837182	7701	3.8865471	7751	3.8893577
7552	3.8780620	7602	3.8809279	7652	3.8837750	7702	3.8866035	7752	3.8894138
7553	3.8781195	7603	3.8809850	7653	3.8838317	7703	3.8866599	7753	3.8894698
7554	3.8781770	7604	3.8810421	7654	3.8838885	7704	3.8867163	7754	3.8895258
7555	3.8782345	7605	3.8810992	7655	3.8839452	7705	3.8867726	7755	3.8895818
7556	3.8782919	7606	3.8811563	7656	3.8840019	7706	3.8868290	7756	3.8896378
7557	3.8783494	7607	3.8812134	7657	3.8840586	7707	3.8868854	7757	3.8896938
7558	3.8784069	7608	3.8812705	7658	3.8841154	7708	3.8869417	7758	3.8897498
7559	3.8784643	7609	3.8813276	7659	3.8841721	7709	3.8869980	7759	3.8898058
7560	3.8785218	7610	3.8813847	7660	3.8842288	7710	3.8870544	7760	3.8898617
7561	3.8785792	7611	3.8814417	7661	3.8842855	7711	3.8871107	7761	3.8899177
7562	3.8786367	7612	3.8814988	7662	3.8843421	7712	3.8871670	7762	3.8899736
7563	3.8786941	7613	3.8815558	7663	3.8843988	7713	3.8872233	7763	3.8900296
7564	3.8787515	7614	3.8816128	7664	3.8844555	7714	3.8872796	7764	3.8900855
7565	3.8788089	7615	3.8816699	7665	3.8845122	7715	3.8873359	7765	3.8901415
7566	3.8788663	7616	3.8817269	7666	3.8845688	7716	3.8873922	7766	3.8901974
7567	3.8788911	7617	3.8817840	7667	3.8846255	7717	3.8874485	7767	3.8902533
7568	3.8789237	7618	3.8818410	7668	3.8846821	7718	3.8875048	7768	3.8903092
7569	3.8790385	7619	3.8818980	7669	3.8847387	7719	3.8875610	7769	3.8903651
7570	3.8790959	7620	3.8819550	7670	3.8847954	7720	3.8876173	7770	3.8904210
7571	3.8791532	7621	3.8820120	7671	3.8848520	7721	3.8876736	7771	3.8904769
7572	3.8792106	7622	3.8820689	7672	3.8849086	7722	3.8877298	7772	3.8905328
7573	3.8792680	7623	3.8821259	7673	3.8849652	7723	3.8877860	7773	3.8905887
7574	3.8793253	7624	3.8821829	7674	3.8850218	7724	3.8878423	7774	3.8906445
7575	3.8793826	7625	3.8822398	7675	3.8850784	7725	3.8878985	7775	3.8907004
7576	3.8794400	7626	3.8822968	7676	3.8851350	7726	3.8879541	7776	3.8907563
7577	3.8794973	7627	3.8823537	7677	3.8851915	7727	3.8880109	7777	3.8908121
7578	3.8795546	7628	3.8824107	7678	3.8852481	7728	3.8880671	7778	3.8908679
7579	3.8796119	7629	3.8824676	7679	3.8853047	7729	3.8881233	7779	3.8909238
7580	3.8796692	7630	3.8825245	7680	3.8853612	7730	3.8881795	7780	3.8909796
7581	3.8797265	7631	3.8825815	7681	3.8854178	7731	3.8882357	7781	3.8910354
7582	3.8797838	7632	3.8826384	7682	3.8854745	7732	3.8882918	7782	3.8910912
7583	3.8798411	7633	3.8826953	7683	3.8855308	7733	3.8883420	7783	3.8911470
7584	3.8798983	7634	3.8827522	7684	3.8855874	7734	3.8884042	7784	3.8912028
7585	3.8799556	7635	3.8828090	7685	3.8856439	7735	3.8884603	7785	3.8912586
7586	3.8800128	7636	3.8828659	7686	3.8857004	7736	3.8885163	7786	3.8913144
7587	3.8800701	7637	3.8829228	7687	3.8857569	7737	3.8885726	7787	3.8913702
7588	3.8801273	7638	3.8829797	7688	3.8858134	7738	3.8886287	7788	3.8914259
7589	3.8801846	7639	3.8830365	7689	3.8858699	7739	3.8886848	7789	3.8914817
7590	3.8802418	7640	3.8830934	7690	3.8859263	7740	3.8887410	7790	3.8915375
7591	3.8802990	7641	3.8831502	7691	3.8859828	7741	3.8887971	7791	3.8915932
7592	3.8803562	7642	3.8832070	7692	3.8860393	7742	3.8888532	7792	3.8916489
7593	3.8804134	7643	3.8832639	7693	3.8860957	7743	3.8889093	7793	3.8917047
7594	3.8804706	7644	3.8833207	7694	3.8861522	7744	3.8889653	7794	3.8917604
7595	3.8805278	7645	3.8833775	7695	3.8862086	7745	3.8890214	7795	3.8918161
7596	3.8805850	7646	3.8834343	7696	3.8862651	7746	3.8890775	7796	3.8918718
7597	3.8806421	7647	3.8834911	7697	3.8863215	7747	3.8891336	7797	3.8919275
7598	3.8806993	7648	3.8835479	7698	3.8863779	7748	3.8891896	7798	3.8919832
7599	3.8807564	7649	3.8836047	7699	3.8864343	7749	3.8892457	7799	3.8920389
7600	3.8808136	7650	3.8836614	7700	3.8864907	7750	3.8893017	7800	3.8920946

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
7801	3.8929503	7851	3.8949250	7901	3.8976821	7951	3.9004218	8001	3.9031443
7802	3.8930059	7852	3.8949803	7902	3.8977370	7952	3.9004764	8002	3.9031985
7803	3.8926616	7853	3.8950356	7903	3.8977920	7953	3.9005310	8003	3.9032528
7804	3.8933173	7854	3.8950909	7904	3.8978469	7954	3.9005856	8004	3.9033071
7805	3.8923729	7855	3.8951462	7905	3.8979019	7955	3.9006402	8005	3.9033613
7806	3.8924285	7856	3.8952015	7906	3.8979578	7956	3.9006948	8006	3.9034156
7807	3.8924842	7857	3.8952568	7907	3.8980117	7957	3.9007494	8007	3.9034698
7808	3.8925398	7858	3.8953120	7908	3.8980667	7958	3.9008039	8008	3.9035241
7809	3.8925954	7859	3.8953673	7909	3.8981216	7959	3.9008585	8009	3.9035783
7810	3.8926510	7860	3.8954225	7910	3.8981765	7960	3.9009131	8010	3.9036325
7811	3.8927066	7861	3.8954778	7911	3.8982314	7961	3.9009676	8011	3.9036867
7812	3.8927622	7862	3.8955330	7912	3.8982863	7962	3.9010222	8012	3.9037409
7813	3.8928178	7863	3.8955883	7913	3.8983412	7963	3.9010767	8013	3.9037951
7814	3.8928734	7864	3.8956435	7914	3.8983960	7964	3.9011313	8014	3.9038493
7815	3.8929290	7865	3.8956987	7915	3.8984509	7965	3.9011858	8015	3.9039035
7816	3.8929846	7866	3.8957539	7916	3.8985058	7966	3.9012403	8016	3.9039577
7817	3.8930401	7867	3.8958092	7917	3.8985606	7967	3.9012948	8017	3.9040119
7818	3.8930957	7868	3.8958644	7918	3.8986155	7968	3.9013493	8018	3.9040661
7819	3.8931512	7869	3.8959195	7919	3.8986703	7969	3.9014038	8019	3.9041202
7820	3.8932068	7870	3.8959748	7920	3.8987252	7970	3.9014583	8020	3.9041744
7821	3.8932623	7871	3.8960299	7921	3.8987800	7971	3.9015128	8021	3.9042285
7822	3.8933178	7872	3.8960851	7922	3.8988348	7972	3.9015673	8022	3.9042827
7823	3.8933733	7873	3.8961403	7923	3.8988897	7973	3.9016218	8023	3.9043368
7824	3.8934288	7874	3.8961954	7924	3.8989445	7974	3.9016762	8024	3.9043909
7825	3.8934843	7875	3.8962506	7925	3.8989993	7975	3.9016307	8025	3.9044450
7826	3.8935398	7876	3.8963057	7926	3.8990541	7976	3.9017851	8026	3.9044992
7827	3.8935953	7877	3.8963608	7927	3.8991089	7977	3.9018396	8027	3.9045533
7828	3.8936508	7878	3.8964160	7928	3.8991636	7978	3.9018940	8028	3.9046074
7829	3.8937063	7879	3.8964711	7929	3.8992184	7979	3.9019485	8029	3.9046615
7830	3.8937618	7880	3.8965262	7930	3.8992732	7980	3.9020029	8030	3.9047155
7831	3.8938172	7881	3.8965813	7931	3.8993279	7981	3.9020573	8031	3.9047696
7832	3.8938727	7882	3.8966364	7932	3.8993827	7982	3.9021117	8032	3.9048237
7833	3.8939281	7883	3.8966915	7933	3.8994375	7983	3.9021661	8033	3.9048778
7834	3.8939836	7884	3.8967466	7934	3.8994922	7984	3.9022205	8034	3.9049318
7835	3.8940392	7885	3.8968017	7935	3.8995469	7985	3.9022749	8035	3.9049859
7836	3.8940944	7886	3.8968568	7936	3.8996017	7986	3.9023293	8036	3.9050393
7837	3.8941498	7887	3.8969118	7937	3.8996564	7987	3.9023837	8037	3.9050940
7838	3.8942053	7888	3.8969669	7938	3.8997111	7988	3.9024381	8038	3.9051480
7839	3.8942607	7889	3.8970220	7939	3.8997658	7989	3.9024924	8039	3.9052020
7840	3.8943161	7890	3.8970770	7940	3.8998205	7990	3.9025468	8040	3.9052560
7841	3.8943715	7891	3.8971320	7941	3.8998752	7991	3.9026011	8041	3.9053101
7842	3.8944268	7892	3.8971871	7942	3.8999299	7992	3.9026555	8042	3.9053641
7843	3.8944822	7893	3.8972421	7943	3.8999846	7993	3.9027098	8043	3.9054181
7844	3.8945376	7894	3.8972971	7944	3.9000392	7994	3.9027641	8044	3.9054721
7845	3.8945929	7895	3.8973521	7945	3.9000939	7995	3.9028185	8045	3.9055260
7846	3.8946483	7896	3.8974071	7946	3.9001486	7996	3.9028728	8046	3.9055800
7847	3.8947037	7897	3.8974621	7947	3.9002032	7997	3.9029271	8047	3.9056340
7848	3.8947590	7898	3.8975171	7948	3.9002579	7998	3.9029814	8048	3.9056880
7849	3.8948143	7899	3.8975721	7949	3.9003125	7999	3.9030357	8049	3.9057419
7850	3.8944697	7900	3.8976271	7950	3.9003671	8000	3.9030900	8050	3.9057959

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
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8052	3.9059038	8102	3.9085922	8152	3.9112642	8202	3.9139198	8252	3.9165592
8053	3.9059577	8103	3.9086458	8153	3.9113174	8203	3.9139727	8253	3.9166118
8054	3.9060116	8104	3.9086994	8154	3.9113707	8204	3.9140257	8254	3.9166645
8055	3.9060655	8105	3.9087530	8155	3.9114240	8205	3.9140785	8255	3.9167171
8056	3.9061195	8106	3.9088063	8156	3.9114772	8206	3.9141315	8256	3.9167697
8057	3.9061734	8107	3.9088602	8157	3.9115305	8207	3.9141844	8257	3.9168223
8058	3.9062273	8108	3.9089137	8158	3.9115837	8208	3.9142373	8258	3.9168749
8059	3.9062812	8109	3.9089673	8159	3.9116369	8209	3.9142903	8259	3.9169275
8060	3.9063350	8110	3.9090209	8160	3.9116902	8210	3.9143432	8260	3.9169800
8061	3.9063889	8111	3.9090744	8161	3.9117434	8211	3.9143961	8261	3.9170326
8062	3.9064428	8112	3.9091279	8162	3.9117966	8212	3.9144489	8262	3.9170852
8063	3.9064967	8113	3.9091815	8163	3.9118498	8215	3.9145018	8263	3.9171878
8064	3.9065505	8114	3.9092350	8164	3.9119030	8214	3.9145547	8264	3.9171903
8065	3.9066044	8115	3.9092885	8165	3.9119562	8215	3.9146076	8265	3.9172429
8066	3.9066582	8116	3.9093420	8166	3.9120094	8216	3.9146604	8266	3.9172954
8067	3.9067121	8117	3.9093955	8167	3.9120626	8217	3.9147134	8267	3.9173479
8068	3.9067659	8118	3.9094490	8168	3.9121157	8218	3.9147661	8268	3.9174005
8069	3.9068197	8119	3.9095025	8169	3.9121689	8219	3.9148190	8269	3.9174530
8070	3.9068735	8120	3.9095560	8170	3.9122221	8220	3.9148718	8270	3.9175055
8071	3.9069273	8121	3.9096095	8171	3.9122752	8221	3.9149246	8271	3.9175580
8072	3.9069812	8122	3.9096630	8172	3.9123284	8222	3.9149775	8272	3.9176105
8073	3.9070350	8123	3.9097165	8173	3.9123815	8223	3.9150303	8273	3.9176630
8074	3.9070887	8124	3.9097599	8174	3.9124346	8224	3.9150831	8274	3.9177155
8075	3.9071425	8125	3.9098234	8175	3.9124878	8225	3.9151359	8275	3.9177680
8076	3.9071963	8126	3.9098768	8176	3.9125409	8226	3.9151887	8276	3.9178205
8077	3.9072501	8127	3.9099303	8177	3.9125940	8227	3.9152415	8277	3.9178730
8078	3.9073038	8128	3.9099837	8178	3.9126471	8228	3.9152943	8278	3.9179254
8079	3.9073576	8129	3.9100371	8179	3.9127002	8229	3.9153471	8279	3.9179779
8080	3.9074114	8130	3.9100905	8180	3.9127533	8230	3.9153998	8280	3.9180303
8081	3.9074651	8131	3.9101440	8181	3.9128064	8231	3.9154526	8281	3.9180828
8082	3.9075188	8132	3.9101974	8182	3.9128595	8232	3.9155054	8282	3.9181352
8083	3.9075726	8133	3.9102508	8183	3.9129126	8233	3.9155581	8283	3.9181877
8084	3.9076263	8134	3.9103042	8184	3.9129656	8234	3.9156109	8284	3.9182401
8085	3.9076801	8135	3.9103576	8185	3.9130187	8235	3.9156636	8285	3.9182925
8086	3.9077337	8136	3.9104109	8186	3.9130717	8236	3.9157163	8286	3.9183449
8087	3.9077874	8137	3.9104643	8187	3.9131248	8237	3.9157691	8287	3.9183973
8088	3.9078411	8138	3.9105177	8188	3.9131778	8238	3.9158218	8288	3.9184497
8089	3.9078948	8139	3.9105710	8189	3.9132309	8239	3.9158745	8289	3.9185021
8090	3.9079485	8140	3.9106244	8190	3.9132839	8240	3.9159272	8290	3.9185545
8091	3.9080022	8141	3.9106778	8191	3.9133369	8241	3.9159799	8291	3.9186069
8092	3.9080559	8142	3.9107311	8192	3.9133899	8242	3.9160326	8292	3.9186593
8093	3.9081095	8143	3.9107864	8193	3.9134430	8243	3.9160853	8293	3.9187117
8094	3.9081632	8144	3.9108378	8194	3.9134960	8244	3.9161380	8294	3.9187640
8095	3.9082169	8145	3.9108911	8195	3.9135490	8245	3.9161907	8295	3.9188164
8096	3.9082705	8146	3.9109444	8196	3.9136019	8246	3.9162433	8296	3.9188687
8097	3.9083241	8147	3.9109977	8197	3.9136549	8247	3.9162960	8297	3.9189211
8098	3.9083778	8148	3.9110510	8198	3.9137079	8248	3.9163487	8298	3.9189734
8099	3.9084314	8149	3.9111043	8199	3.9137609	8249	3.9164013	8299	3.9190258
8100	3.9084850	8150	3.9111576	8200	3.9138139	8250	3.9164540	8300	3.9190781

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
8301	9191501	8351	9217385	8401	9243170	8451	9269081	8501	9294700
8302	9191827	8352	9217905	8402	9243827	8452	9269595	8502	9295211
8303	9192350	8353	9218425	8403	9244344	8453	9270109	8503	9295722
8304	9192873	8354	9218945	8404	9244850	8454	9270622	8504	9296233
8305	9193396	8355	9219465	8405	9245377	8455	9271136	8505	9296743
8306	9193917	8356	9219984	8406	9245894	8456	9271650	8506	9297254
8307	9194412	8357	9220504	8407	9246410	8457	9272163	8507	9297764
8308	9194965	8358	9221024	8408	9246927	8458	9272677	8508	9298275
8309	9195488	8359	9221543	8409	9247444	8459	9273190	8509	9298785
8310	9196010	8360	9222063	8410	9247960	8460	9273704	8510	9299296
8311	9196533	8361	9222582	8411	9248476	8461	9274217	8511	9299806
8312	9197055	8362	9223102	8412	9248993	8462	9274730	8512	9300316
8313	9197578	8363	9223621	8413	9249509	8463	9275243	8513	9300826
4314	9198100	8364	9234140	8414	9250025	8464	9275757	8514	9301336
8315	9198623	8365	9224659	8415	9250541	8465	9276270	8515	9301847
8316	9199145	8366	9225179	8416	9251057	8466	9276783	8516	9302357
8317	9199667	8367	9225698	8417	9252572	8467	9277296	8517	9302866
8318	9200189	8368	9226217	8418	9252089	8468	9277808	8518	9303376
8319	9200711	8369	9226736	8419	9252605	8469	9278321	8519	9303886
4320	9201233	8370	9227255	8420	9253121	8470	9278834	8520	9304396
8321	9201755	8371	9227773	8421	9253637	8471	9279347	8521	9304906
8322	9202277	8372	9228292	8422	9254152	8472	9279859	8522	9305415
8323	9202799	8373	9228811	8423	9254668	8473	9280372	8523	9305925
8324	9203321	8374	9229330	8424	9255184	8474	9280885	8524	9306434
8325	9203842	8375	9229848	8425	9255699	8475	9281397	8525	9306944
8326	9204364	8376	9230367	8426	9256215	8476	9281909	8526	9307453
8327	9204886	8377	7230885	8427	9256730	8477	9282422	8527	9307963
8328	9205407	8378	9231404	8428	9257245	8478	9282934	8528	9308472
8329	9205929	8379	9231922	8429	9257761	8479	9283446	8529	9308981
8330	9206450	8380	9232440	8430	9258276	8480	9283959	8530	9309490
8331	9206971	8381	9232958	8431	9258791	8481	9284472	8531	9309999
8332	9207493	8382	9233477	8432	9259306	8482	9284983	8532	9310508
8333	9208014	8383	9235995	8433	9259821	8483	9285495	8533	9311017
8334	9208535	8384	9234513	8434	9260336	8484	9286007	8534	9311526
8335	9209056	8385	9235031	8435	9260851	8485	9286518	8535	9312035
8336	9209577	8386	9235549	8436	9261365	8486	9287030	8536	9312544
8337	9210098	8387	9236066	8437	9261880	8487	9287542	8537	9313053
8338	9210619	8388	9236584	8438	9262395	8488	9288054	8538	9313562
8339	9211140	8389	9237102	8439	9262910	8489	9288565	8539	9314070
8340	9211661	8390	9237620	8440	9263424	8490	9289077	8540	9314579
8341	9212181	8391	9238137	8441	9263939	8491	9289588	8541	9315087
8342	9212702	8392	9238655	8442	9264443	8492	9290100	8542	9315595
4343	9213222	8393	9239172	8443	9264958	8493	9290611	8543	9816104
8344	9213743	8394	9239690	8444	9265482	8494	9291123	8544	9316612
8345	9214263	8395	9240207	8445	9265997	8495	9291634	8545	9317121
8346	9214784	8396	9240724	8446	9266511	8496	9292145	8546	9317629
8347	9215304	8397	9241242	8447	9267025	8497	9292656	8547	9318137
8348	9215824	8398	9241759	8448	9267539	8498	9293167	8548	9318645
8349	9216345	8399	9242276	8449	9268053	8499	9293678	8549	9319153
8350	9216865	8400	9242792	8450	9268567	8500	9294188	8550	9319661

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
8551	3.9320169	8601	3.9345489	8651	3.9370663	8701	3.9395692	8751	3.9420577
8552	3.9320677	8602	3.9345994	8652	3.9371165	8702	3.9396191	8752	3.9421073
8553	3.9321185	8603	3.9346499	8653	3.9371667	8703	3.9396690	8753	3.9427569
8554	3.9321692	8604	3.9347004	8654	3.9372169	8704	3.9397189	8754	3.9422065
8555	3.9322200	8605	3.9347509	8655	3.9372671	8705	3.9397688	8755	3.9422562
8556	3.9322708	8606	3.9348013	8656	3.9373172	8706	3.9398187	8756	3.9423058
8557	3.9323215	8607	3.9348518	8657	3.9373674	8707	3.9398685	8757	3.9423553
8558	3.9323733	8608	3.9349023	8658	3.9374176	8708	3.9399184	8758	3.9424049
8559	3.9324230	8609	3.9349527	8659	3.9374677	8709	3.9399683	8759	3.9424545
8560	3.9324738	8610	3.9350032	8660	3.9375179	8710	3.9400182	8760	3.9425041
8561	3.9325245	8611	3.9350536	8661	3.9375680	8711	3.9400680	8761	3.9425537
8562	3.9325752	8612	3.9351040	8662	3.9376182	8712	3.9401179	8762	3.9426032
8563	3.9326259	8613	3.9351544	8663	3.9376683	8713	3.9401677	8763	3.9426528
8564	3.9326767	8614	3.9352049	8664	3.9377184	8714	3.9402176	8764	3.9427024
8565	3.9327274	8615	3.9352553	8665	3.9377686	8715	3.9402674	8765	3.9427519
8566	3.9327781	8616	3.9353057	8666	3.9378187	8716	3.9403172	8766	3.9428015
8567	3.9328288	8617	3.9353561	8667	3.9378688	8717	3.9403670	8767	3.9428510
8568	3.9328795	8618	3.9354065	8668	3.9379189	8718	3.9404169	8768	3.9429005
8569	3.9329301	8619	3.9354569	8669	3.9379690	8719	3.9404667	8769	3.9429501
8570	3.9329808	8520	3.9355073	8670	3.9380191	8720	3.9405165	8770	3.9429996
8571	3.9330315	8621	3.9355576	8671	3.9380692	8721	3.9405663	8771	3.9430491
8572	3.9330822	8622	3.9356080	8672	3.9381193	8722	3.9406161	8772	3.9430986
8573	3.9331328	8623	3.9356584	8673	3.9381693	8723	3.9407659	8773	3.9431481
8574	3.9331835	8624	3.9357087	8674	3.9382194	8724	3.9407157	8774	3.9431976
8575	3.9332341	8625	3.9357591	8675	3.9382695	8725	3.9408654	8775	3.9432471
8576	3.9332848	8626	3.9358095	8676	3.9383195	8726	3.9408152	8776	3.9432966
8577	3.9333354	8627	3.9358598	8677	3.9383699	8727	3.9408650	8777	3.9433461
8578	3.9333860	8628	3.9359101	8678	3.9384196	8728	3.9409147	8778	3.9433956
8579	3.9334367	8629	3.9359605	8679	3.9344697	8729	3.9409645	8779	3.9434450
8580	3.9334873	8630	3.9360108	8680	3.9385197	8730	3.9410142	8780	3.9434945
8581	3.9335379	8631	3.9360611	8681	3.9385698	8731	3.9410640	8781	3.9435440
8582	3.9335885	8632	3.9361114	8682	3.9386198	8732	3.9411137	8782	3.9435934
8583	3.9336391	8633	3.9361617	8683	3.9386698	8733	3.9411635	8783	3.9436429
8584	3.9336894	8634	3.9362120	8684	3.9387198	8734	3.9412132	8784	3.9436923
8585	3.9337403	8635	3.9362623	8685	3.9387698	8735	3.9412629	8785	3.9437418
8586	3.9337909	8636	3.9363126	8686	3.9388798	8736	3.9413126	8786	3.9437912
8587	3.9338415	8637	3.9363629	8687	3.9388698	8737	3.9413623	8787	3.9438406
8588	3.9338920	8638	3.9364132	8688	3.9389198	8738	3.9414120	8788	3.9438900
8589	3.9339426	8639	3.9364635	8689	3.9389698	8739	3.9414617	8789	3.9439395
8590	3.9339932	8640	3.9365137	8690	3.9390198	8740	3.9415114	8790	3.9439889
8591	3.9340437	8641	3.9365640	8691	3.9390697	8741	3.9415611	8791	3.9440383
8592	3.9340943	8642	3.9366143	8692	3.9391196	8742	3.9416108	8792	3.9440877
8593	3.9341448	8643	3.9366645	8693	3.9391697	8743	3.9416605	8793	3.9441371
8594	3.9341953	8644	3.9367148	8694	3.9392196	8744	3.9417101	8794	3.9441865
8595	3.9342456	8645	3.9367650	8695	3.9392696	8745	3.9417598	8795	3.9442358
8596	3.9342964	8646	3.9368152	8696	3.9393195	8746	3.9418095	8796	3.9442852
8597	3.9343469	8647	3.9368655	8697	3.9393695	8747	3.9418591	8797	3.9443346
8598	3.9343974	8648	3.9369157	8698	3.9394194	8748	3.9419088	8798	3.9443840
8599	3.9344479	8649	3.9369659	8699	3.9394693	8749	3.9419584	8799	3.9444333
8600	3.9344985	8650	3.9370161	8700	3.9395193	8750	3.9420081	8800	3.9444827

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
8851	3.9445329	8851	3.9469923	8901	3.9494338	8951	3.9518716	9001	3.9542908
8852	3.9445814	8852	3.9470414	8902	3.9494876	8952	3.9519201	9002	3.9543390
8853	3.9446307	8853	3.9470905	8903	3.9495364	8953	3.9519686	9003	3.9543873
8854	3.9446800	8854	3.9471395	8904	3.9495852	8954	3.9520171	9004	3.9544355
8855	3.9447294	8855	3.9471886	8905	3.9496339	8955	3.9520656	9005	3.9544837
8856	3.9447787	8856	3.9472376	8906	3.9496827	8956	3.9521141	9006	3.9544819
8857	3.9448280	8857	3.9472866	8907	3.9497315	8957	3.9521626	9007	3.9545802
8858	3.9448773	8858	3.9473357	8908	3.9497802	8958	3.9522111	9008	3.9546284
8859	3.9449266	8859	3.9473847	8909	3.9498290	8959	3.9522595	9009	3.9546766
8860	3.9449759	8860	3.9474337	8910	3.9498777	8960	3.9523080	9010	3.9547248
8861	3.9450252	8861	3.9473827	8911	3.9499264	8961	3.9523565	9011	3.9547730
8862	3.9450745	8862	3.9475317	8912	3.9499752	8962	3.9524049	9012	3.9548212
8863	3.9451238	8863	3.9475807	8913	3.9500239	8963	3.9524534	9013	3.9548694
8864	3.9451730	8864	3.9476297	8914	3.9500726	8964	3.9525018	9014	3.9549176
8865	3.9452223	8865	3.9476787	8915	3.9501213	8965	3.9525503	9015	3.9549657
8866	3.9452716	8866	3.9477277	8916	3.9501701	8966	3.9525987	9016	3.9550139
8867	3.9453208	7867	3.9477767	8917	3.9502188	8967	3.9526472	9017	3.9550621
8868	3.9453701	8868	3.9478357	8918	3.9502675	8968	3.9526956	9018	3.9551102
8869	3.9454193	8869	3.9478747	8919	3.9503162	8969	3.9527440	9019	3.9551585
8870	3.9454686	8870	3.9479236	8920	3.9503649	4970	3.9527924	9020	3.9552065
8871	3.9455178	8871	3.9479726	8921	3.9504135	8971	3.9528409	9021	3.9552547
8872	3.9455671	8872	3.9480215	8923	3.9504622	8972	3.9528893	9022	3.9553028
8873	3.9456163	8873	3.9480705	8923	3.9505109	8973	3.9529377	9023	3.9553510
8874	3.9456655	8874	3.9481194	8924	3.0505596	8974	3.9529861	9024	3.9553991
8875	3.9457147	8875	3.9481684	8925	3.9506082	8975	3.9530345	9025	3.9554472
8876	3.9457639	8876	3.9482173	8926	3.9506569	8976	3.9530828	9026	3.9554953
8877	3.9458131	8877	3.9482662	8927	3.9507055	8977	3.9531312	9027	3.9555434
8878	3.9458623	8878	3.9483151	8928	3.9507542	8978	3.9531796	9028	3.9555916
8879	3.9459115	8879	3.9483641	8929	3.9508028	8979	3.9532280	9029	3.9556397
8880	3.9459607	8880	3.9484130	8930	3.9508515	8980	3.9532763	9030	3.9556878
8881	3.9460099	8881	3.9484619	8931	3.9509001	8981	3.9533247	9031	3.9557358
8882	3.9460591	8882	3.9485108	8932	3.9509487	8982	3.9533731	9032	3.9557839
8883	3.9461086	8883	3.9485597	8933	3.9509973	8983	3.9534214	9033	3.9558320
8884	3.9461574	8884	3.9486085	8934	3.9510459	8984	3.9534697	9034	3.9558801
8885	3.9462066	8885	3.9486574	8935	3.9510946	8985	3.9535181	9035	3.9559282
8886	3.9462557	8886	3.9487063	8936	3.9511432	8986	3.9535664	9036	3.9559762
8887	3.9463048	8887	3.9487552	8937	3.9511918	8987	3.9536147	9037	3.9560243
8888	3.9463540	8888	3.9488040	8938	3.9512404	8988	3.9536631	9038	3.9560723
8889	3.9464031	8889	3.9488529	8939	3.9512889	8989	3.9537114	9039	3.9561204
8890	3.9464523	8890	3.9489018	8940	3.9513375	8990	3.9537597	9040	3.9561684
8891	3.9465014	8891	3.9489506	8941	3.9513861	8991	3.9538080	9041	3.9562165
8892	3.9465505	8892	3.9489995	8942	3.9514347	8992	3.9538563	9042	3.9562645
8893	3.9465996	8893	3.9490483	8943	3.9514832	8993	3.9539046	9043	3.9563125
8894	3.9466487	8894	3.9490971	8944	3.9515318	8994	3.9539529	9044	3.9563606
8895	3.9466978	8895	3.9491460	8945	3.9515803	8995	3.9540012	9045	3.9564086
8896	3.9467469	8896	3.9491948	8946	3.9516289	8996	3.9540494	9046	3.9564566
8897	3.9467960	8897	3.9492436	8947	3.9516774	8997	3.9540977	9047	3.9565046
8898	3.9468451	8898	3.9492924	8948	3.9517260	8998	3.9541460	9048	3.9565526
8899	3.9468942	8899	3.9493412	8949	3.9517745	8999	3.9541943	9049	3.9566006
8900	3.9469433	8900	3.9493900	8950	3.9518230	9000	3.9542425	9050	3.9566486

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
9051	3.9566966	9101	3.9590891	9151	3.9614686	9201	3.9638350	9251	3.9661887
9052	3.9567445	9102	3.9591368	9152	3.9615160	9202	3.9638822	9252	3.9662356
9053	3.9567925	9103	3.9591845	9153	3.9615635	9203	3.9639294	9253	3.9662826
9054	3.9568405	9104	3.9592322	9154	3.9616109	9204	3.9639766	9254	3.9663295
9055	3.9568885	9105	3.9592800	9155	3.9616583	9205	3.9640238	9255	3.9663764
9056	3.9569364	9106	3.9593276	9156	3.9617058	9206	3.9640710	9256	3.9664233
9057	3.9569844	9107	3.9593753	9157	3.9617532	9207	3.9641181	9257	3.9664703
9058	3.9570323	9108	3.9594230	9158	3.9618006	9208	3.9641653	9258	3.9665172
9059	3.9570803	9109	3.9594707	9159	3.9618481	9209	3.9642125	9259	3.9665641
9060	3.9571282	9110	3.9595184	9160	3.9618955	9210	3.9642596	9260	3.9666110
9061	3.9571761	9111	3.9595660	9161	3.9619429	9211	3.9643068	9261	3.9666579
9062	3.9572241	9112	3.9596137	9162	3.9619903	9212	3.9643539	9262	3.9667048
9063	3.9572720	9113	3.9596614	9163	3.9620337	9213	3.9644011	9263	3.9667517
9064	3.9573199	9114	3.9597090	9164	3.9620851	9214	3.9644482	9264	3.9667985
9065	3.9573678	9115	3.9597567	9165	3.9621325	9215	3.9644953	9265	3.9668454
9066	3.9574157	9116	3.9598043	9166	3.9621799	9216	3.9645425	9266	3.9668923
9067	3.9574636	9117	3.9598520	9167	3.9622272	9217	3.9645896	9267	3.9669392
9068	3.9575115	9118	3.9598996	9168	3.9622746	9218	3.9646367	9268	3.9669860
9069	3.9575594	9119	3.9599472	6169	3.9623220	9219	3.9646838	9269	3.9670329
9070	3.9576073	9120	3.9599948	9170	3.9623993	9220	3.9647209	9270	3.9670797
9071	3.9576552	9121	3.9600425	9171	3.9624167	9221	3.9647780	9271	3.9671266
9072	3.9577030	9122	3.9600901	9172	3.9624640	9222	3.9648251	9272	3.9671734
9073	3.9577509	9123	3.9601377	9173	3.9625114	9223	3.9648722	9273	3.9672203
9074	3.9577988	9124	3.9601853	9174	3.9625587	9224	3.9649193	9274	3.9672671
9075	3.9578466	9125	3.9602329	9175	3.9626061	9225	3.9649664	9275	3.9673139
9076	3.9578945	9126	3.9602805	9176	3.9626534	9226	3.9650135	9276	3.9673607
9077	3.9579423	9127	3.9603281	9177	3.9627007	9227	3.9650605	9277	3.9674076
9078	3.9579902	9128	3.9603756	9178	3.9627481	9228	3.9651076	9278	3.9674544
9079	3.9580380	9129	3.9604232	9179	3.9627954	9229	3.9651547	9279	3.9675012
9080	3.9580858	9130	3.9604708	9180	3.9628427	9230	3.9652017	9280	3.9675480
9081	3.9581337	9131	3.9605183	9181	3.9628900	9231	3.9652488	9281	3.9675948
9082	3.9581815	9132	3.9605659	9182	3.9629373	9232	3.9652958	9282	3.9676416
9083	3.9582293	9133	3.9606135	9183	3.9629846	9233	3.9653428	9283	3.9676884
9084	3.9582771	9134	3.9606610	9184	3.9630319	9234	3.9653899	9284	3.9677351
9085	3.9583249	9135	3.9607086	9185	3.9630792	9235	3.9654369	9285	3.9677819
9086	3.9583727	9136	3.9607561	9186	3.9631264	9236	3.9654839	9286	3.9678287
9087	3.9584205	9137	3.9608036	9187	3.9631737	9237	3.9655309	9287	3.9678754
9088	3.9584683	9138	3.9608512	9188	3.9632210	9238	3.9655780	9288	3.9679222
9089	3.9585161	9139	3.9608987	9189	3.9632683	9239	3.9656250	9289	3.9679689
9090	3.9585639	9140	3.9609462	9190	3.9633155	6240	3.9656720	9290	3.9680157
9091	3.9586116	9141	3.9609937	9191	3.9633628	9241	3.9657190	9291	3.9680625
9092	3.9586594	9142	3.9610412	9192	3.9634100	9242	3.9657660	9292	3.9681092
9093	3.9587072	9143	3.9610887	9193	3.9634573	9243	3.9658130	9293	3.9681559
9094	3.9587549	9144	3.9611362	9194	3.9635045	9244	3.9658599	9294	3.9682027
9095	3.9588027	9145	3.9611837	9195	3.9635517	9245	3.9659069	9295	3.9682494
9096	3.9588505	9146	3.9612312	9196	3.9635990	9246	3.9659539	9296	3.9682961
9097	3.9588982	9147	3.9612887	9197	3.9636462	9247	3.9660009	9297	3.9683428
9098	3.9589459	9148	3.9613262	9198	3.9636934	9248	3.9660478	9298	3.9683895
9099	3.9589937	9149	3.9613736	9199	3.9637406	9249	3.9660948	9299	3.9684362
9100	3.9590414	9150	3.9614211	9200	3.9637878	9250	3.9661417	9300	3.9684829

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9303	3.9686230	9353	3.9709509	9403	3.9732664	9453	3.9755697	9503	3.9778607
9304	3.9686697	9354	3.9709974	9404	3.9733126	9454	3.9756156	9504	3.9779064
9305	3.9687164	9355	3.9710438	9405	3.9733588	9455	3.9756615	9505	3.9779521
9306	3.9687630	9356	3.9710902	9406	3.9734050	9456	3.9757075	9506	3.9779978
9307	3.9688097	9357	3.9711366	9407	3.9734511	9457	3.9757534	9507	3.9780435
9308	3.9688564	9358	3.9711830	9408	3.9734973	9458	3.9757993	9508	3.9780892
9309	3.9689030	9359	3.9712294	9409	3.9735435	9459	3.9758452	9509	3.9781348
9310	3.9689497	9360	3.9712758	9410	3.9735896	9460	3.9758911	9510	3.9781805
9311	3.9689963	9361	3.9713222	9411	3.9736358	9461	3.9759370	9511	3.9782263
9312	3.9690430	9362	3.9713686	9412	3.9736819	9462	3.9759829	9512	3.9782718
9313	3.9690896	9363	3.9714150	9413	3.9737281	9463	3.9760288	9513	3.9783175
9314	3.9691362	9364	3.9714614	9414	3.9737742	9464	3.9760747	9514	3.9783631
9315	3.9691829	9365	3.9715078	9415	3.9738203	9465	3.9761206	9515	3.9784088
9316	3.9692295	9366	3.9715542	9416	3.9738664	9466	3.9761665	9516	3.9784544
9317	3.9692761	9367	3.9716005	9417	3.9739126	9467	3.9762124	9517	3.9785001
9318	3.9693227	9368	3.9716469	9418	3.9739587	9468	3.9762582	9518	3.9785457
9319	3.9693694	9369	3.9716932	9419	3.9740048	9469	3.9763041	9519	3.9785913
9320	3.9694159	9370	3.9717396	9420	3.9740509	9470	3.9763500	9520	3.9786369
9321	3.9694625	9371	3.9717859	9421	3.9740970	9471	3.9763958	9521	3.9786826
9322	3.9695091	9372	3.9718323	9422	3.9741431	9472	3.9764417	9522	3.9787282
9323	3.9695557	9373	3.9718786	9423	3.9741892	9473	3.9764875	9523	3.9787738
9324	3.9696023	9374	3.9719249	9424	3.9742353	9474	3.9765334	9524	3.9788194
9325	3.9696488	9375	3.9719713	9425	3.9742814	9475	3.9765792	9525	3.9788650
9326	3.9696954	9376	3.9720176	9426	3.9743274	9476	3.9766251	9526	3.9789106
9327	3.9697420	9377	3.9720639	9427	3.9743735	9477	3.9766709	9527	3.9789562
9328	3.9697885	9378	3.9721102	9428	3.9744166	9478	3.9767167	9528	3.9790017
9329	3.9698351	9379	3.9721565	9429	3.9744656	9479	3.9767625	9529	3.9790473
9330	3.9698816	9380	3.9722028	9430	3.9745117	9480	3.9768083	9530	3.9790929
9331	3.9699282	9381	3.9722491	9431	3.9745577	9481	3.9768541	9531	3.9791385
9332	3.9699747	9382	3.9722954	9432	3.9746038	9482	3.9769000	9532	3.9791840
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9334	3.9700678	9384	3.9723880	9434	3.9746959	9484	3.9769915	9534	3.9792751
9335	3.9701143	9385	3.9724343	9435	3.9747419	9485	3.9770377	9535	3.9793207
9336	3.9701608	9386	3.9724805	9436	3.9747879	9486	3.9770831	9536	3.9793662
9337	3.9702074	9387	3.9725268	9437	3.9748340	9487	3.9771289	9537	3.9794118
9338	3.9702539	9388	3.9725731	9438	3.9748800	9488	3.9771747	9538	3.9794573
9339	3.9703004	9389	3.9726193	9439	3.9749260	9489	3.9772204	9539	3.9795028
9340	3.9703469	9390	3.9726656	9440	3.9749720	9490	3.9772662	9540	3.9795484
9341	3.9703934	9391	3.9727118	9441	3.9750180	9491	3.9773120	9541	3.9795939
9342	3.9704399	9392	3.9727581	9442	3.9750640	9492	3.9773577	9542	3.9796394
9343	3.9704863	9393	3.9728043	9443	3.9751100	9493	3.9774035	9543	3.9796849
9344	3.9705328	9394	3.9728506	9444	3.9751560	9494	3.9774492	9544	3.9797304
9345	3.9705793	9395	3.9728968	9445	3.9752020	9495	3.9774950	9545	3.9797759
9346	3.9706258	9396	3.9729430	9446	3.9752479	9496	3.9775407	9546	3.9798214
9347	3.9706722	9397	3.9729892	9447	3.9752939	9497	3.9775864	9547	3.9798669
9348	3.9707187	9398	3.9730354	9448	3.9753399	9498	3.9776322	9548	3.9799124
9349	3.9707652	9399	3.9730816	9449	3.9753858	9499	3.9776779	9549	3.9799579
9350	3.9708116	9400	3.9731279	9450	3.9754318	9500	3.9777236	9550	3.9800034

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9553	3.9801398	9603	3.9824069	9653	3.9846623	9703	3.9869060	9753	3.9891382
9554	3.9801852	9604	3.9824522	9654	3.9847073	9704	3.9869508	9754	3.9891828
9555	3.9802307	9605	3.9824974	9655	3.9847523	9705	3.9869955	9755	3.9892273
9556	3.9802761	9606	3.9825426	9656	3.9847973	9706	3.9870403	9756	3.9892719
9557	3.9803216	9607	3.9825878	9657	3.9848422	9707	3.9870850	9757	3.9893163
9558	3.9803670	9608	3.9826330	9658	3.9848872	9708	3.9871298	9758	3.9893608
9559	3.9804125	9609	3.9826782	9659	3.9849322	9709	3.9871745	9759	3.9894053
9560	3.9804579	9610	3.9827234	9660	3.9849771	9710	3.9872192	9760	3.9894498
9561	3.9805033	9611	3.9827686	9661	3.9850221	9711	3.9872640	9761	3.9894943
9562	3.9805487	9612	3.9828138	9662	3.9850670	9712	3.9873087	9762	3.9895388
9563	3.9805942	9613	3.9828589	9663	3.9851120	9713	3.9873534	9763	3.9895833
9564	3.9806396	9614	3.9829041	9664	3.9851569	9714	3.9873981	9764	3.9896278
9565	3.9806850	9615	3.9829493	9665	3.9852019	9715	3.9874428	9765	3.9896722
9566	3.9807304	9616	3.9829945	9666	3.9852468	9716	3.9874875	9766	3.9897167
9567	3.9807758	9617	3.9830396	9667	3.9852917	9717	3.9875322	9767	3.9897612
9568	3.9808212	9618	3.9830848	9668	3.9853366	9718	3.9875769	9768	3.9898057
9569	3.9808666	9619	3.9831299	9669	3.9853816	9719	3.9876216	9769	3.9898501
9570	3.9809119	9620	3.9831751	9670	3.9854265	9720	3.9876663	9770	3.9898946
9571	3.9809573	9621	3.9832202	9671	3.9854714	9721	3.9877109	9771	3.9899390
9572	3.9810027	9622	3.9832654	9672	3.9855163	9722	3.9877556	9772	3.9899835
9573	3.9810481	9623	3.9833105	9673	3.9855612	9723	3.9878003	9773	3.9900279
9574	3.9810934	9624	3.9833556	9674	3.9856061	9724	3.9878450	9774	3.9900723
9575	3.9811388	9625	3.9834007	9675	3.9856510	9725	3.9878896	9775	3.9901168
9576	3.9811841	9626	3.9834459	9676	3.9856959	9726	3.9879343	9776	3.9901612
9577	3.9812295	9627	3.9834910	9677	3.9857407	9727	3.9879789	9777	3.9902056
9578	3.9812748	9628	3.9835361	9678	3.9857856	9728	3.9880236	9778	3.9902500
9579	3.9813202	9629	3.9835812	9679	3.9858304	9729	3.9880682	9779	3.9902944
9580	3.9813655	9630	3.9836263	9680	3.9858753	9730	3.9881128	9780	3.9903389
9581	3.9814108	9631	3.9836714	9681	3.9859202	9731	3.9881575	9781	3.9903833
9582	3.9814562	9632	3.9837165	9682	3.9859651	9732	3.9882021	9782	3.9904277
9583	3.9815015	9633	3.9837616	9683	3.9860099	9733	3.9882467	9783	3.9904721
9584	3.9815468	9634	3.9838066	9684	3.9860548	9734	3.9882913	9784	3.9905164
9585	3.9815921	9635	3.9838517	9685	3.9860996	9735	3.9883360	9785	3.9905608
9586	3.9816374	9636	3.9838968	9686	3.9861445	9736	3.9883806	9786	3.9906052
9587	3.9816827	9637	3.9839419	9687	3.9861893	9737	3.9884252	9787	3.9906496
9588	3.9817280	9638	3.9839869	9688	3.9862341	9738	3.9884698	9788	3.9906940
9589	3.9817733	9639	3.9440320	9689	3.9862790	9739	3.9885144	9789	3.9907383
9590	3.9818186	9640	3.9840770	9690	3.9863238	9740	3.9885590	9790	3.9907827
9591	3.9818639	9641	3.9841221	9691	3.9863686	9741	3.9886035	9791	3.9908271
9592	3.9819092	9642	3.9841671	9692	3.9864134	9742	3.9886481	9792	3.9908714
9593	3.9819544	9643	3.9842122	9693	3.9864582	9743	3.9886927	9793	3.9909158
9594	3.9819997	9644	3.9842572	9694	3.9865030	9744	3.9887373	9794	3.9909601
9595	3.9820450	9645	3.9843022	9695	3.9865478	9745	3.9887818	9795	3.9910044
9596	3.9820902	9646	3.9843473	9696	3.9865926	9746	3.9888264	9796	3.9910486
9597	3.9821355	9647	3.9843923	9697	3.9866374	9747	3.9888710	9797	3.9914931
9598	3.9821807	9648	3.9844373	9698	3.9866822	9748	3.9889155	9798	3.9911374
9599	3.9822260	9649	3.9844823	9699	3.9867270	9749	3.9889601	9799	3.9911818
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9803	3.9913590	9853	3.9935685	9903	3.9957668	9953	3.9979540
9804	3.9914033	9854	3.9936125	9904	3.9958106	9954	3.9979976
9805	3.9914476	9855	3.9936566	9905	3.9958545	9955	3.9980413
9806	3.9914919	9856	3.9937007	9906	3.9958983	9956	3.9980849
9807	3.9915362	9857	3.9937447	9907	3.9959422	9957	3.9981285
9808	3.9915805	9858	3.9937888	9908	3.9959860	9958	3.9981721
9809	3.9916247	9859	3.9938329	9909	3.9960298	9959	3.9982157
9810	3.9916690	9860	3.9938769	9910	3.9960736	9960	3.9982593
9811	3.9917133	9861	3.9939209	9911	3.9961175	9961	3.9983029
9812	3.9917575	9862	3.9939650	9912	3.9961613	9962	3.9983465
9813	3.9918018	9863	3.9940090	9913	3.9962052	9963	3.9983901
9814	3.9918461	9864	3.9940531	9914	3.9962489	9964	3.9984337
9815	3.9918903	9865	3.9940971	9915	3.9962927	9965	3.9984773
9816	3.9919345	9866	3.9941411	9916	3.9963365	9966	3.9985209
9817	3.9919788	9857	3.9941851	9917	3.9963803	9967	3.9985645
9818	3.9920230	9868	3.9942291	9918	3.9964241	9968	3.9986080
9819	3.9920673	9869	3.9942731	9919	3.9964679	9969	3.9986516
9820	3.9921115	9870	3.9943171	9920	3.9965117	9970	3.9986951
9821	3.9921557	9871	3.9943611	9921	3.9965554	9971	3.9987387
9822	3.9921999	9872	3.9944051	9922	3.9965992	9972	3.9987823
9823	3.9922441	9873	3.9944491	9923	3.9966430	6973	3.9988258
9824	3.9922884	9874	3.9944931	9924	3.9966867	9974	3.9988694
9825	3.9923326	9875	3.9945371	9925	3.9967305	9975	3.9989129
9826	3.9923768	9876	3.9945811	9926	3.9967743	9976	3.9989564
9827	3.9924210	9877	3.9946250	9927	3.9968180	9977	3.9990000
9828	3.9924651	9878	3.9946690	9928	3.9968618	9978	3.9990435
9829	3.9925093	9879	3.9947130	9929	3.9969055	9979	3.9990870
9830	3.9925535	9880	3.9947569	9930	3.9969492	9980	3.9991305
9831	3.9925977	9881	3.9948009	9931	3.9969930	9981	3.9991740
9832	3.9926419	9882	3.9948448	9932	3.9970367	9982	3.9992176
9833	3.9926860	9883	3.9948888	9933	3.9970804	9983	3.9992611
9834	3.9927302	9884	3.9949327	9934	3.9971241	9984	3.9993046
9835	3.9927744	9885	3.9949767	9935	3.9971679	9985	3.9993481
9836	3.9928185	9886	3.9950206	9936	3.9972116	9986	3.9993916
9837	3.9928627	9887	3.9950645	9937	3.9972553	9987	3.9994350
9838	3.9929068	9888	3.9951 85	9938	3.9972990	9988	3.9994785
9839	3.9929510	9889	3.9951524	9939	3.9373427	9989	3.9995220
9840	3.9929951	9890	3.9951963	9940	3.9973864	9990	3.9995655
9841	3.9930392	9891	3.9952402	9941	3.9974301	9991	3.9996089
9842	3.9930834	9892	3.9952841	9942	3.9974737	9992	3.9996524
9843	3.9931275	9893	3.9953280	9943	3.9975174	9993	3.9996959
9844	3.9931716	9894	3.9953799	9944	3.9975619	9994	3.9997393
9845	3.9932157	9895	3.9954158	9945	3.9976048	9995	3.9997828
9846	3.9932598	9896	3.9954597	9946	3.9976484	9996	3.9998662
9847	3.9933039	9897	3.9955036	9947	3.9976921	9997	3.9998697
9848	3.9933480	9898	3.9955474	9948	3.9977358	9998	3.9999131
9849	3.9933921	9899	3.9955913	9949	3.9977794	9999	3.9999566
9850	3.9934362	9900	3.9956352	9950	3.9978231	10000	4.0000000

APPENDIX.

THE *hyperbolick curve* was found also, by Lord Napier, the celebrated inventor of *Logarithms*, to afford another source for developing and illustrating the properties and construction of logarithms. For the hyperbolick areas lying between the curve and one asymtote, when they are bounded by ordinates parallel to the other asymtote, are analagous to the logarithms of their abscissas or parts of the asymtote. And although Napier's logarithms are commonly said to be the same as hyperbolick logarithms, it is not to be understood that hyperbolas exhibit Napier's logarithms *only*, but indeed all other possible systems of logarithms whatever. For it has been found that the logarithms of the same number in two different systems are to one another as the reciprocals of the logarithms of the radical numbers of those systems, these latter logarithms being taken according to any system whatever:

Thus,

Brigg's logarithm of any number:
is to *Napier's* logarithm of the same number::
as *Brigg's* logarithm of 10:
is to *Napier's* logarithm of 10:

But Brigg's logarithm, or the common logarithm of 10 is 1, and Napier's logarithm of 10 is 2.302585093; whence if Brigg's, or the common logarithm of any number, be denoted by *c. l.*

Note. This curve greatly facilitates the conception of logarithms to the imagination, and affords almost an intuitive proof of the very important property of their fluctions, or very small increments, viz. that the fluction of the number is to the fluction of the logarithm, as the number is to the subtangent. And it is evident, that in the beginning of the generation of these areas from the vertex of the hyperbola, the rascent increment of the abscisse drawn into the altitude 1, is to the increment of the area, as radius is to the angle of the ordinate and abscisse, or of the asymtotes: and at the beginning of the logarithms, the rascent increment of the natural numbers is to the increment of the logarithms as 1 is to the modulus of the system.

Hence, we easily discover, that the angle formed by the asymtotes of the hyperbola, exhibiting Brigg's System of Logarithms, will be 25° 44' 25"; this being the angle whose sine is 0.4342944819, &c. the modulus of the common system.

and Napier's or the hyperbolick logarithm of the same number, by *H. L.* we shall have 2.302585093 : 1 :: *H. L.* : *C. L.*; or

$$\frac{1}{H.\,L. \times 2.302585093} \quad H.\,L. \times .4342944819 = C.\,L.$$ as was required.

In comparing the different systems of logarithms, Lord Napier's is evidently the most simple in respect of facility of construction, because its modulus is unity.* Thus, suppose it were required to find the Napierean logarithm of the number 2.

Here employing the formula,

$$\log. 2 = 2\left\{\frac{1}{3} + \frac{1}{3.3^3} + \frac{1}{5.3^5} + \frac{1}{7.3^7} + \frac{1}{9.3^9} + \&c.\right\}$$

$= A + \frac{1}{3}B + \frac{1}{5}C + \frac{1}{7}D + \frac{1}{9}E + \&c.$ where A is put for $\frac{2}{3}$, B for $\frac{1}{9}$ A, C for $\frac{1}{9}$ B, D for $\frac{1}{9}$ C, &c.

The calculation will be as follows:—

$A = \frac{2}{3}$	$= .666666666$	$A =$	$.666666666$
$B = \frac{1}{9}A$	$= .074074074$	$\frac{1}{3}B =$	$.024691358$
$C = \frac{1}{9}B$	$= .008230453$	$\frac{1}{5}C =$	$.001646091$
$D = \frac{1}{9}C$	$= .000914495$	$\frac{1}{7}D =$	$.000130642$
$E = \frac{1}{9}D$	$= .000101611$	$\frac{1}{9}E =$	$.000011290$
$F = \frac{1}{9}E$	$= .000011290$	$\frac{1}{11}F =$	$.000001026$
$G = \frac{1}{9}F$	$= .000001254$	$\frac{1}{13}G =$	$.000000096$
$H = \frac{1}{9}G$	$= .000000139$	$\frac{1}{15}H =$	$.000000009$
$I = \frac{1}{9}H$	$= .000000015$	$\frac{1}{17}I =$	$.000000001$

Nap. log. 2 = .693144179

Or, retaining only eight figures, Nap. log. 2 = .69314718

Having obtained the log. of 2, we can easily find the logarithm of 4, 8, and in general of any power of 2.

Ex. 2. Required the Napierean logarithm of 5.

By employing the same formula as before, and proceeding exactly as in the last example, by taking the sum of a sufficient

* Note. Mr. Baron Maseres gives the following definition of the *Modulus*, namely, "that it is the limit of the magnitude of a fourth proportional to these three quantities viz. the difference of any two natural numbers that are very nearly equal to each other, either of the said numbers and the logarithm or measure of the ratio they have to each other." Or we may define the *modulus* to be the natural number at that part of the system of logarithms, where the fluction of the number is equal to the fluction of the logarithm, or where the numbers and logarithms are have equal differences. And hence it follows, that the logarithms of equal numbers, or of equal ratios, in different systems, are to one another, as the moduli of those systems.

number of the terms of the series, we shall find the Napierean log. of 5 = 1. 609437912.

The Napierean logarithms of 2 and 5 being found, the Nap. log. of 10 = 2 X 5 becomes known.

Thus, to log. 2 = 693147179
Add log. 5 = 1.609437912
The sum is log. 10 = 2.302585091

Or, retaining eight figures, log. 10 = 2.30258509.

Whence also the *modulus* of the common system of logarithms is known, for it is the reciprocal of the Napierean logarithm of 10, or $\frac{1}{2.30258509} = .434294482$

We can now easily find the *common* logarithms of the numbers 2 and 5 ; for we have only to multiply the Napierean log. already found by the modulus .434294482, or divide them by its reciprocal 2.30258509, and the products, or quotients, are the logarithms sought.

Thus retaining only seven decimal places of the products, we have

Com. Log. 2 = .69314718 X .4342944 = .3010300
Com. Log. 5 = 1.60943791 X .4342944 = .6989700
Com. Log. 10 = 2.30258509 X .4342944 = 1.0000000

Or, the Common Logarithm may be found by putting M for .4342944, as in the following expression.

$$\text{Log. } 3 = \log. 2 + 2\,M \left\{ \frac{1}{5} + \frac{1}{3.5^3} + \frac{1}{5.5^5} + \frac{1}{7.5^7} + \&c. \right\}$$

And the calculation will stand thus :

Log. 2 = 30103000

$\frac{2M}{5}$ = .17371779

$\frac{2M}{3.5^3}$ = .00231624

$\frac{2M}{5.5^5}$ = .00000559

$\frac{2M}{7.5^7}$ = .00000159

$\frac{2M}{9.5^9}$ = .00000005

Com. Log. 3 = .47712126

And .47712126 ÷ .43429448 = Nap. log. 3 = 1.0986049.

Another Method, to find the Logarithm of any of the natural numbers, 1, 2, 3, 4, &c.

RULE.*

1. Take the geometrical series, 1, 10, 100, 1000, 10000, &c. and apply to it the corresponding arithmetical series 1, 2, 3, 4, &c. as logarithms.

2. Find a geometrick mean between 1 and 10, 10 and 100, or any other two adjacent terms of the series betwixt which the proposed number lies.

3. Between the mean, thus found, and the nearest extreme, find another geometrical mean, in the same manner; and so on, till you are arrived within the proposed limit of the number whose logarithm is sought.

4. Find as many arithmetical means, in the same order as you found the geometrical ones, and the last of these will be the logarithm answering to the number requered.

EXAMPLES.

Let it be required to find the logarithm of the number 9.

Here the numbers between which 9 lies are 1 and 10.

First, then, the logarithm of 10 is 1, and the log. of 1 is 0; therefore $\frac{1+0}{2} = .5$ is the arithmetical mean, and $\sqrt{(1\times10)} = \sqrt{10} = 3.1622777 =$ geometrick mean: whence the logarithm of 3.1622777 is .5

Secondly, the log. of 10 is 1, and the log. of 3.1622777 is 5; therefore $\frac{1+.5}{2} = .75 =$ arithmetical mean, and $\sqrt{(10 \times 3.1622777)} = 5.6234132 =$ geometrick mean: whence the log. of 5.6234132 is .75.

Thirdly, the log. of 10 is 1, and the log. of 5.6234132 is .75; therefore $\frac{1+.75}{2} = .875 =$ arithmetical mean; and $\sqrt{(10 \times 5.6234132)} = 7.4989421 =$ geometrick mean: whence the log. of 7.4989421 is .875.

Fourthly, the log of 10 is 1, and the log. of 7.4989421 is .875; therefore $\frac{1+.875}{2} = .9375 =$ arithmetical mean, and $\sqrt{(10 \times 7.4989421)} = 8.6596431 =$ geometrick mean: whence the log. of 8.6596431 is .9375.

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* Note. The reader who wishes to inform himself more particularly concerning the history, nature, and construction of logarithms, may consult Hutton's Mathematical Tables, published a few years since, where he will find his curiosity amply gratified.

Fifthly, the log. of 10 is 1, and the log. of 8.6596431 is .9375; therefore $\frac{1 + .9375}{2} = .96875$ = arithmetical mean, and $\sqrt{(10 \times 8.6596431)} = 9.3057204$ = geometrick mean: whence the log. of 9.3057204 is .96875.

Sixthly, the log. of 8.6596431 is .9375, and the logarithm of 9.3057204 is .96875; therefore $\frac{.9375 + .96875}{2} = .953125$ = arithmetical mean, and $\sqrt{(8.6596431 \times 9.3057204)} = 8.9768713$ = geometrick mean: whence the log. of 8.9768713 is .953125

And, proceeding in this manner, after 25 extractions the logarithm of 8.9999998 will be found to be .9542425; which may be taken for the logarithm of 9, because it differs from it only by $\frac{1}{10000000}$, and is therefore sufficiently exact for all practical purposes.

And in the same manner were the logarithms of almost all the prime numbers found by Lord Napier; a work so incredibly laborious, that the unremitted industry of several years was scarcely sufficient for its achievement.

The reader is requested to correct the following which are the principal

ERRATA.

Page	15,	26th line from the top,	for 126 gals, read 128.
	16,	9	for $136,58 read $163,07
	26,	27	for $ 428,50 read $ 429.50
	27,	28	for 182 yds. read 132 yds.
	32,	26	for $12114\frac{62}{77}$ read $1270\frac{52}{77}$
	33,	8	for £481 read £431
	40,	33	for 15' read 3'
	41,	3	for 12 h. 2 m. read 11 h. 58m.
	ibid.	7	for 5 o'clock 19m. 37sec. read 6 o'clock 40m. 23. sec.
	ibid.	14	for $ 1940 read $ 1946
	ibid.	34	for $ 50 read $ 51.
	67,	26	for $ 975,99cts. read $ 1042,86 cts.
	73,	26	for $73\frac{1}{4}$ gal. read $7\frac{1}{4}$ gal.
	ibid.	32	for 22 ib. read 2 lb.
	74,	26	erase - 2d.
	ibid.	42	for *ad infinetum* read ad infinitum.
	77,	15	for DUBLE read DOUBLE.
	79,	24	for fully read briefly.
	84,	11	for 3, the root read 3758, the root.
	87,	18	for square read cube.
	95,	7	for *From the square of the height, subtract the height, read * From the square of the distance divided by the height, subtract the height.
	110,	19	for £ 1 read £ 7
	134,	9	for 64 read 40
143—147*p.*			for minuets read minutes.
	168	11	for *South-Descending* read 6 Signs *South-(Descending.*
	191,	26	for North Ascending read North Descend-(ing.
	201,	16	for right read left.
	202,	13	erase totally.